

C12 January 2018 (MA)

Q1 a) $y = \frac{1}{3}x^{\frac{2}{3}} + \frac{1}{2}$

$$\frac{dy}{dx} = \frac{2}{3} \left(\frac{1}{3} \right) x^{-\frac{1}{3}} + 0$$

$$\therefore \boxed{\frac{dy}{dx} = \frac{2}{9}x^{-\frac{1}{3}}}$$

b) $\int \left(\frac{1}{3}x^{\frac{2}{3}} + \frac{1}{2} \right) dx = \frac{1}{3}x^{\frac{5}{3}} + \frac{1}{2}x + c$

$$= \boxed{\frac{1}{5}x^{\frac{5}{3}} + \frac{1}{2}x + c}$$

Q2 a) $u_2 = 2 - 3(1) = \boxed{-1}$

$$u_3 = 2 - 3(-1) = \boxed{5}$$

b) $\sum_{r=1}^4 r - u_r = (1+2+3+4) - (u_1+u_2+u_3+u_4)$

$\swarrow \Sigma r$ $\downarrow \Sigma u_r$

$$= (10) - (1 - 1 + 5 + 2 - 3(5))$$

$$= 10 - -8 = \boxed{18}$$

$$\bullet \text{ (Q3a)} \quad (3x^{\frac{1}{2}})^4 = 3^4 \times x^{\frac{1}{2} \times 4} = \boxed{81x^2}$$

$$\text{b) } \frac{2y^7 \times (4y)^{-2}}{3y} = \frac{2y^7 \times \frac{1}{16y^2}}{3y}$$

$$= \frac{\frac{2y^7}{16y^2}}{3y} = \frac{\frac{1}{8}y^5}{3y} = \frac{y^5}{24y} = \boxed{\frac{y^4}{24}}$$

$$\text{(Q4a)} \quad (p-2)x^2 + (8)x + (p+4) = 0$$

no real roots so $b^2 - 4ac < 0$

$$\left. \begin{array}{l} a = p-2 \\ b = 8 \\ c = p+4 \end{array} \right\} \begin{array}{l} b^2 - 4ac < 0 \\ (8)^2 - 4(p-2)(p+4) < 0 \end{array}$$

$$64 - 4(p^2 + 2p - 8) < 0$$

$$64 - 4p^2 - 8p + 32 < 0$$

$$-4p^2 - 8p + 96 < 0$$

$$\div 4 : -p^2 - 2p + 24 < 0$$

$$\underline{x-1} : p^2 + 2p - 24 > 0$$

$$b) \quad p^2 + 2p - 24 = 0$$

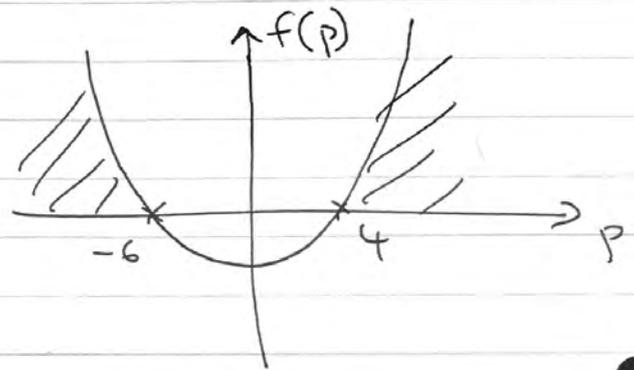
finding critical values ... By Quadratic Formula

$$\left. \begin{array}{l} a = 1 \\ b = 2 \\ c = -24 \end{array} \right\} p = \frac{-2 \pm \sqrt{4 - 4(-24)}}{2}$$

$$p = \frac{-2 \pm \sqrt{100}}{2}$$

$$\text{so } p = 4 \quad \text{or} \quad p = -6$$

we want
where $y \geq 0$.



so

$$\boxed{\begin{array}{l} p > 4 \\ p < -6 \end{array}}$$

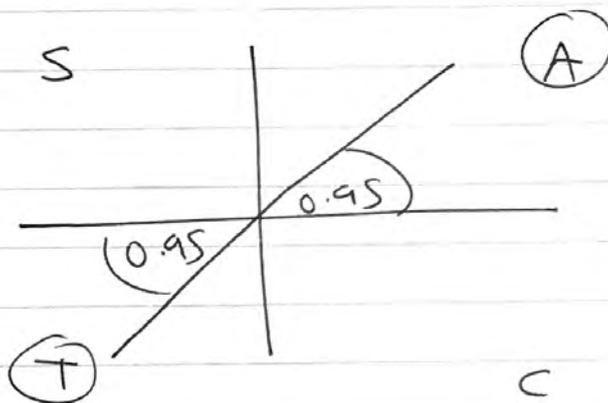
$$\bullet \text{ (Q5 i)} \quad 5 \sin 3\theta - 7 \cos 3\theta = 0$$

$$\div \underline{\cos 3\theta} : 5 \tan 3\theta = 7$$

$$\tan 3\theta = \frac{7}{5}$$

$$3\theta = \tan^{-1}\left(\frac{7}{5}\right) = 0.9505\dots$$

$$\bullet \text{ range : } \underline{0 < 3\theta < \frac{3\pi}{2}}$$



$$3\theta = 0.9505, \pi + 0.9505$$

$$3\theta = 0.9505, 4.092$$

$$\bullet \text{ so } \theta = \boxed{0.317, 1.36}$$

$$\text{ii) } 9 \cos^2 x + 5 \cos x = 3 - 3 \cos^2 x$$

$$12 \cos^2 x + 5 \cos x - 3 = 0$$

$$(3 \cos x - 1)(4 \cos x + 3) = 0$$

$$3 \cos x - 1 = 0$$

$$\cos x = \frac{1}{3}$$

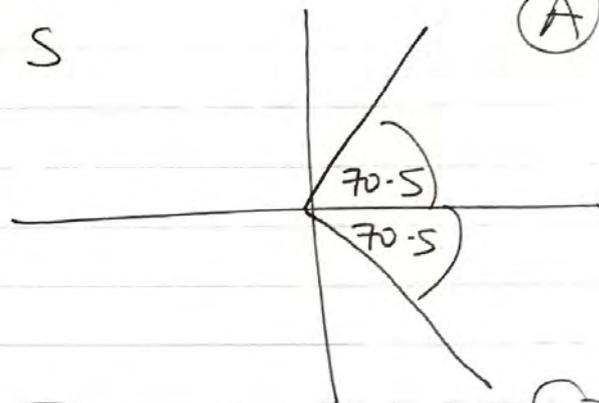
$$4 \cos x + 3 = 0$$

$$\cos x = -\frac{3}{4}$$

$$\alpha = \cos^{-1}\left(\frac{1}{3}\right) = 70.53^\circ \dots$$

$$\alpha = 70.5^\circ, 360 - 70.5^\circ \text{ S}$$

$$\alpha = 70.5^\circ, 289.5^\circ$$

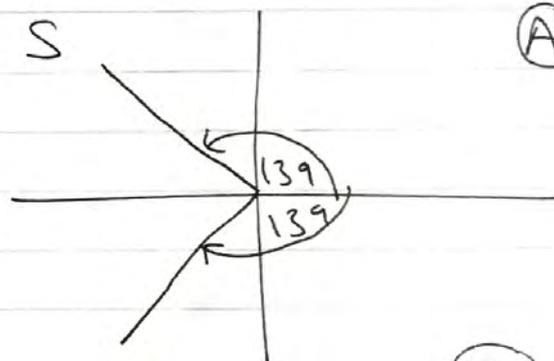


T

$$\alpha = \cos^{-1}\left(-\frac{3}{4}\right) = 138.6^\circ$$

$$\alpha = 138.6^\circ, 360 - 138.6^\circ \text{ S}$$

$$\alpha = 138.6^\circ, 221.4^\circ$$



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Q6a)

$f(x)$ div by $(x+1) \rightarrow$ no remainder

$$\text{so } f(-1) = 0.$$

$$f(-1) = -a - 8 - b + 6 = 0$$

$$a + b = -2 //$$

$f(x)$ div by $(x-2) \rightarrow$ remainder = -12

$$\text{so } f(2) = -12$$

$$\hookrightarrow f(2) = 8a - 32 + 2b + 6 = -12 //$$

$$\therefore 8a + 2b = 14$$

$$\div 2: 4a + b = 7$$

$$- (a + b = -2)$$

$$\hline 3a + 0 = 9$$

$$\therefore 3a = 9 \quad \text{so} \quad \boxed{a = 3}$$

$$b = -2 - a = -2 - 3 = \boxed{-5}$$

b)

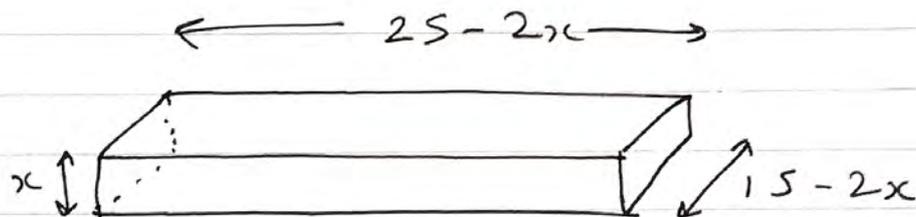
$$\begin{array}{r} 3x^2 - 11x + 6 \\ x+1 \overline{) 3x^3 - 8x^2 - 5x + 6} \\ \underline{3x^3 + 3x^2} \\ 0 - 11x^2 - 5x \\ \underline{-11x^2 - 11x} \\ 0 + 6x + 6 \\ \underline{6x + 6} \\ 0 \quad 0 \end{array} //$$

$$\therefore f(x) = (x+1)(3x^2 - 11x + 6)$$

$$\text{but } (3x^2 - 11x + 6) = (3x - 2)(x - 3) //$$

$$\therefore f(x) = \boxed{(x+1)(3x-2)(x-3)}$$

(Q7a)



$$\text{Volume} = (25 - 2x) \times (15 - 2x) \times (x)$$

$$V = (375 - 50x - 30x + 4x^2) \times x$$

$$V = (375 - 80x + 4x^2) \times x$$

$$V = 4x^3 - 80x^2 + 375x$$

$$b) \quad \frac{dV}{dx} = 12x^2 - 160x + 375$$

at max value, $\frac{dV}{dx} = 0$.

$$\Rightarrow 12x^2 - 160x + 375 = 0$$

By Quadratic formula...

$$\left. \begin{array}{l} a = 12 \\ b = -160 \\ c = 375 \end{array} \right\} x = 3.03, 10.3$$

but $x \neq 10.3$ since side length is $15 - 2x$

$$\text{so } \boxed{x = 3.03}$$

c) $\frac{d^2V}{dx^2} = 24x - 160$

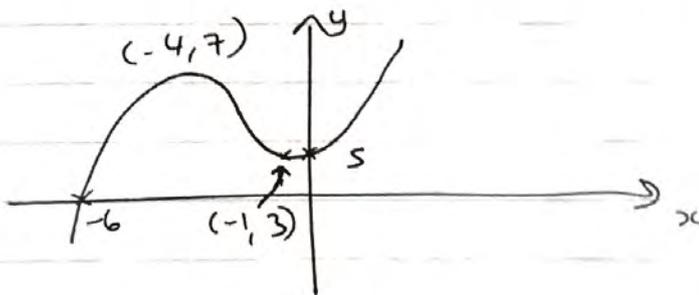
$$x = 3.03 \rightarrow \left(\frac{d^2V}{dx^2}\right)_{x=3.03} = 24(3.03) - 160 \approx -87 < 0$$

hence value is a maximum.

d) $V = 4(3.03)^3 - 80(3.03)^2 + 375(3.03)$

$$\approx \boxed{513} \text{ cm}^3$$

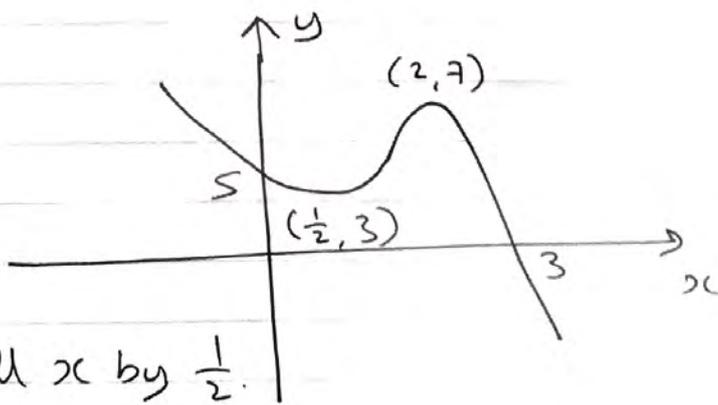
(Q8a) $f(-x) \rightarrow$ multiply all x by -1



b) $f(2x)$



multiply all x by $\frac{1}{2}$.



$$(Q9a) \quad \begin{array}{ccc} a & ar & ar^2 \\ 20 & 20(0.9) & 20(0.9)^2 \end{array}$$

$$S^{\text{th}} \text{ term} = ar^4 = 20 \times (0.9)^4 \\ = \boxed{13.122}$$

$$b) S_8 = \frac{20(1-0.9^8)}{1-0.9} = \boxed{113.9}$$

$$c) S_{\infty} = \frac{a}{1-r} = \frac{20}{1-0.9} = 200$$

$$S_{\infty} - S_N = 200 - \frac{20(1-0.9^N)}{0.1}$$

$$S_{\infty} - S_N = 200 - 200(1-0.9^N) \\ = 200 - 200 + 200(0.9^N) \\ = 200(0.9^N) //$$

$$\text{hence } 200(0.9^N) < 0.04$$

$$\div \underline{200} : \quad 0.9^N < \frac{0.04}{200}$$

$$\therefore 0.9^N < 0.0002 //$$

$$\bullet \text{ d) } \log(0.9^N) < \log(0.0002)$$

$$N \log(0.9) < \log(0.0002)$$

$$N > \frac{\log 0.0002}{\log 0.9}$$

$$\bullet \quad N > 80.84 \dots$$

∵ by a -ve value
flips the signs

$$\text{So } \boxed{N_{\min} = 81}$$

$$\text{(Q10.) } 3\log_8(2) + \log_8(7-x) = 2 + \log_8(x)$$

$$\log_8(2^3) + \log_8(7-x) - \log_8(x) = 2$$

$$\log_8(8) + \log_8\left(\frac{7-x}{x}\right) = 2$$

$$1 + \log_8\left(\frac{7-x}{x}\right) = 2$$

$$\therefore \log_8\left(\frac{7-x}{x}\right) = 1$$

$$\therefore 8^1 = \frac{7-x}{x}$$

$$\text{So } 8x = 7-x$$

$$9x = 7 \quad : \quad \boxed{x = \frac{7}{9}}$$

$$\text{ii) } 3^{2y} + 3^{y+1} = 10$$

$$3^{2y} + (3^y)(3) - 10 = 0$$

$$\text{let } 3^y = u,$$

$$\text{then } u^2 + 3u - 10 = 0$$

$$(u+5)(u-2) = 0$$

$$u = -5$$

$$3^y = -5$$

$$y \log 3 = \log(-5)$$

\downarrow
 no valid solutions....

($a^x > 0$ for all x)

$$u = 2$$

$$3^y = 2$$

$$\log(3^y) = \log(2)$$

$$y \log 3 = \log 2$$

$$y = \boxed{\frac{\log 2}{\log 3}}$$

● Q11ai)
/ii)

$$x^2 - 8x + y^2 - 10y + 16 = 0$$

$$(x-4)^2 - 16 + (y-5)^2 - 25 + 16 = 0$$

$$\therefore \boxed{(x-4)^2 + (y-5)^2 = 25}$$

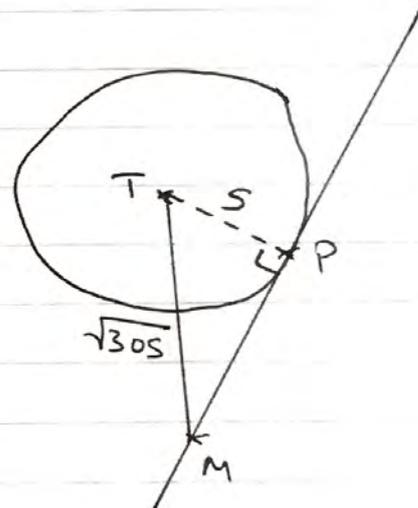
$$\text{so } \boxed{\begin{array}{l} T(4, 5) \\ r = 5 \end{array}}$$

$$\text{b) } MT = \sqrt{(4-20)^2 + (5-12)^2} = \boxed{\sqrt{305}}$$

$$\text{c) } PM = \sqrt{(305) - (5)^2} = 2\sqrt{70} //$$

$$\therefore \Delta PMT \text{ area} = \frac{1}{2}(2\sqrt{70})(5)$$

$$= \boxed{5\sqrt{70}}$$



$$\text{Q12a) } \underline{x = -1} : -1 + 3y - 11 = 0$$

$$3y = 12 \quad \therefore y = \boxed{4 = P}$$

$$\underline{y = 2} : x + 6 - 11 = 0$$

$$x = 5 \quad \therefore \boxed{Q = 5}$$

$$b) \quad (-1, 4) \quad (5, 2)$$

$$AB = \sqrt{(5 - (-1))^2 + (2 - 4)^2} = \boxed{2\sqrt{10}}$$

$$c) \text{ midpoint: } \left(\frac{-1+5}{2}, \frac{4+2}{2} \right)$$

$$\Rightarrow (2, 3)$$

$$m_{AB} = \frac{2-4}{5-(-1)} = \frac{-2}{6} = -\frac{1}{3}$$

$$\text{so } m_{L_2} = 3 \quad (3 \times -\frac{1}{3} = -1)$$

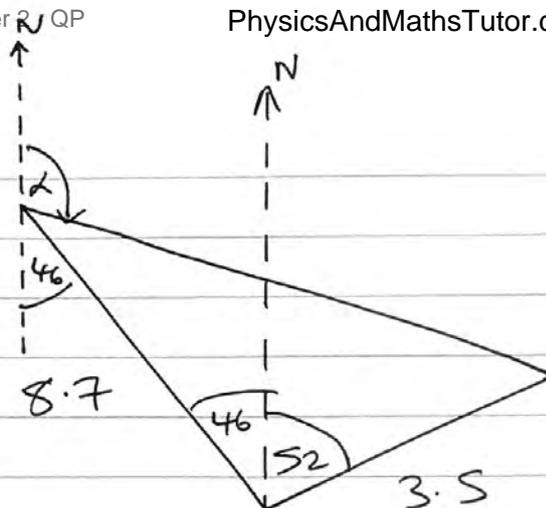
since L_2 is perp to L_1 .

$$\Rightarrow y - 3 = 3(x - 2)$$

$$\Rightarrow y = 3x - 6 + 3$$

$$\Rightarrow \boxed{y = 3x - 3}$$

Q13a)



$$360 - 314 = 46$$

$$\angle APB = 46 + 52 = \boxed{98^\circ}$$

b) cosine rule

$$AB^2 = (8.7)^2 + (3.5)^2 - 2(8.7)(3.5)\cos 98$$

$$\approx 96.42 \dots$$

$$\text{so } AB = \boxed{9.8 \text{ km}}$$

c) sine rule to find $\angle PAB$.

$$\frac{9.8}{\sin 98} = \frac{3.5}{\sin \angle PAB}$$

$$\therefore \sin \angle PAB = \frac{3.5 \sin 98}{9.8}$$

$$\therefore \angle PAB = \sin^{-1}\left(\frac{3.5 \sin 98}{9.8}\right) \approx \underline{\underline{20.66^\circ}}$$

$$\text{Bearing} = d = 180 - 46 - 20.66$$

$$= \boxed{113^\circ}$$

Q14a)

$$8 - x = 14 + 3x - 2x^2$$

$$2x^2 - 4x - 6 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$x = 3$$

$$x = -1$$



$$y = 8 - 3 = 5$$

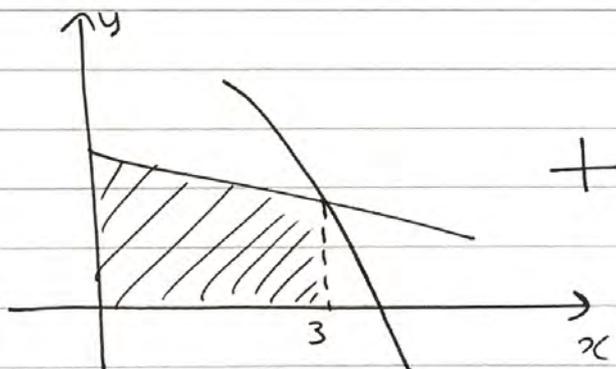
$$y = 8 - (-1) = 9$$

so $B(3, 5)$

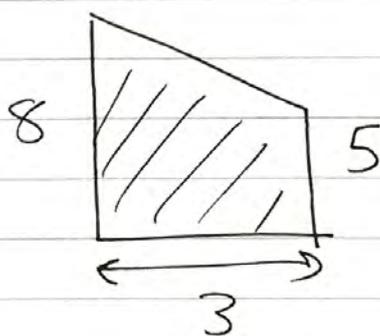
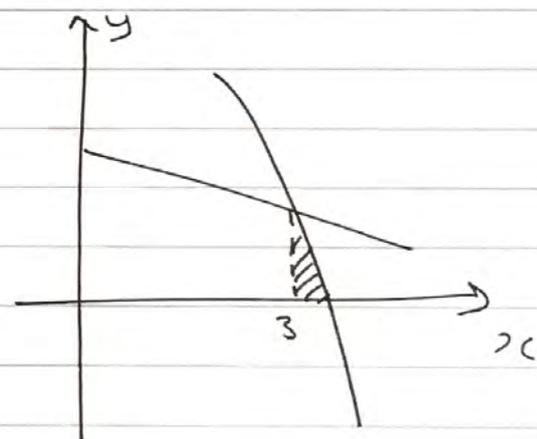
so $A(-1, 9)$

b)

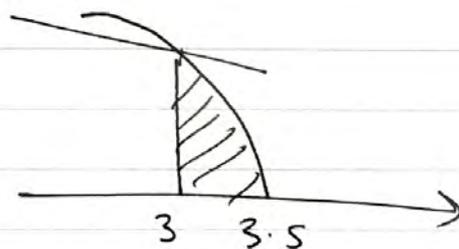
R =



+



+



$$14 + 3x - 2x^2 = 0 \rightarrow x = 3.5, x = \frac{2}{7}$$

$$\text{Trapezium Area} = \frac{(8+5)(3)}{2} = \frac{39}{2} //$$

$$\text{Area (2)} = \int_3^{3.5} [14 + 3x - 2x^2] dx$$

$$= \left[14x + \frac{3}{2}x^2 - \frac{2x^3}{3} \right]_3^{3.5}$$

$$= \left[\frac{931}{24} \right] - \left[\frac{75}{2} \right] = \frac{31}{24} //$$

$$\therefore R = \frac{39}{2} + \frac{31}{24} = \boxed{\frac{499}{24}}$$

$$\text{Q15a) } (1+kx)^n \approx 1 + nkx + \frac{n(n-1)k^2x^2}{2}$$

$$= 1 + 36x + 126kx^2$$

$$\text{so... } 36 = nk \quad \text{and} \quad \frac{n(n-1)k^2}{2} = 126k$$

$$\therefore \frac{n(n-1)k}{2} = 126$$

$$\text{hence } nk(n-1) = 252$$

(x2)

$$\text{b) } nk(n-1) = 252$$

$$\text{but } nk = 36$$

$$\therefore 36(n-1) = 252$$

$$n-1 = \frac{252}{36}$$

$$n-1 = 7 \quad \therefore \boxed{n=8}$$

$$\text{and } k = \frac{36}{n} = \frac{36}{8} = \boxed{\frac{9}{2}}$$

$$\text{c) } x^3 \text{ term} = \frac{n(n-1)(n-2)}{6} (k^3 x^3)$$

$$\text{so coefficient} = \frac{8(8-1)(8-2)}{6} \times \left(\frac{9}{2}\right)^3 = \boxed{5103}$$