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1. Find the first 4 terms, in ascending powers of x , of the binomial expansion of

$$\left(3 - \frac{1}{3}x\right)^5$$

giving each term in its simplest form.

(4)

$$3^5 - 5(3)^4\left(\frac{1}{3}x\right) + 10(3)^3\left(\frac{1}{3}x\right)^2 - 10(3)^2\left(\frac{1}{3}x\right)^3$$

$$= 243 - 135x + 30x^2 - \frac{10}{3}x^3$$

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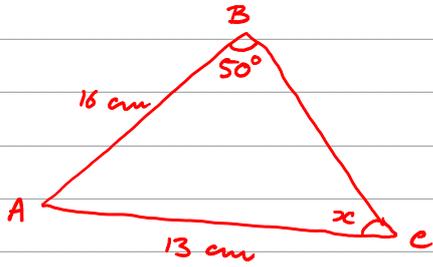
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2. In the triangle ABC , $AB = 16$ cm, $AC = 13$ cm, angle $ABC = 50^\circ$ and angle $BCA = x^\circ$

Find the two possible values for x , giving your answers to one decimal place.

(4)



$$\frac{\sin x}{16} = \frac{\sin 50^\circ}{13}$$

$$\sin x = \frac{16}{13} \sin 50^\circ$$

$$= 0.9428$$

$$x = 70.5^\circ, 109.5^\circ$$

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3. (a) $y = 5^x + \log_2(x + 1), \quad 0 \leq x \leq 2$

Complete the table below, by giving the value of y when $x = 1$

x	0	0.5	1	1.5	2
y	1	2.821	6	12.502	26.585

(1)

- (b) Use the trapezium rule, with all the values of y from the completed table, to find an approximate value for

$$\int_0^2 (5^x + \log_2(x + 1)) dx$$

giving your answer to 2 decimal places.

(4)

- (c) Use your answer to part (b) to find an approximate value for

$$\int_0^2 (5 + 5^x + \log_2(x + 1)) dx$$

giving your answer to 2 decimal places.

(1)

(b) $A = \frac{1}{2} \times 0.5 (1 + 26.585 + 2(2.821 + 6 + 12.502))$
 $= 17.56$

(c) $17.56 + 5 \times 2 = 27.56$

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4.

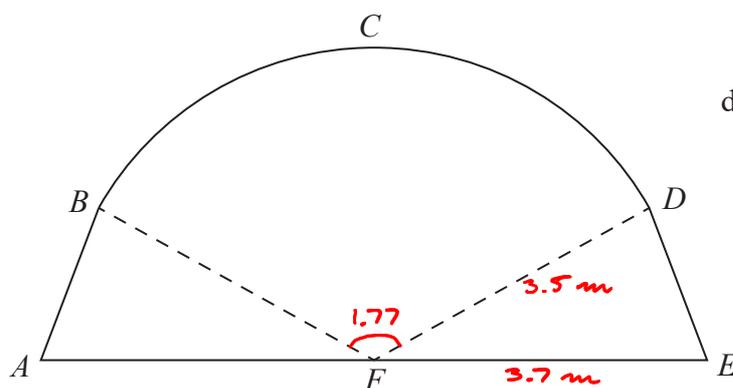


Diagram not drawn to scale

Figure 1

Figure 1 is a sketch representing the cross-section of a large tent $ABCDEF$.
 AB and DE are line segments of equal length.
 Angle FAB and angle DEF are equal.
 F is the midpoint of the straight line AE and FC is perpendicular to AE .
 BCD is an arc of a circle of radius 3.5 m with centre at F .
 It is given that

$$AF = FE = 3.7\text{ m}$$

$$BF = FD = 3.5\text{ m}$$

$$\text{angle } BFD = 1.77 \text{ radians}$$

Find

- (a) the length of the arc BCD in metres to 2 decimal places, (2)
- (b) the area of the sector $FBCD$ in m^2 to 2 decimal places, (2)
- (c) the total area of the cross-section of the tent in m^2 to 2 decimal places. (4)

(a) $l = r\theta$ (b) $A = \frac{1}{2}r^2\theta$

$= 3.5 \times 1.77$ $= \frac{1}{2} \times 3.5^2 \times 1.77$

$= 6.20 \text{ m}$ $= 10.84 \text{ m}^2$

(c) $\angle DFE = \frac{1}{2}(\pi - 1.77)$

$= 0.6858$

$\therefore A = 2 \times \frac{1}{2} \times 3.5 \times 3.7 \times \sin 0.6858 + 10.84$

$= 8.201 + 10.84$

$= 19.04 \text{ m}^2$

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5. The circle C has equation

$$x^2 + y^2 - 10x + 6y + 30 = 0$$

Find

- (a) the coordinates of the centre of C , (2)
- (b) the radius of C , (2)
- (c) the y coordinates of the points where the circle C crosses the line with equation $x = 4$, giving your answers as simplified surds. (3)

(a) $x^2 - 10x + 25 + y^2 + 6y + 9 = -30 + 25 + 9$
 $(x - 5)^2 + (y + 3)^2 = 4$
 $\therefore C(5, -3)$

(b) $r = 2$

(c) $(4 - 5)^2 + (y + 3)^2 = 4$
 $(y + 3)^2 = 3$
 $y = -3 \pm \sqrt{3}$

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6.

$$f(x) = -6x^3 - 7x^2 + 40x + 21$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$

(2)

(b) Factorise $f(x)$ completely.

(4)

(c) Hence solve the equation

$$6(2^{3y}) + 7(2^{2y}) = 40(2^y) + 21$$

giving your answer to 2 decimal places.

(3)

$$\begin{aligned} (a) \quad f(-3) &= -6(-3)^3 - 7(-3)^2 + 40(-3) + 21 \\ &= 0 \end{aligned}$$

$\therefore x + 3$ is a factor

$$\begin{aligned} (b) \quad -6x^3 - 7x^2 + 40x + 21 &= (x+3)(-6x^2 + 11x + 7) \\ &= -(x+3)(6x^2 - 11x - 7) \\ &= -(x+3)(6x^2 - 14x + 3x - 7) \\ &= -(x+3)(2x(3x-7) + 3x-7) \\ &= -(x+3)(3x-7)(2x+1) \\ &= (x+3)(7-3x)(2x+1) \end{aligned}$$

$$\begin{aligned} (c) \quad 2^y &= \frac{7}{3} \quad (-3 \text{ \& } -\frac{1}{2} \text{ are extraneous}) \\ y &= \log_2 \frac{7}{3} \\ &= 1.22 \end{aligned}$$

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7. (i) $2\log(x+a) = \log(16a^6)$, where a is a positive constant

Find x in terms of a , giving your answer in its simplest form.

(3)

(ii) $\log_3(9y+b) - \log_3(2y-b) = 2$, where b is a positive constant

Find y in terms of b , giving your answer in its simplest form.

(4)

(i) $2\log(x+a) = 2\log(4a^3)$

$x+a = 4a^3$

$x = 4a^3 - a$

$= a(2a+1)(2a-1)$

(ii) $\log_3(9y+b) - \log_3(2y-b) = 2$

$\frac{9y+b}{2y-b} = 3^2$

$9y+b = 9(2y-b)$

$= 18y - 9b$

$10b = 9y$

$y = \frac{10}{9}b$

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8. (a) Show that the equation

$$\cos^2 x = 8 \sin^2 x - 6 \sin x$$

can be written in the form

$$(3 \sin x - 1)^2 = 2 \quad (3)$$

- (b) Hence solve, for $0 \leq x < 360^\circ$,

$$\cos^2 x = 8 \sin^2 x - 6 \sin x$$

giving your answers to 2 decimal places.

(5)

$$\begin{aligned} (a) \quad 1 - \sin^2 x &= 8 \sin^2 x - 6 \sin x \\ 2 &= 9 \sin^2 x - 6 \sin x + 1 \\ 2 &= (3 \sin x - 1)^2, \quad \text{QED.} \end{aligned}$$

$$\begin{aligned} (b) \quad 3 \sin x - 1 &= \pm \sqrt{2} \\ \sin x &= \frac{1 \pm \sqrt{2}}{3} \\ x &= 53.58^\circ, 126.42^\circ, 187.94^\circ, 352.06^\circ \end{aligned}$$

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9. The first three terms of a geometric sequence are

$$7k - 5, 5k - 7, 2k + 10$$

where k is a constant.

(a) Show that $11k^2 - 130k + 99 = 0$

(4)

Given that k is not an integer,

(b) show that $k = \frac{9}{11}$

(2)

For this value of k ,

(c) (i) evaluate the fourth term of the sequence, giving your answer as an exact fraction,

(ii) evaluate the sum of the first ten terms of the sequence.

(6)

(a) $\frac{5k-7}{7k-5} = \frac{2k+10}{5k-7}$

$$(5k-7)^2 = (2k+10)(7k-5)$$

$$25k^2 - 70k + 49 = 14k^2 + 60k - 50$$

$$11k^2 - 130k + 99 = 0, \quad \text{QED.}$$

(b) $11k^2 - 121k - 9k + 99 = 0$

$$11k(k-11) - 9(k-11) = 0$$

$$(k-11)(11k-9) = 0$$

$$\therefore k = \frac{9}{11}, \quad \text{QED.}$$

(c) First term = $7\left(\frac{9}{11}\right) - 5 = \frac{8}{11}$ Second term = $5\left(\frac{9}{11}\right) - 7 = -\frac{32}{11}$

$$r = -\frac{32}{8} = -4$$

$$\text{Third term} = -\frac{32}{11} \times -4 = \frac{128}{11}$$

$$\therefore \text{Fourth term} = \frac{128}{11} \times -4 = -\frac{512}{11}$$

(ii) $S_n = \frac{a(r^n - 1)}{r - 1}$
 $S_{10} = \frac{\frac{8}{11}((-4)^{10} - 1)}{-4 - 1}$
 $= -152520$

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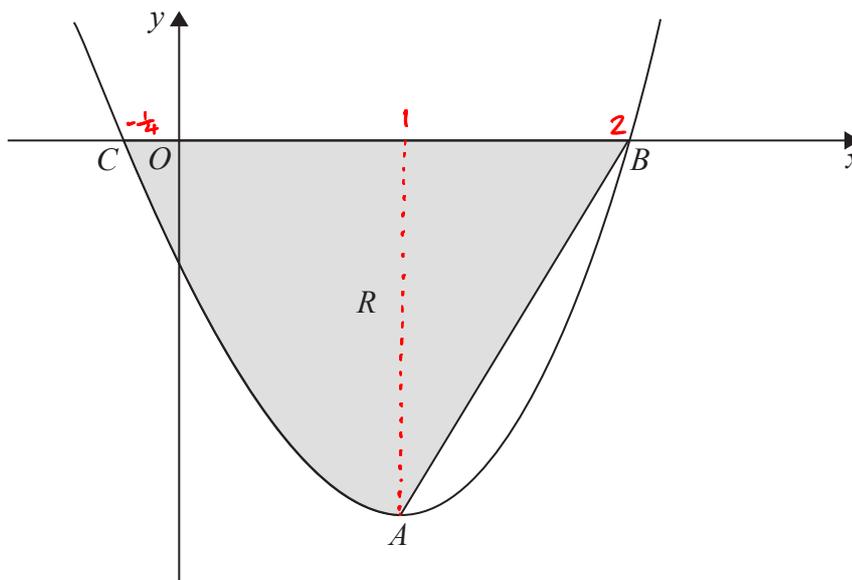


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 4x^3 + 9x^2 - 30x - 8, \quad -0.5 \leq x \leq 2.2$$

The curve has a turning point at the point A .

- (a) Using calculus, show that the x coordinate of A is 1 (3)

The curve crosses the x -axis at the points $B(2, 0)$ and $C\left(-\frac{1}{4}, 0\right)$

The finite region R , shown shaded in Figure 2, is bounded by the curve, the line AB , and the x -axis.

- (b) Use integration to find the area of the finite region R , giving your answer to 2 decimal places. (7)

(a) $\frac{dy}{dx} = 12x^2 + 18x - 30$

$0 = 2x^2 + 3x - 5$

$= 2x^2 - 2x + 5x - 5$

$= 2x(x-1) + 5(x-1)$

$= (x-1)(2x+5)$

$x = -\frac{5}{2}, 1$

$\therefore x = 1, \text{ QED.}$

$y(1) = 4 + 9 - 30 - 8$

$= -25$

$\therefore A(1, -25)$

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Question 10 continued

$$\begin{aligned}
 (6) \quad & \int_{-\frac{1}{4}}^1 (4x^3 + 9x^2 - 30x - 8) \, dx \\
 & = \left[x^4 + 3x^3 - 15x^2 - 8x \right]_{-\frac{1}{4}}^1 \\
 & = (1 + 3 - 15 - 8) - \left(\left(-\frac{1}{4}\right)^4 + 3\left(-\frac{1}{4}\right)^3 - 15\left(-\frac{1}{4}\right)^2 - 8\left(-\frac{1}{4}\right) \right) \\
 & = -19 - \frac{261}{256} \\
 & = -\frac{5125}{256}
 \end{aligned}$$

$$\begin{aligned}
 \text{Area of triangle} &= \frac{1}{2} \times 1 \times 25 \\
 &= \frac{25}{2}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Area of R} &= \frac{5125}{256} + \frac{25}{2} \\
 &= \frac{8325}{256} \\
 &= 32.52
 \end{aligned}$$

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