(1) Geometric series with first term a and common ratio $r = 3/4$.
Sum of first four terms is 175.

(a) $\dfrac{a}{1-3/4}\left(1-(3/4)^4\right) = 175 \Leftrightarrow a\left(\dfrac{175}{64}\right) = 175 \Leftrightarrow a = 64$ [2]

(b) $S_\infty = \dfrac{a}{1-r} = \dfrac{64}{1-3/4} = 256.$ [2]

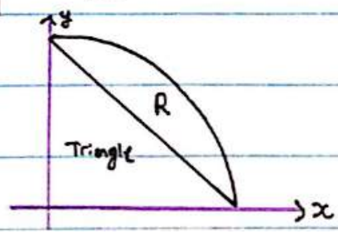
(c) Let $u_n = 64 \cdot (3/4)^{n-1}$ then $|u_9 - u_{10}| = 64\,|(3/4)^8 - (3/4)^9| = 1.602$ [3]

(2) (a)

x	0	1	2	3	4
y	7.5	7	6	4	0

where $y = 8 - 2^{x-1}$ [1]

(b) $\displaystyle\int_0^4 8 - 2^{x-1}\,dx \approx \frac{1}{2}\cdot\frac{(4-0)}{4}\left[7.5 + 2(7+6+4) + 0\right] = 20.75$ [3]



(c) $R \approx 20.75 - \frac{1}{2}\times 4 \times 7.5 = 5.75$ [2]

③ ⓐ $|PQ| = \sqrt{(10-7)^2 + (13-8)^2} = \sqrt{9 + 25} = \sqrt{34}$ [2]

where $P(7,8)$ and $Q(10,13)$ and $|PQ|$ is the exact length of PQ.

ⓑ C is a circle centre P and passes through Q so has radius $r = \sqrt{34}$.

$\therefore\ C: \ (x-7)^2 + (y-8)^2 = 34.$ [2]

ⓒ Gradient of radius is $m_r = \frac{13-8}{10-7} = \frac{5}{3}$. [4]

So gradient of tangent is $m_T = \frac{-3}{5}$ (perpendicular).

l passes through Q so : $l: \ y - 13 = \frac{-3}{5}(x-10)$

$\Leftrightarrow\ 5y - 65 = -3x + 30 \ \Leftrightarrow\ 5y + 3x - 95 = 0.$

[Note: $\pm k(5y + 3x - 95) = 0$ ($k \neq 0$) are valid answers. $k \in \mathbb{Z}$.]

④ $f(x) = 6x^3 + 13x^2 - 4$

ⓐ When $f(x)$ is divided by $(2x+3)$ the remainder is $f(-3/2) = 5$. [2]

ⓑ $(x+2)$ is a factor of $f(x)$ iff $f(-2) = 0$. Since $f(-2) = 6(-2)^3 + 13(-2)^2 - 4$

$= -48 + 52 - 4 = 0$ then $(x+2)$ is indeed a factor of $f(x)$ [2]

ⓒ Note that: [4]

$$
\begin{array}{r l}
 & 6x^2 + x - 2 \\
x+2 \,\big) & 6x^3 + 13x^2 + 0x - 4 \\
 & 6x^3 + 12x^2 \\
 & \quad x^2 + 0x \\
 & \quad x^2 + 2x \\
 & \quad\quad -2x - 4 \\
 & \quad\quad -2x - 4 \\
 & \quad\quad\quad 0
\end{array}
$$

So $f(x) = (x+2)(6x^2 + x - 2)$

$= (x+2)(3x+2)(2x-1)$ //

5 (a) first three terms of $(2-9x)^4 = 2^4(1-\frac{9}{2}x)^4$ [4]

is $2^4\left[1 - 18x + \frac{243}{2}x^2 + \cdots\right] = 2^4 - 288x + 1944x^2$

$= 16 - 288x + 1944x^2$ //

$f(x) = (1+kx)(2-9x)^4 = A - 232x + Bx^2 + \cdots$

(b) $A = 2^4 = 16$ // [1]

(c) $f(x) = (1+kx)(16 - 288x + 1944x^2 + \cdots) = 16 - 288x + 1944x^2$ [2]

$+ 16kx - 288kx^2 + \cdots$

so $f(x) = 16 + x(16k - 288) + x^2(1944 - 288k) + \cdots$

Comparing coefficients of x : $16k - 288 = -232 \Leftrightarrow k = 7/2$ /

(d) Comparing coefficients of x^2 : $B = 1944 - 288 \times 7/2 = 936$ [2]

6 (i) $-\pi < \theta \leq \pi$, $1 - 2\cos\left(\theta - \frac{\pi}{5}\right) = 0$ [3]

$\Leftrightarrow \cos\left(\theta - \frac{\pi}{5}\right) = \frac{1}{2}$

$\Leftrightarrow \theta - \frac{\pi}{5} = \frac{\pi}{3}, \cancel{\frac{5\pi}{3}}, \cancel{\frac{\pi}{3} - 2\pi}, \frac{5\pi}{3} - 2\pi$

$\Rightarrow \theta = \frac{\pi}{5} + \frac{\pi}{3}$ and $\frac{\pi}{5} + \frac{5\pi}{3} - 2\pi$

$\Rightarrow \theta = \frac{8\pi}{15}$ and $\frac{-2\pi}{15}$

(ii) $0 \leq x < 360°$: $4\cos^2 x + 7\sin x - 2 = 0$ [6]

$\Leftrightarrow 4(1 - \sin^2 x) + 7\sin x - 2 = 0$

$\Leftrightarrow 4 - 4\sin^2 x + 7\sin x - 2 = 0$

$\Leftrightarrow 4\sin^2 x - 7\sin x - 2 = 0$

$\Leftrightarrow \underbrace{(4\sin x + 1)}\underbrace{(\sin x - 2)}_{\text{no solutions}} = 0$

$\sin x = -\frac{1}{4} \Rightarrow x = 180° - \sin^{-1}(-1/4)$

$x = 360° + \sin^{-1}(-1/4)$

$\therefore x = 194.5°, 345.5°$

(7) (a) Find $\int 3x - x^{3/2} \, dx$. [3]

$$\int 3x - x^{3/2} \, dx = \frac{3x^2}{2} - \frac{x^{5/2}}{(5/2)} + c$$

$$= \frac{3x^2}{2} - \frac{2x^{5/2}}{5} + c \; //$$

(b) Area $S = \int_0^9 3x - x^{3/2} \, dx = \left[\frac{3x^2}{2} - \frac{2x^{5/2}}{5} + \cancel{c} \right]_0^9$ [3]

$$= \frac{243}{2} - \frac{486}{5} - 0 + 0$$

$$= \frac{243}{10} \; //$$

(8) (i) $\log_3 (3b+1) - \log_3 (a-2) = -1$. [3]

$\Leftrightarrow \log_3 \left(\frac{3b+1}{a-2}\right) = -1$

$\Leftrightarrow \left(\frac{3b+1}{a-2}\right) = 3^{-1}$

$\Leftrightarrow \frac{3b+1}{a-2} = \frac{1}{3}$

$\Leftrightarrow 3(3b+1) = a-2$

$\Leftrightarrow 9b + 3 = a-2$

$\Leftrightarrow 9b = a-5$

$\Leftrightarrow b = (a-5)/9 \; //$

(ii) $2^{2x+5} - 7(2^x) = 0$ [4]

Let ~~so~~ $u = 2^x$

$2^5 (2^x)^2 - 7(2^x) = 0$

$\Leftrightarrow 32u^2 - 7u = 0$

$\Rightarrow u = 0$ or $u = \frac{7}{32}$

$\Rightarrow \underbrace{2^x = 0}_{\text{no solutions}}$ or $2^x = 7/32$

$\Rightarrow x = \log_2 7/32$

$= -2.19$ (2 dp)

(9) (a) Angle $\angle EFA$ is $2\pi/3$ so area sector is $\frac{1}{2} \cdot x^2 \cdot \frac{2\pi}{3} = \frac{\pi x^2}{3}$ m². [2]

(b) $BC = y$ metres and area of enclosure is 1000 m². [3]

Area equilateral triangle + Area rectangle + Area sector = 1000

so $\frac{x^2\sqrt{3}}{4} + 2xy + \frac{\pi x^2}{3} = 1000 \Leftrightarrow 2yx = 1000 - \frac{x^2\sqrt{3}}{4} - \frac{x^2\pi}{3}$

$\Leftrightarrow y = \frac{1000}{2x} - \frac{x\sqrt{3}}{8} - \frac{x\pi}{6} = \frac{500}{x} - \frac{x}{24}(4\pi + 3\sqrt{3})$

[3]

(c) $P = 2x + 2y + x + 2\pi x/3$

$$= x\left(3 + \frac{2\pi}{3}\right) + \frac{1000}{x} - \frac{x}{12}\left(4\pi + 3\sqrt{3}\right)$$

$$= \frac{1000}{x} - \frac{x}{12}\left(4\pi + 3\sqrt{3} - 36 - 8\pi\right)$$

$$= \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right)$$

[5]

(d) $$\frac{dP}{dx} = -\frac{1000}{x^2} + \frac{1}{12}\left(4\pi + 36 - 3\sqrt{3}\right) = 0$$

$$\Leftrightarrow x^2 = \frac{12 \times 1000}{4\pi + 36 - 3\sqrt{3}} \Rightarrow x = 16.6339$$ (taking the positive root)

$\therefore P_{min} = 120.2 \approx 120$ m (3 s.f)

[2]

(e) $$\frac{d^2P}{dx^2} = \frac{2000}{x^3} > 0 \text{ for all } x \in \mathbb{R}^+$$