C12 October 2017 (MA)

Q1a) $8x + 2y - 15 = 0$

$$2y = 15 - 8x$$

$\div 2 :$ $y = \dfrac{15}{2} - 4x$

So $\boxed{m = -4}$ = gradient

b) $m_{l_2} = -4$ (parallel) $\left(-\dfrac{3}{4}, 16\right)$

$\therefore$ $y - 16 = -4\left(x + \dfrac{3}{4}\right)$

$$y = -4x - 3 + 16$$

$$\boxed{y = -4x + 13}$$

Q2a) $f(x + 2)$ indicates a translation of $+2$ in the negative x-direction

So $\boxed{(0, 3)}$

b) $-f(x) \longrightarrow$ multiply all y by -1

So $\boxed{(2, -3)}$

c) $y = \dfrac{1}{2} f(x) \longrightarrow$ multiply all y by $\dfrac{1}{2}$

So $\boxed{\left(2, \dfrac{3}{2}\right)}$

d) $y = f(x) - 4 \rightarrow$ translation of $+4$ in the negative y-dir.

$$\boxed{(2, -1)}$$

Q3a) $\dfrac{x^3 + 4}{2x^2} = \dfrac{x^3}{2x^2} + \dfrac{4}{2x^2} = \dfrac{x}{2} + \dfrac{2}{x^2}$

$$= \boxed{\dfrac{1}{2}x^1 + 2x^{-2}}$$

b) $\displaystyle\int \left[\dfrac{x^3 + 4}{2x^2}\right] dx = \int \left(\dfrac{1}{2}x + 2x^{-2}\right) dx$

$$= \dfrac{x^2}{4} + \dfrac{2x^{-1}}{-1} (+c) = \boxed{\dfrac{1}{4}x^2 - \dfrac{2}{x} + c}$$

Q4a) Area $\Delta = \dfrac{1}{2} ab\sin C = \dfrac{1}{2}(3x)(x)\sin 60$

$\Rightarrow \dfrac{3x^2 \sin 60}{2} = 24\sqrt{3}$
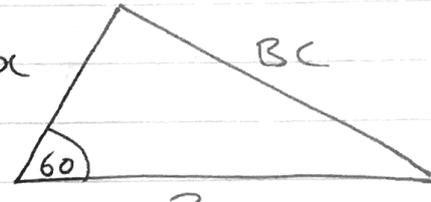
$\Rightarrow \dfrac{3x^2}{2} \times \dfrac{\sqrt{3}}{2} = 24\sqrt{3}$

$\Rightarrow x^2\left(\dfrac{3\sqrt{3}}{4}\right) = 24\sqrt{3}$

$\Rightarrow x^2 = \dfrac{24\sqrt{3}}{\dfrac{3\sqrt{3}}{4}} = 32 \text{ //}$

So $x = \sqrt{32} = \boxed{4\sqrt{2}}$

b) <u>cosine rule</u>

$BC^2 = x^2 + (3x)^2 - 2(x)(3x)\cos 60$

$BC^2 = 10x^2 - 6x^2 \left(\frac{1}{2}\right)$

$BC^2 = 7x^2 = 7\left(4\sqrt{2}\right)^2 = 224 \,//$

so $\quad BC = \sqrt{224} = \boxed{4\sqrt{14}}$

Q5a) $\quad y = 27x^{\frac{1}{2}} - 2x^2$

$\dfrac{dy}{dx} = \boxed{\dfrac{27}{2}x^{-\frac{1}{2}} - 4x}$

b) $\quad \dfrac{dy}{dx} = 0 \quad$ at P

$\dfrac{27}{2}x^{-\frac{1}{2}} - 4x = 0$

$4x = \dfrac{27}{2}x^{-\frac{1}{2}}$

$\therefore \quad 4x^{\frac{3}{2}} = \dfrac{27}{2}$

So $\quad x^{\frac{3}{2}} = \dfrac{27}{8}$

$x = \sqrt[3/2]{\dfrac{27}{8}} = \dfrac{9}{4} \,//$

at $x = \dfrac{9}{4}, \quad y = 27\left(\dfrac{9}{4}\right)^{\frac{1}{2}} - 2\left(\dfrac{9}{4}\right)^2 = \dfrac{243}{8}$

so $\boxed{P\left(\dfrac{9}{4},\ \dfrac{243}{8}\right)}$

Q6a)

year 1	year 2	year 3
600	680	760

$\left(\begin{array}{l} a = 600 \\ d = 80 \end{array}\right)$

n^{th} term $= a + (n-1)d$

N^{th} term $= 600 + (N-1) \times 80 = 1000$

$$80(N-1) = 400$$

$$N - 1 = 5$$

$$\therefore \boxed{N = 6}$$

b) we require sum of the ... first 15 terms.

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$n = 15$: $S_{15} = \dfrac{15}{2}\left(2(600) + (14)(80)\right)$

$$= \boxed{£17400}$$

c)

year 1	year 2	year 3
A	2A	3A

$\left(\begin{array}{l} a = A \\ d = A \end{array}\right)$

Sum of first 15 terms is equal for Saima and Lin.

$$S_{15} = \frac{15}{2}(2A + (15-1)A) = 17400 \; /\!/$$

So $\dfrac{15}{2}(2A + 14A) = 17400$

$120A = 17400$

$A = \dfrac{17400}{120} = \boxed{145}$

(Q7a) $g(x) = 2x^3 + ax^2 - 18x - 8$

$(x+2)$ is a factor of $g(x)$ so $g(-2) = 0$

$g(-2) = 2(-2)^3 + a(-2)^2 - 18(-2) - 8 = 0$

$-16 + 4a + 36 - 8 = 0$

$4a = -12 \qquad \therefore \boxed{a = -3}$

b)

$$
\begin{array}{r}
2x^2 - 7x - 4 \\
x+2 \,\overline{\smash{\big)}\, 2x^3 - 3x^2 - 18x - 8} \\
\underline{2x^3 + 4x^2} \\
0 - 7x^2 - 18x \\
\underline{-7x^2 - 14x} \\
0 - 4x - 8 \\
\underline{-4x - 8} \\
0 \quad 0
\end{array}
$$

$\therefore g(x) = (x+2)(2x^2 - 7x - 4)$

but $2x^2 - 7x - 4 = (2x+1)(x-4)$

so $g(x) = \boxed{(x+2)(2x+1)(x-4)}$

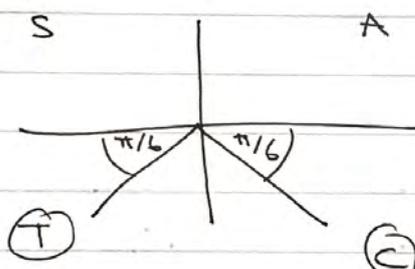c) $2\sin^3\theta - 3\sin^2\theta - 18\sin\theta - 8 = 0$

but notice this is exactly the same as
$[g(x) = 0]$ if we let $x = \sin\theta$.

so let $x = \sin\theta$:

$$2x^3 - 3x^2 - 18x - 8 = 0 \quad (= g(x))$$

$$\therefore \quad (x+2)(2x+1)(x-4) = 0$$

$x + 2 = 0$	$2x + 1 = 0$	$x - 4 = 0$
$x = -2$	$x = -\frac{1}{2}$	$x = 4$
$\sin\theta = -2$	$\sin\theta = -\frac{1}{2}$	$\sin\theta = 4$
NO SOLUTIONS		NO SOLUTIONS
$-1 \le \sin\theta \le 1$	$\theta = \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$	$-1 \le \sin\theta \le 1$

S   A

π/6   π/6

Ⓣ   Ⓒ

$$\theta = \pi + \frac{\pi}{6}, \quad 2\pi - \frac{\pi}{6}$$

$$\boxed{\theta = \frac{7\pi}{6}, \quad \frac{11\pi}{6}}$$

Q8a) $L = \boxed{r\theta = 6}$
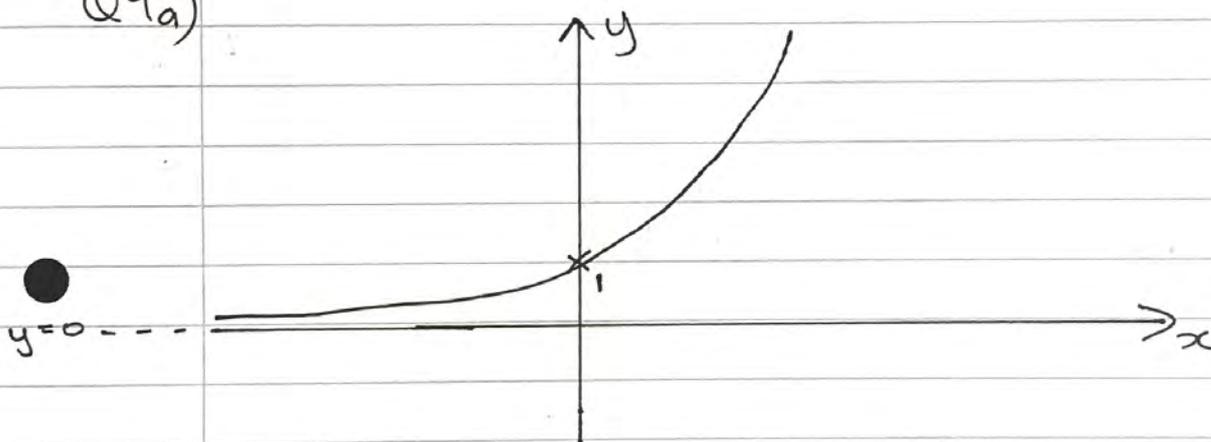
$$\frac{1}{2}r^2\theta = \text{Area } OAB = 20$$

$$20 = \frac{1}{2}r^2\theta \longrightarrow \boxed{40 = r^2\theta}$$

b) $r\theta = 6 \longrightarrow 40 = 6 \times r$

$$r = \frac{40}{6} = \boxed{\frac{20}{3}}$$

$$\theta = \frac{6}{r} = \frac{6}{\frac{20}{3}} = \boxed{\frac{9}{10}}$$

Q9a)

b) $h = \dfrac{b-a}{n} = \dfrac{4--4}{4} = 2$

$$\text{Area} \simeq \frac{1}{2} \times 2 \left[0.0625 + 16 + 2(0.25 + 1 + 4) \right]$$

$$\simeq \boxed{26.56}$$

ci) $2^{x+2} = 2^x(2^2) = 2^2 \times 2^x$

so $\displaystyle\int_{-4}^{4} 2^{x+2}\, dx = 2^2 \times 26.56 \approx \boxed{106}$

or $\dfrac{425}{16}$ (exact)

ii) $\displaystyle\int_{-4}^{4}(3+2^x)\, dx = \int_{-4}^{4}(3)\, dx + \int_{-4}^{4}(2^x)\, dx$

$= \big[3x\big]_{-4}^{4} + 26.56$

$= \big[3\times4\big] + \big[3\times4\big] + 26.56 \approx \boxed{50.6}$

or $\dfrac{809}{16}$ (exact)

Q10a) $A(7, -3)$ and $D(10, 5)$
$C(p, q)$

midpoint $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

since D is the midpoint...

$\hookrightarrow \left(\dfrac{7+p}{2}, \dfrac{q-3}{2}\right) = (10, 5)$

$\dfrac{7+p}{2} = 10 \qquad\therefore p = 20 - 7 = \boxed{13}$

$\dfrac{q-3}{2} = 5 \qquad\therefore q = 10 + 3 = \boxed{13}$

b) $m_{AC} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{13 - -3}{13 - 7} = \dfrac{16}{6} = \dfrac{8}{3}$

but L is perp to AC so $M_L = \dfrac{-3}{8}$ //

$$\left(\dfrac{-3}{8} \times \dfrac{8}{3} = -1 \right)$$ //

$\Rightarrow y - 5 = -\dfrac{3}{8}(x - 10)$

$\Rightarrow y = -\dfrac{3}{8}x + \dfrac{30}{8} + 5$

$\Rightarrow y = -\dfrac{3}{8}x + \dfrac{35}{4}$

$\underrightarrow{\times 8}:$ $8y = -3x + 70$

$\longrightarrow \boxed{3x + 8y = 70}$

c) AB is simply the line $x = 7$ (for $-3 \leq y \leq 20$)

So sub $x = 7$: $3(7) + 8y = 70$

$8y = 49$

$\boxed{y = \dfrac{49}{8}}$

So $\boxed{E\left(7, \dfrac{49}{8}\right)}$ //

Q11a) $(3+ax)^5 \simeq (3)^5 + \binom{5}{1}(3)^4(ax)^1$

$$+ \binom{5}{2}(3)^3(ax)^2$$

$$\simeq \boxed{243 + 405ax + 270a^2x^2}$$

b) $f(x) = (a-x)(243 + 405ax + 270a^2x^2)$

$$= 243 + 405a^2x + 270a^3x^2$$
$$- 243x - 405ax^2 - 270a^2x^3$$

x terms are $(+405a^2x)$ and $(-243x)$

coeff. of x is 0 so $405a^2 - 243 = 0$

$$\Rightarrow a^2 = \frac{243}{405} = \frac{3}{5}$$

$$\therefore a = \boxed{\sqrt{\frac{3}{5}}}$$

Q(2i) $3\sin(\theta + 30) = 2\cos(\theta + 30)$

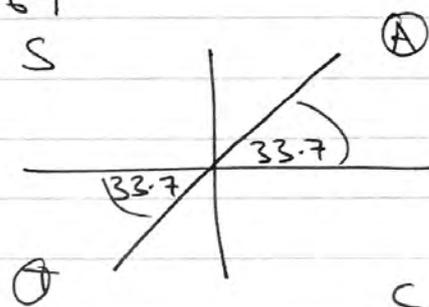$\div \cos(\theta+30)$: $3\tan(\theta+30) = 2$

$\tan(\theta+30) = \dfrac{2}{3}$

so $\theta + 30 = \tan^{-1}\left(\dfrac{2}{3}\right) = 33.69°..$

range ... $30° < \theta + 30 \le 390°$

$\theta + 30° = 33.69°, \; 180 + 33.69°$

$\theta + 30 = 33.69°, \; 213.69°$

$\boxed{\theta = 3.69°, \; 183.69°}$

iia) $\dfrac{\cos^2 x + 2\sin^2 x}{1 - \sin^2 x} = 5$

$\dfrac{\cos^2 x + 2\sin^2 x}{\cos^2 x} = 5$

$1 + 2\tan^2 x = 5$

$2\tan^2 x = 4$

$\therefore \boxed{\tan^2 x = 2}$ $(k = 2)$

$$0 < x \leq 2\pi$$

b) we need to solve: $\tan^2 x = 2$
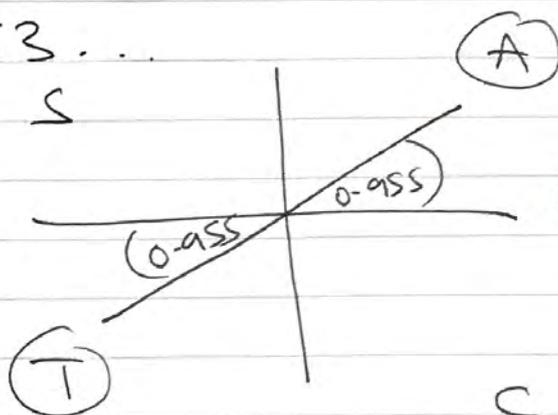
$$\tan x = \pm\sqrt{2}$$

$\tan x = \sqrt{2}$

$x = \tan^{-1}(\sqrt{2}) = 0.9553\ldots$

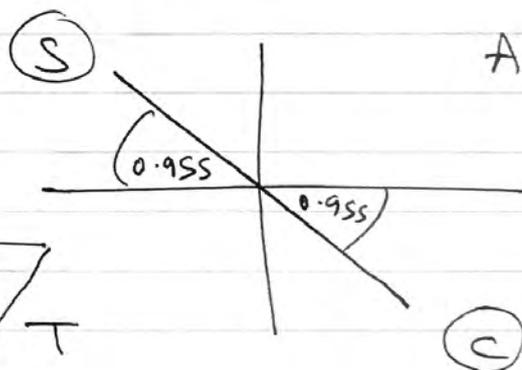$x = 0.955^c, 4.10\ldots^c, \ldots$

$\boxed{x = 0.955^c, 4.097^c}$

$\tan x = -\sqrt{2}$

$x = \tan^{-1}(-\sqrt{2}) = -0.9553\ldots$

$x = \pi - 0.9553,$
$\quad 2\pi - 0.9553$

$\boxed{x = 2.186^c, 5.328^c}$

1

Q13ai) $(x-3)^2 + (y+4)^2 = 30$

centre $\boxed{(3, -4)}$

ii) radius $\boxed{\sqrt{30}}$

b) since P lies inside the circle we know that the distance from the centre to P will be less than the radius.

so $\sqrt{(6-3)^2 + (4+u)^2} < \sqrt{30}$

square both sides ...

$(6-3)^2 + (u+4)^2 < 30$

$9 + u^2 + 16 + 8u < 30$

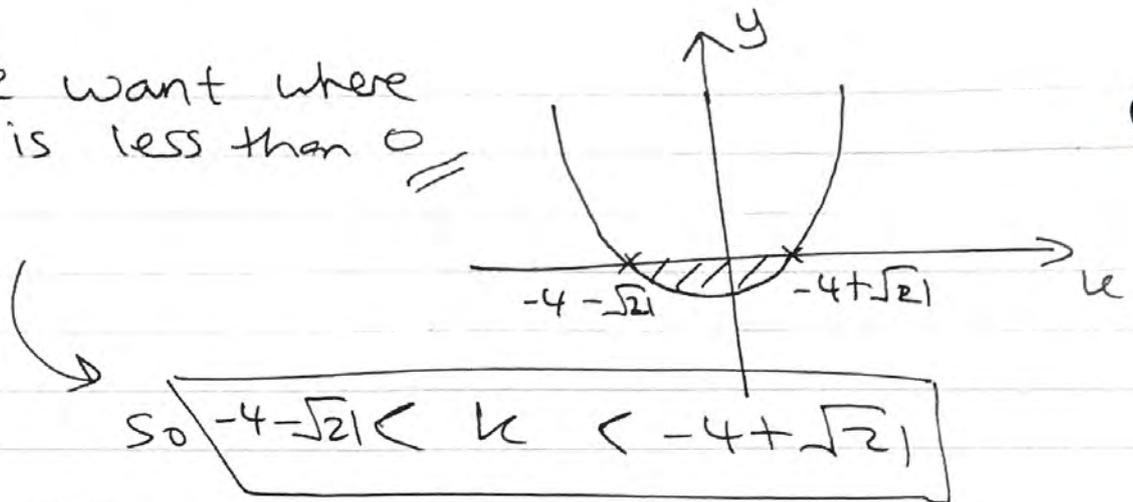$u^2 + 8u + 25 - 30 < 0$

$u^2 + 8u - 5 < 0$

c) we want the values of u for which

$u^2 + 8u - 5 < 0$

critical values: $u^2 + 8u - 5 = 0$

By Quadratic formula, $u = -4 \pm \sqrt{21}$

we want where
y is less than 0

So $\boxed{-4-\sqrt{21} < k < -4+\sqrt{21}}$

(Q14a)   year 1   year 2   year 3
          8000   8000 × (0.85)   8000 × (0.85)²

$$\left(\begin{array}{c} a = 8000 \\ r = 0.85 \end{array} \right)$$

So 6th year $\longrightarrow$ ar^{n-1} = 8000 $(0.85)^5$

$$= 3549.6$$

$$\simeq \boxed{3550}$$

b) $\boxed{r < 1}$ as $n \to \infty$, $S_n \to$ constant.
                                    (ie S_∞)

c) $S_\infty = \dfrac{a}{1-r} = \dfrac{8000}{1-0.85}$ $\simeq \boxed{53333}$

d) $S_N = \dfrac{a(1-r^n)}{1-r}$ = 40000

$$\dfrac{8000(1-0.85^N)}{0.15} = 40000$$

$$8000\left(1 - 0.85^N\right) = 6000$$

$$1 - 0.85^N = \frac{3}{4}$$

$$0.85^N = \frac{1}{4}$$

$$\log\left(0.85^N\right) = \log\left(\frac{1}{4}\right)$$

$$N \log 0.85 = \log \frac{1}{4}$$

$$N = \frac{\log \frac{1}{4}}{\log 0.85} = 8.53\ldots$$

So $\boxed{N = 9}$ (in the 8th year, total mass < 40000)

so $N = 9$

Q15ai) $x = 0$ ∴ $y = \dfrac{(-3)^2(4)}{2} = \boxed{18}$

ii) $y = 0$: $\dfrac{(x-3)^2(x+4)}{2} = 0$

$$\boxed{x = 3}   (x > 0)$$

b) $y = 18$: $18 = \dfrac{(x-3)^2(x+4)}{2}$

$$36 = (x-3)^2(x+4)$$

$$(x^2 - 6x + 9)(x + 4) = 36$$

$$x^3 - 2x^2 - 15x + 36 = 36$$

$$x^3 - 2x^2 - 15x = 0$$

$$x(x^2 - 2x - 15) = 0$$

$$x > 0 \quad \therefore \quad \boxed{x^2 - 2x - 15 = 0}$$
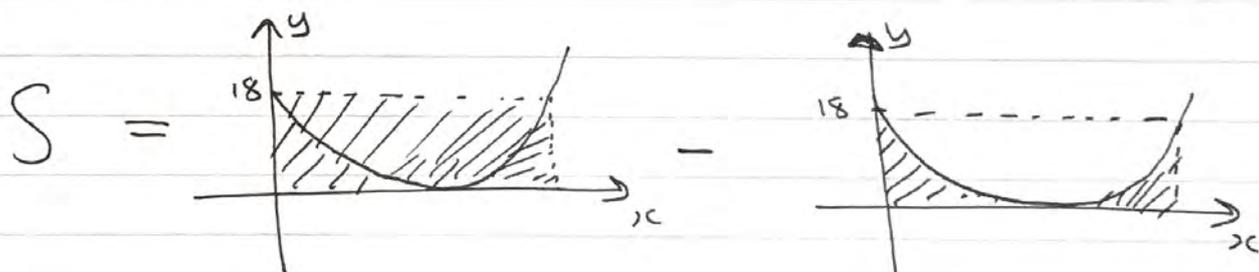
c) $\quad (x - 5)(x + 3) = 0$

$$x = 5$$
$$x = -3$$

$$x > 0 \quad \text{so} \quad \boxed{x = 5}$$

$$\text{hence} \quad \boxed{Q(5, 18)}$$

d)

$$S = \text{[graph with shaded region, } y \text{ axis marked } 18] \quad - \quad \text{[graph with shaded region, } y \text{ axis marked } 18]$$

$$S = \text{[rectangle, height } 18, \text{ width } S] \quad - \quad \int_0^5 f(x)\, dx$$

$$\therefore S = (18 \times 5) - \frac{1}{2}\int_0^5 [x^3 - 2x^2 - 15x + 36]\, dx$$

$$= 90 - \frac{1}{2}\left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{15x^2}{2} + 36x\right]_0^5$$

$$= 90 - \frac{1}{2}\left[\frac{785}{12}\right] = \boxed{\frac{1375}{24}}$$