Write your name here

Surname Solution.	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

Core Mathematics C3

Advanced

Tuesday 20 June 2017 – Afternoon

Time: 1 hour 30 minutes

Paper Reference **6665/01**

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

ND052066720

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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Pearson

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1. Express $\dfrac{4x}{x^2-9} - \dfrac{2}{x+3}$ as a single fraction in its simplest form.

(4)

$$\frac{4x}{(x+3)(x-3)} - \frac{2}{x+3}$$

$$\frac{4x}{(x+3)(x-3)} - \frac{2x-6}{(x+3)(x-3)}$$

$$\frac{4x-2x+6}{(x+3)(x-3)}$$

$$\frac{2x+6}{(x+3)(x-3)} = \frac{2(x+3)}{(x+3)(x-3)} = \frac{2}{x-3}$$

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2. Find the exact solutions, in their simplest form, to the equations

(a) $e^{3x-9} = 8$

(3)

(b) $\ln(2y + 5) = 2 + \ln(4 - y)$

(4)

a) $e^{3x-9} = 8$

$3x-9 = \ln(8)$

$x = \dfrac{\ln(8)+9}{3}$

b) $\ln(2y+5) = 2+\ln(4-y)$

$\ln(2y+5) - \ln(4-y) = 2$

$\ln\left(\dfrac{2y+5}{4-y}\right) = 2$

$\dfrac{2y+5}{4-y} = e^2$

$2y+5 = 4e^2 - e^2y$

$2y + e^2y = 4e^2 - 5$

$y(2+e^2) = 4e^2-5$

$y = \dfrac{4e^2-5}{2+e^2}$

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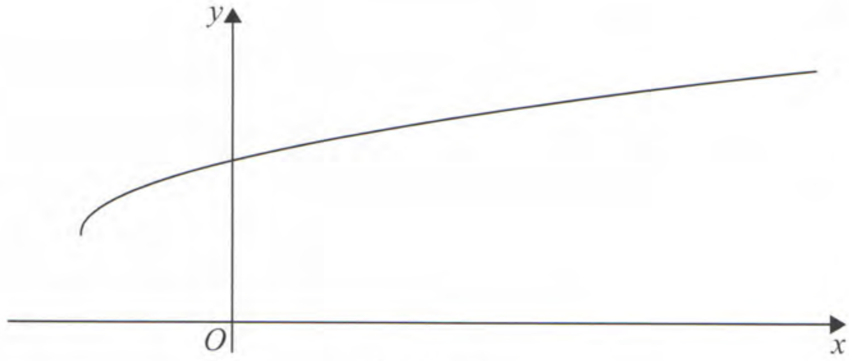

Figure 1

Figure 1 shows a sketch of part of the graph of $y = g(x)$, where

$$g(x) = 3 + \sqrt{x+2}, \qquad x \geqslant -2$$

(a) State the range of g. **(1)**

(b) Find $g^{-1}(x)$ and state its domain. **(3)**

(c) Find the exact value of x for which

$$g(x) = x$$

(4)

(d) Hence state the value of a for which

$$g(a) = g^{-1}(a)$$

(1)

a) $g(x) \geq 3$

b) $x = 3 + \sqrt{y+2}$

$x - 3 = \sqrt{y+2}$

$(x-3)^2 = y+2$

$y = (x-3)^2 - 2, \quad x \geq 3$

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Question 3 continued

c) $x = 3 + \sqrt{x+2}$

$(x-3)^2 = x+2$

$x^2 - 6x + 9 = x + 2$

$x^2 - 7x + 7 = 0$

$$\frac{7 \pm \sqrt{49 - (4 \times 1 \times 7)}}{2}$$

$\frac{7 \pm \sqrt{21}}{2}$ So only $\frac{7+\sqrt{21}}{2}$ as $\frac{7-\sqrt{21}}{2} < 3$!!

$x = \frac{7+\sqrt{21}}{2}$

d) $\frac{7+\sqrt{21}}{2}$

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4. (a) Write $5\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants, $R > 0$ and $0 \leqslant \alpha < \dfrac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

(b) Show that the equation

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

can be rewritten in the form

$$5\cos 2x - 2\sin 2x = c$$

where c is a positive constant to be determined.

(2)

(c) Hence or otherwise, solve, for $0 \leqslant x < \pi$,

$$5\cot 2x - 3\operatorname{cosec} 2x = 2$$

giving your answers to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

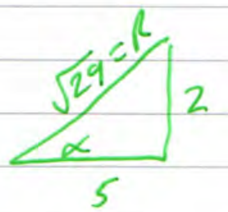
(4)

a) $5\cos\theta - 2\sin\theta \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$

$R\cos\alpha = 5$

$R\sin\alpha = 2$

$\tan\alpha = 2/5$



$R = \sqrt{29}$

$\alpha = 0.380506377 \text{ rads} = 0.381 \text{ rads}$

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Question 4 continued

b) $5\cot(2x) - 3\operatorname{cosec}(2x) = 2$

$\frac{5\cos(2x)}{\sin(2x)} - \frac{3}{\sin(2x)} = 2$

$\frac{5\cos(2x) - 3}{\sin(2x)} = 2$

$5\cos(2x) - 3 = 2\sin(2x)$

$5\cos(2x) - 2\sin(2x) = 3$

$c = 3$

c) $5\cos(2x) - 2\sin(2x) = 3$

~~$5\cos^2(x) - 5\sin^2(x) - 2(2\sin(x)\cos(x)) = 3$~~

~~$5(1-\sin^2(x)) - 5\sin^2(x) - 4\sin(x)\cos(x) = 3$~~

~~$5 - 5\sin^2(x) - 5\sin^2(x) - 4\sin(x)\cos(x) = 3$~~

~~$5 - 10\sin^2(x) - 4\sin(x)\cos(x) = 3$~~

~~$0 = 10\sin^2(x) + 4\sin(x)\cos(x) - 2$~~

$\sqrt{29}\cos(2x + 0.381) = 3$

$\cos(2x + 0.381) = \frac{3}{\sqrt{29}}$

$2x + 0.381... = 56.1454851 9°, 303.8545148°, 416.1454852°, 663.38545148°$

$x = 2.46$ rads and 0.30 rads

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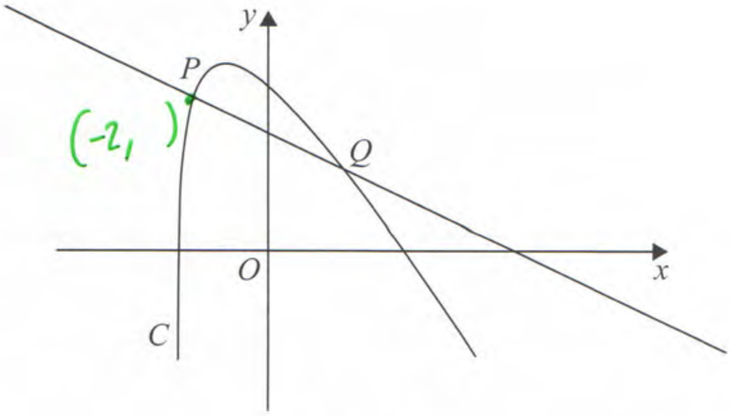

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = 2\ln(2x + 5) - \frac{3x}{2}, \qquad x > -2.5$$

The point P with x coordinate -2 lies on C.

(a) Find an equation of the normal to C at P. Write your answer in the form $ax + by = c$, where a, b and c are integers.

(5)

The normal to C at P cuts the curve again at the point Q, as shown in Figure 2.

(b) Show that the x coordinate of Q is a solution of the equation

$$x = \frac{20}{11}\ln(2x + 5) - 2$$

(3)

The iteration formula

$$x_{n+1} = \frac{20}{11}\ln(2x_n + 5) - 2$$

can be used to find an approximation for the x coordinate of Q.

(c) Taking $x_1 = 2$, find the values of x_2 and x_3, giving each answer to 4 decimal places.

(2)

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Question 5 continued

a) $\frac{dy}{dx} = \frac{4}{2x+5} - \frac{3}{2}$

at $x=-2$ $\frac{dy}{dx} = \frac{4}{1} - \frac{3}{2} = 2\frac{1}{2} = \frac{5}{2}$

When $x=-2, y = 2\ln(1) - -\frac{6}{2} = 3$ $(-2, 3)$

So $y = mx + c$

$3 = \frac{-2}{5}x - 2 + c$

$c = 2.2 = 2\frac{1}{5} = \frac{11}{5}$

$y = \frac{-2}{5}x + \frac{11}{5}$

$5y = -2x + 11$

$5y + 2x = 11$

b) $y = \frac{11-2x}{5}$ and $y = 2\ln(2x+5) - \frac{3x}{2}$

So $\frac{11-2x}{5} = 2\ln(2x+5) - \frac{3x}{2}$

$11 - 2x = 10\ln(2x+5) - \frac{15x}{2}$

$22 - 4x = 20\ln(2x+5) - 15x$

$11x = 20\ln(2x+5) - 22$

$x = \frac{20}{11}\ln(2x+5) - 2$

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Question 5 continued

c) $x_1 = 2$

$x_2 = \frac{20}{11}\ln(9) - 2 = 1.994953777 = \cancel{1.994}\ 1.9950$

$x_3 = \frac{20}{11}\ln(8.989907554) - 2 = 1.992913755 = 1.9929$

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6. Given that a and b are positive constants,

(a) on separate diagrams, sketch the graph with equation

(i) $y = |2x - a|$

(ii) $y = |2x - a| + b$

Show, on each sketch, the coordinates of each point at which the graph crosses or meets the axes.

(4)

Given that the equation

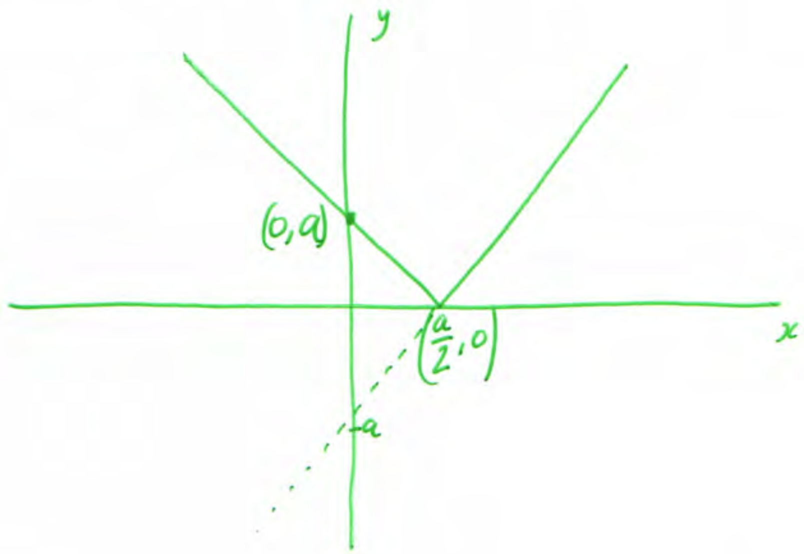
$$|2x - a| + b = \frac{3}{2}x + 8$$

has a solution at $x = 0$ and a solution at $x = c$,

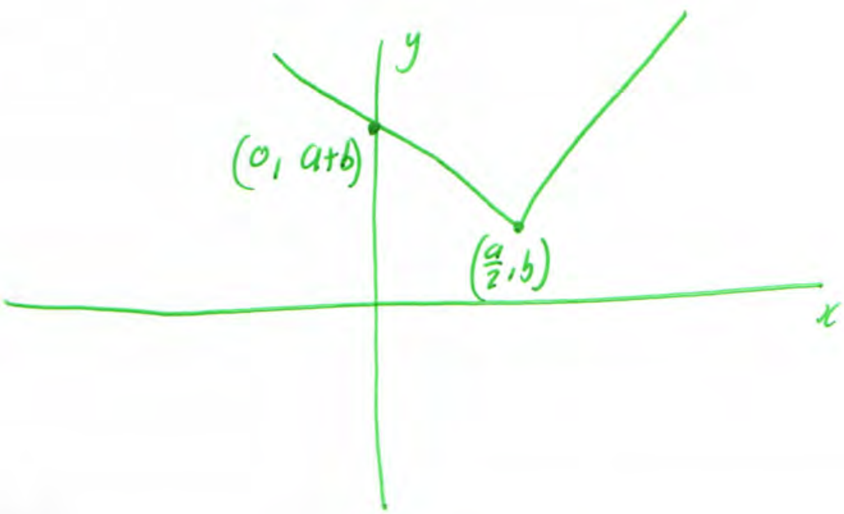
(b) find c in terms of a.

(4)

a) i)



ii)



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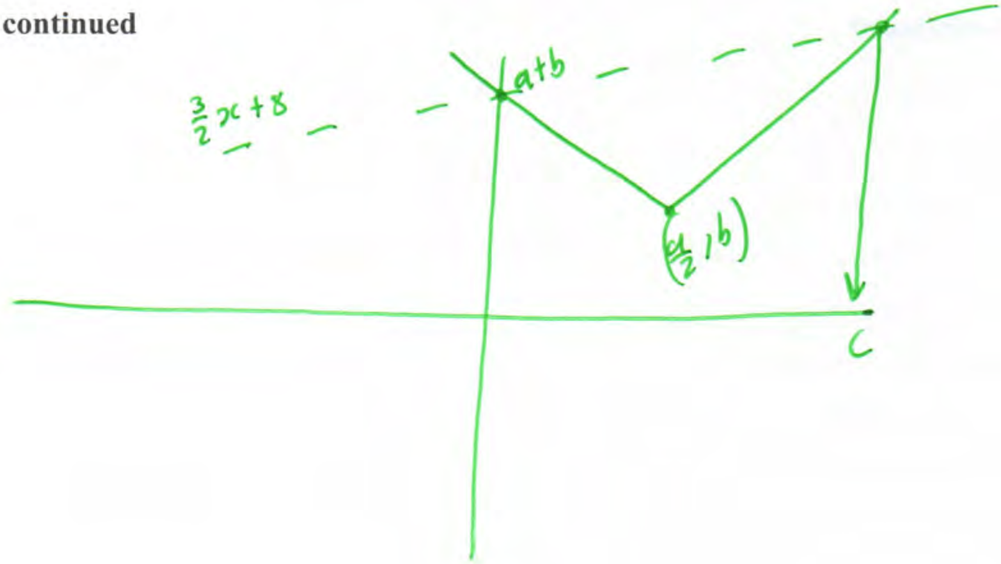
Question 6 continued

b)



$2x - a + b = \frac{3}{2}x + 8$ AND $-2x + a + b = \frac{3}{2}x + 8$

$2c - a + b = \frac{3}{2}c + 8$

$\frac{1}{2}c - a + b = 8$

$c - 2a + 2b = 16$

$c = 2a - 2b + 16$

$c = 2(8-b) - 2b + 16$

$c = 16 - 2b - 2b + 16$

$c = 32 - 4b$

$c = 32 - 4(8-a)$

$c = 32 - 32 + 4a$ So $c = 4a$

but when $x = 0$

$a + b = 8$

$a = 8 - b$

OR

$b = 8 - a$

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7. (i) Given $y = 2x(x^2 - 1)^5$, show that

(a) $\dfrac{dy}{dx} = g(x)(x^2 - 1)^4$ where $g(x)$ is a function to be determined. **(4)**

(b) Hence find the set of values of x for which $\dfrac{dy}{dx} \geqslant 0$ **(2)**

(ii) Given

$$x = \ln(\sec 2y), \qquad 0 < y < \frac{\pi}{4}$$

find $\dfrac{dy}{dx}$ as a function of x in its simplest form. **(4)**

i) a) $u = 2x$ $\quad du = 2$

$v = (x^2-1)^5$ $\quad dv = 5 \times 2x \times (x^2-1)^4 = 10x(x^2-1)^4$

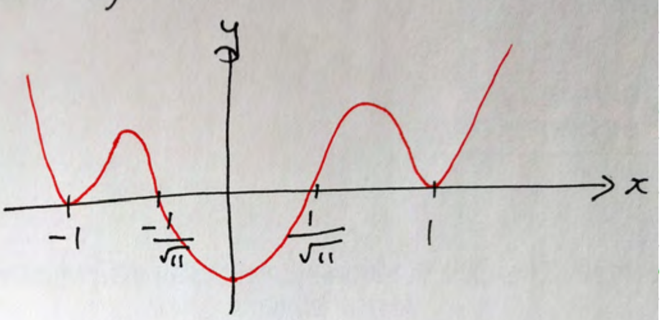
$$\frac{dy}{dx} = 20x^2(x^2-1)^4 + 2(x^2-1)^5$$

$$= (x^2-1)^4[20x^2 + 2(x^2-1)]$$

$$= \cancel{2}(x^2-1)^4[20x^2 + 2x^2 - 2]$$

$$= \cancel{2x}(x^2-1)^4[22x^2 - 2]$$

b) $(x^2-1)^4(22x^2-2) \geqslant 0$



$\{x \leqslant -\frac{1}{\sqrt{11}}\} \cup \{x \geqslant \frac{1}{\sqrt{11}}\}$

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Question 7 continued

ii)

let $u = \sec(2y)$ $\quad \frac{du}{dy} = \sec 2y \tan 2y \times 2$

$= 2\sec 2y \tan 2y$

$x = \ln(u)$ $\quad \frac{dx}{du} = \frac{1}{u} \times 2\sec 2y \tan 2y$

$\frac{dx}{dy} = \frac{2\sec 2y \tan 2y}{\sec 2y} = \frac{2\tan 2y}{\cancel{\ }}$

$\frac{dy}{dx} = \frac{1}{2\tan(2y)}$

If $x = \ln(\sec 2y)$

$e^{x} = \sec 2y$

$e^{2x} = \sec^2 2y$

$e^{2x} = \tan^2 2y + 1$

So $\frac{dy}{dx} = \frac{1}{2\times\sqrt{e^{2x}-1}}$

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8.

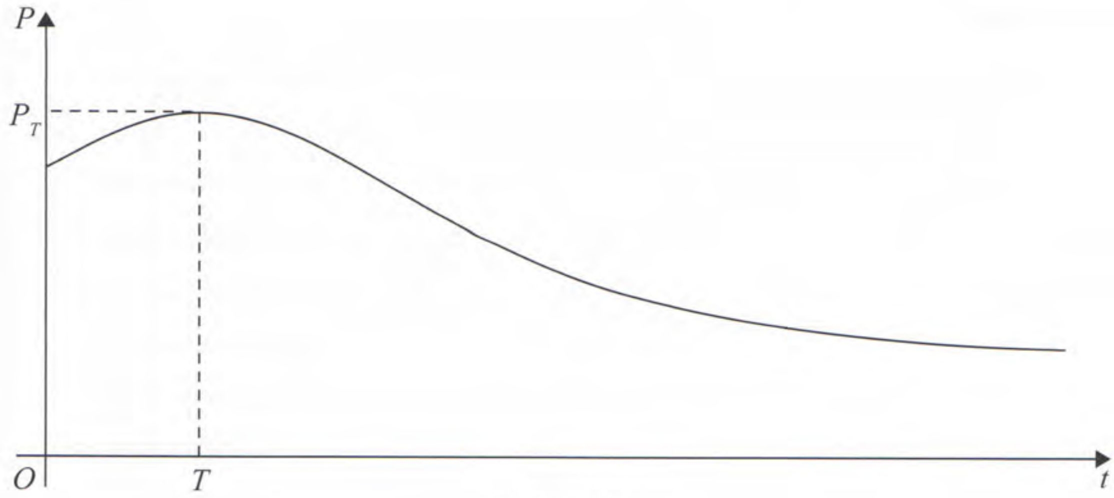

Figure 3

The number of rabbits on an island is modelled by the equation

$$P = \frac{100e^{-0.1t}}{1 + 3e^{-0.9t}} + 40, \qquad t \in \mathbb{R}, t \geqslant 0$$

where P is the number of rabbits, t years after they were introduced onto the island.

A sketch of the graph of P against t is shown in Figure 3.

(a) Calculate the number of rabbits that were introduced onto the island.

(1)

(b) Find $\frac{dP}{dt}$

(3)

The number of rabbits initially increases, reaching a maximum value P_T when $t = T$

(c) Using your answer from part (b), calculate

(i) the value of T to 2 decimal places,

(ii) the value of P_T to the nearest integer.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

For $t > T$, the number of rabbits decreases, as shown in Figure 3, but never falls below k, where k is a positive constant.

(d) Use the model to state the maximum value of k.

(1)

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Question 8 continued

a) at $t=0$ $P = \frac{100e^0}{1+3e^0} + 40 = 65$

b) $\frac{dP}{dt} =$

$u = 100e^{-0.1t}$ $du = -10e^{-0.1t}$

$v = 1+3e^{-0.9t}$ $dv = -2.7e^{-0.9t}$

$$\frac{-10e^{-0.1t} + -30e^{-1t} - -270e^{-t}}{(1+3e^{-0.9t})^2} = \frac{dP}{dt}$$

$$\frac{-10e^{-0.1t} + 240e^{-t}}{(1+3e^{-0.9t})^2} = \frac{dP}{dt}$$

c) $0 = -10e^{-0.1t} + 240e^{-t}$

i) $\cancel{10e^{-0.1t} = 240e^{-t}}$

$0 = e^{-t}(-10e^{0.9t} + 240)$

ii) $P = 102.4439939$
$= 102.$

$e^{-t} = 0$ or $-10e^{0.9t} + 240 = 0$

$\Downarrow$

No solutions

$t = \frac{\ln(24)}{0.9} = 3.531170923 = 3.53$

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Question 8 continued

d) As $t \to \infty$, $P \to 40$

$k = 40$

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9. (a) Prove that

$$\sin 2x - \tan x \equiv \tan x \cos 2x, \qquad x \neq (2n+1)90°, \qquad n \in \mathbb{Z}$$

(4)

(b) Given that $x \neq 90°$ and $x \neq 270°$, solve, for $0 \leqslant x < 360°$,

$$\sin 2x - \tan x = 3\tan x \sin x$$

Give your answers in degrees to one decimal place where appropriate.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

a) $\sin 2x - \tan x$

$2\sin x\cos x - \dfrac{\sin x}{\cos x}$

$\dfrac{2\sin(x)\cos^2(x) - \sin(x)}{\cos(x)}$

$\dfrac{\sin(x)[2\cos^2(x) - 1]}{\cos(x)}$

$\cos(2x) = \cos^2(x) - \sin^2(x)$

$\cos(2x) = \cos^2(x) - 1 + \cos^2(x)$

$\underbrace{\cos(2x)} = \underbrace{2\cos^2(x) - 1}$

$\dfrac{\sin(x)[\cos(2x)]}{\cos(x)}$

$\tan(x)\cos(2x)$

b) $\tan(x)\cos(2x) = 3\tan(x)\sin(x)$

$\tan(x)[\cos(2x) - 3\sin(x)] = 0$

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Question 9 continued

$\tan(x)=0 \quad \therefore x = 0°, 180°,$

$\cos(2x) - 3\sin(x) = 0$

$\cos^2(x) - \sin^2(x) - 3\sin(x) = 0$

$1 - 2\sin^2(x) - 3\sin(x) = 0$

$2\sin^2(x) + 3\sin(x) - 1 = 0$

$\sin(x) = \dfrac{-3 \pm \sqrt{9 - 4 \times 2 \times -1}}{4}$

$\sin(x) = 0.2807764064 \qquad x = 16.3°, 163.7°$

$\sin(x) = -1.780776406 \qquad x =$ No solution.

So $x = 0°, 180°, 16.3°, 163.7°$