# Mark Scheme (Results)

October 2022

Pearson Edexcel International Advanced Level
In Pure Mathematics P3 (WMA13) Paper 01

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October 2022
Question Paper Log Number P71379A
Publications Code WMA13_01_MS_2210

## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

**PEARSON EDEXCEL IAL MATHEMATICS**

**General Instructions for Marking**

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

'M' marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC – special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp – decimal places
- sf – significant figures
- ∗ The answer is printed on the paper
- ⎣ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Pure Mathematics Marking

*(But note that specific mark schemes may sometimes override these general principles)*

**Method mark for solving 3 term quadratic:**

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c,\quad q \neq 0$, leading to $x = \ldots$

**Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

**Use of a formula**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

**Method mark** for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

**Exact answers**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

**Answers without working**

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

| Question | Scheme | Marks |
|---|---|---|
| **1(a)** | $2x^3-4x-15=(Ax+B)(x^2+3x+4)+C(2x+3)\Rightarrow A=...$ or $x^2+3x+4\overline{)2x^3+0x-4x-15}$ with quotient $2x...$, $2x^3+...$ | **M1** |
| | $A=2$ | **A1** |
| | $B=-6, C=3$ | **A1A1** |
| | | **(4)** |
| **(b)** | $\int f(x)\,dx = "A"\frac{x^2}{2}+"B"x+"C"\ln\left(x^2+3x+4\right)$ | **M1** **A1ft** |
| | $\int f(x)\,dx = x^2-6x+3\ln\left(x^2+3x+4\right)$ | |
| | $\int_3^5 f(x)\,dx=\left(5^2-6\times5+3\ln\left(5^2+3\times5+4\right)\right)-\left(3^2-6\times3+3\ln\left(3^2+3\times3+4\right)\right)=..$ | **dM1** |
| | $=-5+3\ln 44+9-3\ln 22=4+3\ln\left(\frac{44}{22}\right)$ | **M1** |
| | $=4+\ln 8$ | **A1** |
| | | **(5)** |
| | | **(9 marks)** |

**Notes:**

**(a)**

**M1:** A correct method leading to at least one of the constants. May attempt long division getting the term $A$, or multiply through and compare coefficients or substitute values. Note that achieving $A = 2$ implies this mark.

**A1:** $A=2$ May be seen in the long division or in the expression.

**A1:** Either $B=-6$ or $C=3$ Stated or as part of the expression

**A1:** Both $B=-6$ and $C=3$ Stated or as part of the expression

**(b) Note: use of letters *A*, *B* and *C* can score max 3 marks M1A1ftdM0M1A0.**

**M1:** Attempts to integrate their answer to (a), look for $x^n\to x^{n+1}$ at least once and

$$\frac{2x+3}{x^2+3x+4}\to k\ln(x^2+3x+4)$$ Allow with letters used if values have not been found.

**A1ft:** Correct integration following through their $A$, $B$ and $C$ (or with letters).

**dM1:** Applies the limits to their integral and subtracts. *Must have numerical values for this mark..*

**M1:** Uses correct log laws to combine the log terms into a single log term. (Need not have used the power law at this stage, unless earlier error leads to mismatch of coefficients.) Allow if letters used, so $3C\ln(44/22)$ can score this mark.

**A1cao:** Correct answer in the form specified.

| Question | Scheme | Marks |
|---|---|---|
| 2(a) | Either $\mathrm{f}(x) < 5$ or $\mathrm{f}(x) \geqslant 3$ | M1 |
| | $3 \leqslant \mathrm{f}(x) < 5$ | A1 |
| | | (2) |
| (b) | (i) $y = 5 - \dfrac{4}{3x+2} \Rightarrow \dfrac{4}{3x+2} = 5 - y \Rightarrow \dfrac{4}{5-y} = 3x+2 \Rightarrow x = \ldots$ | M1 |
| | $\left(\mathrm{f}^{-1}(x)\right) = \dfrac{1}{3}\left(\dfrac{4}{5-x} - 2\right)$ oe such as $\dfrac{4}{15-3x} - \dfrac{2}{3}$ or $\dfrac{2x-6}{15-3x}$ | A1 |
| | (ii) Domain is $3 \leqslant x < 5$ | B1ft |
| | | (3) |
| (c) | $\mathrm{fg}(-\pi) = \mathrm{f}\left(\left\lvert 4\sin\left(-\dfrac{\pi}{3} + \dfrac{\pi}{6}\right)\right\rvert\right) = \mathrm{f}\left(\left\lvert 4\sin\left(-\dfrac{\pi}{6}\right)\right\rvert\right) = \mathrm{f}(2) = \ldots$ | M1 |
| | $= 5 - \dfrac{4}{6+2} = \dfrac{9}{2}$ | A1 |
| | | (2) |
| | | **(7 marks)** |

**Notes:**

**(a)**
**M1:** One correct end of range, though allow with $\leqslant$ or $<$ and $\geqslant$ or $>$ (in correct direction) at the respective ends, and accept with $y$ instead of f($x$), or even with $x$ for the M1.
**A1:** Correct range, allow with $y$ instead of f($x$) or with other correct set notation (but use of $x$ is A0 unless in formal set notation). Accept as two separate inequalities.

**(b)(i)**
**M1:** Attempts to make $x$ or a swapped $y$ the subject of the equation. Allow sign slips, but there should be a correct order of operations.
**A1:** Correct rule, must be in terms of $x$, accept equivalents and isw after a correct answer is seen. Do not be concerned with the lhs (accept $y = \ldots$ and even condone f($x$) =...)

**(b)(ii)**
**B1ft:** Follow through on their answer to (a). Accept intervals or set notation answers. Do not accept e.g. $3 \leqslant \mathrm{f}^{-1}(x) < 5$ or with $y$

**(c)**
**M1:** Attempts to evaluate g at $-\pi$ and substitutes into f. (Allow even if a negative value is found.) Evaluation of $\mathrm{g}(-\pi)$ must be attempted, not just substitution into the expression. They may find the expression for fg($x$) first and substituted, in which case the trigonometric expression must be evaluated before the mark is awarded.
**A1:** Correct answer. Accept as decimal. Dot not isw if they try to change to degrees.

| Question | Scheme | Marks |
|---|---|---|
| 3(a) | $f'(x) = 2(x-2)\times e^{3x} + (x-2)^2 \times 3e^{3x}$ | M1 A1 |
| | $f'(x) = 0 \Rightarrow e^{3x}(x-2)(2+3(x-2)) = 0 \Rightarrow x = ...$ | M1 |
| | | |
| | $\Rightarrow x = \frac{4}{3}$ | A1 |
| | $y = \frac{4}{9}e^4$ $\left(\text{so } A \text{ is } \left(\frac{4}{3}, \frac{4}{9}e^4\right)\right)$ | A1 |
| | | (5) |
| (b) | $k > 0$ or $k \leqslant$ their $\frac{4}{9}e^4$ or with … or < respectively. | M1 |
| | $0 < k \leqslant$ their $\frac{4}{9}e^4$ | A1ft |
| | | (2) |
| | | (7 marks) |

**Notes:**

**(a)**

**M1:** Differentiates using the product rule reaching the form $\alpha(x-2)\times e^{3x} + (x-2)^2 \times \beta e^{3x}$

**A1:** Correct derivative, need not be simplified.

**M1:** Sets their derivative equal to zero and attempts to cancel or factorise out the exponential term and the $(x-2)$ term (or multiple thereof) and solve for $x$. Alternatively, they may expand to a 3 term quadratic $(3x^2 - 10x + 8)$ and attempt to solve this via correct method. Allow recovery on slips in the exponent for this mark (and the A's if a correct quadratic expression is solved).

**A1:** Correct $x$ value.

**A1:** Correct simplified $y$ value. Allow if seen in part (b). Isw, after a correct answer, but the follow through in (b) will not apply on an incorrectly simplified answer if this mark has been awarded.

**(b)**

**M1:** Identifies one correct boundary for $k$ (the direction of inequality must be correct). Accept in terms of $y$ or f($x$) for this mark.

**A1ft:** Correct range for $k$ following their **positive** $y$ coordinate in (a). Must have correct inequalities. Accept awrt 24.3 for the upper bound. Allow as separate inequalities, or set or interval notation.

| Question | Scheme | Marks |
|---|---|---|
| 4(a) | $y=\log_{10}(2x+1)\Rightarrow 10^y=2x+1\Rightarrow x=...$ | M1 |
| | $\Rightarrow x=\dfrac{10^y-1}{2}$ | A1 |
| | | (2) |
| (b) | $\left(\dfrac{dx}{dy}=\right)\dfrac{1}{2}10^y\ln 10$ | M1 |
| | $\dfrac{dy}{dx}=1\Big/\dfrac{dx}{dy}=\dfrac{1}{\frac{1}{2}10^y\ln 10}$ | M1 |
| | $\dfrac{dy}{dx}=\dfrac{2}{(2x+1)\ln 10}$ | A1 |
| | | (3) |
| | | **(5 marks)** |

**Notes:**

**(a)**
**M1:** Correctly undoes the logarithm and attempts to rearrange to make $x$ the subject.
**A1:** Correct expression for $x$. Apply isw after a correct answer.
**(b)**
**M1:** Attempts to differentiate $10^y$. Accept $\alpha 10^y\to\beta 10^y\ln 10$ (ignore any extra terms). Do not be concerned about the lhs for this mark.
**M1:** Applies reciprocal $\dfrac{dy}{dx}=1\Big/\dfrac{dx}{dy}$. Variables must be consistent.
**A1:** Correct answer. Accept equivalents in terms of $x$, e.g. with $10^{\log(2x+1)}$ isw after a correct answer.

Alt for (b): Allow use of change of base formula.
**M1:** $y=\dfrac{\ln(2x+1)}{\ln 10}$ (effectively a B mark via this method)
**M1:** $\dfrac{dy}{dx}=\left(\dfrac{1}{\ln 10}\times\right)\dfrac{1}{2x+1}\times...$ Attempts to differentiate using the chain rule.
**A1:** Correct answer.

Note: Candidates who write $\dfrac{dy}{dx}=\dfrac{1}{2x+1}\times...\left(\text{ or }\dfrac{dy}{dx}=\dfrac{1}{10^y}\times...\right)$ with no ln 10 term can score M0M1A0 under the Alt.
Alt 2: Implicit Differentiation
**M1:** As main scheme, attempts to differentiate as part of their work, so $\alpha 10^y\to\beta 10^y\ln 10\dfrac{dy}{dx}$
**M1:** Makes $\dfrac{dy}{dx}$ the subject. **A1:** Correct answer.

| Question | Scheme | Marks |
|---|---|---|
| 5(a) | $P(1)=\frac{4-1}{10}+\frac{3}{4}\ln\left(\frac{2}{3^2}\right)\quad (=-0.828...)$ | M1 |
| | $P(1)=-0.828...$ (which is negative so a loss of £0.828 million,) so approximately £830 000 loss. | A1* |
| | | (2) |
| (b) | $P(6)=-0.08799\ldots$ and $P(7)=0.1975\ldots$ | M1 |
| | There is a sign change and hence as $P$ is continuous on [6,7], so the root for $t$ lies in [6,7]. | A1 |
| | | (2) |
| (c) | $P=0\Rightarrow\frac{4t-1}{10}=-\frac{3}{4}\ln\left(\frac{t+1}{(2t+1)^2}\right)\Rightarrow t=...$ | M1 |
| | $\Rightarrow t=\dfrac{10\left(-\frac{3}{4}\ln\left(\frac{t+1}{(2t+1)^2}\right)\right)+1}{4}=\frac{1}{4}+\frac{30}{16}\ln\left(\frac{t+1}{(2t+1)^2}\right)^{-1}$ <br> $\Rightarrow t=\frac{1}{4}+\frac{15}{8}\ln\left(\frac{(2t+1)^2}{t+1}\right)$ * | A1* |
| | | (2) |
| (d) | $t_2=\frac{1}{4}+\frac{15}{8}\ln\left(\frac{13^2}{7}\right)=...(=6.219978...)$ | M1 |
| | $t_2=\text{awrt }6.220$ | A1 |
| | $t_6=6.314$ | A1 |
| | | (3) |
| (e) | $"6.3..."\times 12=...$ months, or repeated iteration to root 6.31487 gives 75.7785… or allow $"0.314..."\times 12="3.768..."$ | M1 |
| | So it will take 76 months. (Accept 75 or awrt 76 months) | A1 |
| | | (2) |
| | | **(11 marks)** |

**Notes:**

**(a)**

**M1:** Attempts to substitute $t=1$ into the given formula. Allow if there is a slip but an attempt at substitution is seen, or allow for sight of awrt $-0.828...$

**A1*:** Correct value for $P(1)$ seen to at least 2.s.f. if substitution has been shown, or at least 3 s.f. if no substitution was shown, followed by suitable conclusion that it is a loss of approximately £830 000, though accept awrt £830 000. Must mention "loss" and include units (£ or pounds). Negative value given is A0.

**(b)**

**M1:** Attempts both $P(6)$ and $P(7)$ with at least one correct to 1 s.f. rounded or truncated. A tighter interval could be used but must contain the root (6.31487)

**A1:** Both correct to1 s.f. rounded or truncated with suitable conclusion made. Must mention sign change and continuity as well as conclusion about root in the interval.

**(c)**

**M1:** Attempts to isolate $\alpha t$ from the $\frac{4t-1}{10}$ after setting equal to zero.

**A1*:** Correct work to reach the given answer with no incorrect work seen and at least one intermediate step with either $\alpha t = ...$ reached or with the power law applied on the ln term (need not see the power explicitly used, but must have been applied correctly).

**(d)**

**M1:** Attempts to use the formula with $t_1$ =6. Accept with 6 embedded in formula followed by a value, or awrt 6.2 (or even 6.3) as implying the attempt.

**A1:** awrt 6.220 Accept 6.22.

**A1:** Correct and given to 3 d.p.

**(e)**

**M1:** Multiplies their final root from (d) by 12, or uses repeated iteration to narrow further then multiplies by 12. Allow the method if they multiply the fractional part only by 12 and get, e.g., 6 years 3 months.

**A1:** Accept 75 or 76 months, or anything that rounds to 76 months.

| Question | Scheme | Marks |
|---|---|---|
| 6 | $\frac{d}{dx}(\cos x+\sin x)=-\sin x+\cos x$ | B1 |
| | $\frac{dy}{dx}=\frac{(\cos x+\sin x)(3\cos x)-(-\sin x+\cos x)(2+3\sin x)}{(\cos x+\sin x)^2}$ | M1<br>A1 |
| | $=\frac{3\cos^2 x+\cancel{3\sin x\cos x}+2\sin x+3\sin^2 x-2\cos x-\cancel{3\sin x\cos x}}{\cos^2 x+2\sin x\cos x+\sin^2 x}$<br>$=\frac{3+2\sin x-2\cos x}{1+2\sin x\cos x}$ | M1 |
| | $=\frac{3+2\sin x-2\cos x}{1+2\sin x\cos x}\times\frac{\sec x}{\sec x}=\frac{3\sec x+2\sec x\sin x-2}{\sec x+2\sin x}$ | M1 |
| | $=\frac{2\tan x+3\sec x-2}{\sec x+2\sin x}$ | A1 |
| | | **(6)** |
| | | **(6 marks)** |

**Notes:**

**B1:** Correct differentiation of $\cos x+\sin x\to-\sin x+\cos x$ seen somewhere in the proof. This can be scored if seen in workings for quotient rule, or even in the denominator of an incorrect attempt at *u*'/*v*'.

**M1:** Differentiates using the quotient rule or product rule. For quotient rule look for

$$\frac{(\cos x+\sin x)(\pm a\cos x)-(\pm\sin x\pm\cos x)(2+3\sin x)}{(\cos x+\sin x)^2}$$

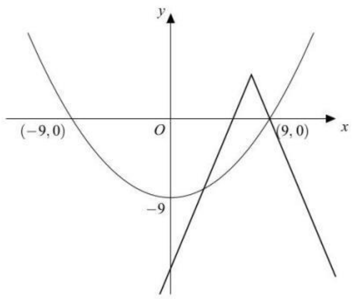
For product rule look for $(\cos x+\sin x)^{-1}(\pm a\cos x)\pm(\cos x+\sin x)^{-2}(\pm\cos x\pm\sin x)(2+3\sin x)$

**A1:** Fully correct derivative.

**M1:** Expands numerator or denominator and applies $\sin^2 x+\cos^2 x=1$ or other appropriate correct Pythagorean identity at least once in the proof.

**M1:** Attempts to multiply through by sec *x* in numerator and denominator (to achieve the 2sin *x* term). Not dependent and may be scored before the previous M.

**A1:** Correct answer. Terms may be in different order. Allow minor slips in notation (e.g. a single missing *x*) and recovery from missing brackets if the intent is clear, but A0 for persistent incorrect notation throughout (e.g no *x*'s in the trig terms).

| Question | Scheme | Marks |
|---|---|---|
| 7(a) | (i) $\left(\frac{22}{3}, 5\right)$ | B1 |
| | (ii) $(0, -17)$ | B1 |
| | | (2) |
| (b) | $5-(3x-22)=0 \Rightarrow x=...$ **and** $5+(3x-22)=0 \Rightarrow x=...$ | M1 |
| | $x=9$ and $x=\frac{17}{3}$ | A1 |
| | | (2) |
| (c) |  Correct U shape symmetric about $y$-axis with vertex on negative $y$-axis. | B1 |
| | Graphs meet $(9,0)$ with $(-9,0)$ also shown. | B1 |
| | Intercept at $(0,-9)$ stated or labelled. | B1 |
| | | (3) |
| (d) | Intersect at $(9,0)$ | B1 |
| | Or when $5+3x-22=\frac{1}{9}x^2-9$ | M1 |
| | $\Rightarrow (x-3)(x-24)=0 \Rightarrow x=...$ | dM1 |
| | Need smaller root $x=3 \Rightarrow y=...$ | dM1 |
| | $(3,-8)$ | A1 |
| | | (5) |
| | | **(12 marks)** |

**Notes:**

**(a)**

**Mark (a) and (b) as a whole**

**(i) B1:** Correct coordinates. May be listed as separate coordinates, $x = \ldots, y = \ldots$

**(ii) B1:** Correct $y$ intercept (and no other). Accept as coordinates, or stated as $y = \ldots$ (allow without the $x = 0$) or seen on graph as the $y$ intercept. However just −17 on its own is B0.

**(b)**

**M1:** Attempts to solve **both** equations, though allow sign slips when expanding the brackets.

**A1:** Both values correct.

**(c)**

**B1:** Correct U shape symmetric about $y$-axis with vertex on negative $y$-axis. Allow a little tolerance with shape, but the curve must not clearly bend back on itself, and the minimum should be clearly intended as on the $y$-axis.

**B1:** Graphs meet at $(9,0)$ both $x$ intercepts labelled or clearly stated (and no others). Shape need not be correct for this mark e.g. inverted parabola may be shown). Must pass through the same point as the modulus graph at (9,0).

**B1:** States or labels the $y$ intercept at $(0,-9)$ and intercept must be on the negative $y$-axis.

**(d)**

**B1:** Deduces one point of intersection is (9,0). Must be seen in (d), or clearly stated as the answer if parts are not labelled, do not accept just this marked on the diagram.

**M1:** Sets up equation for the intersection of the quadratic with the positive gradient line segment, accept $5+3x\pm 22=\frac{1}{9}x^2-9$

**dM1:** Solves their equation, any valid means. After the equation is seen you may see just the correct root, or 24, which implies the M. Alternatively, expanding and solving the quadratic by usual rules.

**dM1: Depends on first M mark.** Selects the correct (smaller) root of their quadratic and attempts to find the $y$ value (accept any $y$ value appearing after choosing the correct root as an attempt). The larger root must be rejected. Following a correct equation, if no working is shown for finding the root (see above) they must have achieved $x = 3$ and go on to find $y$ for this mark. From an incorrect equation/incorrect method to solve, if only one root is given it must correspond to the smaller root of their equation/method.

**A1:** $(3,-8)$ only

Alternative by squaring.

**B1:** As main scheme.

**M1:** Rearranges to $|3x-22|=a+bx^2\left(=14-\frac{1}{9}x^2\right)$ and squares both sides to reach a quartic equation. If correct it is $\frac{1}{81}x^4-\frac{109}{9}x^2+132x-288=0$

**dM1:** Solves the resulting quartic (any means). If no method is shown, a correct value from the correct equation will imply the mark.

**dM1: Depends on first M mark** Selects correct root (second of the four in ascending order) and finds the $y$ coordinate. If no method shown, but the quartic was correct, they must be using $x = 3$.

**A1:** As scheme.

| Question | Scheme | Marks |
|---|---|---|
| 8(a) | $R = 17$ | B1 |
| | $\tan\alpha = \frac{15}{8}$ | M1 |
| | $\alpha = 1.081$ | A1 |
| | | (3) |
| (b) | (i) Min f($x$) $= \frac{15}{41+2\times"17"}$ | M1 |
| | $= \frac{1}{5}$ | A1 |
| | (ii) Occurs when $\sin(x-1.081)=1 \Rightarrow x-1.081=\frac{\pi}{2} \Rightarrow x=...$ | M1 |
| | $x =$ awrt 2.65 | A1 |
| | | (4) |
| (c) | $-\frac{23}{5}$ (or −4.6) | B1ft |
| | | (1) |
| (d) | Awrt 1.33 | B1ft |
| | | (1) |
| | | **(9 marks)** |

**Notes:**

**(a)**

**B1:** For 17 only.

**M1:** Attempts an equation in $\alpha$. Accept $\tan\alpha = \pm\frac{15}{8}$ or $\tan\alpha = \pm\frac{8}{15}$. If using $R$ accept $\cos\alpha = \pm\frac{8}{"17"}$ or $\sin\alpha = \pm\frac{15}{"17"}$. Implied by a correct value for $\alpha$

**A1:** Awrt 1.081. Must be in radians.

**(b)(i)**

**M1:** Attempts to apply the result from (a) to find the minimum. Allow for $\frac{15}{41\pm 2\times"\text{their } R"}$

**A1:** cao.

**(ii)**

**M1:** Attempts to solve $x \pm "\alpha" = \frac{\pi}{2}$

**A1:** cao

**(c)**

**B1ft:** Correct answer or follow through $2\times\left(\text{their}\frac{1}{5}\right)-5$ and no other solutions.

**(d)**

**B1ft:** For awrt 1.33 or follow through $0.5\times\text{their } 2.65$ and no other solutions.

| Question | Scheme | Marks |
|---|---|---|
| 9(a) | $\dfrac{\cos^2\theta}{\cos 2\theta - \sin 3\theta} = \dfrac{\cos^2\theta}{\cos 2\theta - (\sin 2\theta\cos\theta + \cos 2\theta\sin\theta)}$ | M1 |
| | $= \dfrac{\cos^2\theta}{1-2\sin^2\theta - 2\sin\theta\cos^2\theta - \sin\theta(1-2\sin^2\theta)}$ | M1 |
| | $= \dfrac{1-\sin^2\theta}{1-3\sin\theta-2\sin^2\theta+4\sin^3\theta} = \dfrac{(1-\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1-2\sin\theta-4\sin^2\theta)}$ | M1 |
| | $= \dfrac{1+\sin\theta}{1-2\sin\theta-4\sin^2\theta}$ * | A1* |
| | | (4) |
| (b) | $\dfrac{1+\sin\theta}{1-2\sin\theta-4\sin^2\theta} = 2\operatorname{cosec}\theta \Rightarrow (1+\sin\theta)\sin\theta = 2(1-2\sin\theta-4\sin^2\theta)$ | M1 |
| | $\Rightarrow 9\sin^2\theta + 5\sin\theta - 2 = 0$ | A1 |
| | $\Rightarrow \sin\theta = \dfrac{-5\pm\sqrt{25-4\times9\times-2}}{18} = \ldots \Rightarrow \theta = \sin^{-1}\ldots$ | M1 |
| | Two of $\theta = 15.6°, 164.4°, 235.6°, 304.4°$ | A1 |
| | All of $\theta = 15.6°, 164.4°, 235.6°, 304.4°$ | A1 |
| | | (5) |
| | | **(9 marks)** |

**Notes:**

**(a)**
**M1:** Applies the compound angle formula to $\sin 3\theta$. Accept $\pm\sin\theta\cos 2\theta \pm \sin 2\theta\cos\theta$ Allow for students who apply $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ directly. May be seen anywhere (e.g. in separate working).
**M1:** Uses correct double angle formula for $\cos 2\theta$ **and** $\sin 2\theta$ (or assumed used correctly if correct $\sin 3\theta$ formula was used) to achieve all terms in single angle. (May be scored after the third M.)
**M1:** Uses Pythagorean identity to identify the factor of $1-\sin\theta$ in numerator. They need not have achieved single angle arguments for all terms for this mark. (May be scored before the second M.) There must be an intermediate step before the given answer with $1-\sin\theta$ cancelled if the factor is not explicitly seen in the numerator.
**A1:** Cancels the $1-\sin\theta$ from numerator and denominator and simplifies to the given result with no incorrect steps or consistently incorrect notation.
**(b)**
**M1:** Substitutes the result from (a) and cross multiplies to get an equation in $\sin\theta$ only.
**A1:** Correct simplified quadratic in $\sin\theta$
**M1:** Solves the resulting quadratic by formula, completing square or calculator (allow by factorisation if their quadratic factorises) and applies inverse sine to at least one root. May be implied by any correct arcsin of one of their solutions, such as −55.6°. My be implied by a radians answer (e.g. 0.272, 2.869…)
**A1:** Any two correct solutions in the range, accept awrt. Answers in radians scores A0.
**A1:** Awrt all four solutions correct and no others in the range.

| Question | Scheme | Marks |
|---|---|---|
| 9(a) | $\dfrac{\cos^2\theta}{\cos 2\theta - \sin 3\theta} = \dfrac{\cos^2\theta}{\cos 2\theta - (\sin 2\theta\cos\theta + \cos 2\theta\sin\theta)}$ | M1 |

| | | |
|---|---|---|
| **Example Alt** | $=\dfrac{1-\sin^2\theta}{\cos 2\theta(1-\sin\theta)-2\sin\theta(1-\sin^2\theta)}$<br>$=\dfrac{(1-\sin\theta)(1+\sin\theta)}{(1-\sin\theta)\cos 2\theta-2\sin\theta(1+\sin\theta)(1-\sin\theta)}$ | **3rd M1** |
| | $=\dfrac{(1-\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1-2\sin^2\theta-2\sin\theta-2\sin^2\theta)}$ | **2nd M1** |
| | $=\dfrac{1+\sin\theta}{1-2\sin\theta-4\sin^2\theta}$ * | **A1*** |
| | | **(4)** |
| **Notes**<br>The above approach can be marked via the main scheme but scores the 3rd M before the 2nd M.<br>Alternative approaches will hopefully fit the scheme similarly. Other approaches can be similarly applied as each of the three method marks will generally be required. E.g. see below (allowing first M for students who apply $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ directly)<br>Note in Alt 2 below it is more likely the 1st M will be gained by aside work to expand sin3$\theta$ with substitution later seen into the equation. | | |
| **9(a) Alt2 In reverse** | $\dfrac{1+\sin\theta}{1-2\sin\theta-4\sin^2\theta}\times\dfrac{1-\sin\theta}{1-\sin\theta}=\dfrac{1-\sin^2\theta}{1-3\sin\theta-2\sin^2\theta+4\sin^3\theta}$<br>$=\dfrac{\cos^2\theta}{1-3\sin\theta-2\sin^2\theta+4\sin^3\theta}$ | **3rd M1** |
| | $=\dfrac{\cos^2\theta}{1-2\sin^2\theta-3\sin\theta+4\sin^3\theta}=\dfrac{\cos^2\theta}{\cos 2\theta-(3\sin\theta-4\sin^3\theta)}$ | **2nd M1** |
| | $=\dfrac{\cos^2\theta}{\cos 2\theta-\sin\theta(3-4\sin^2\theta)}=\dfrac{\cos^2\theta}{\cos 2\theta-\sin\theta(1+2(1-\sin^2\theta)-2\sin^2\theta)}$<br>$=\dfrac{\cos^2\theta}{\cos 2\theta-\sin\theta(1+2\cos^2\theta-(1-\cos 2\theta))}$<br>$=\dfrac{\cos^2\theta}{\cos 2\theta-(2\sin\theta\cos^2\theta+\sin\theta\cos 2\theta)}$<br>$=\dfrac{\cos^2\theta}{\cos 2\theta-(\sin 2\theta\cos\theta+\cos 2\theta\sin\theta)}$ | **1st M1** |
| | $=\dfrac{\cos^2\theta}{\cos 2\theta-\sin 3\theta}$ * | **A1*** |
| | | **(4)** |

| | | |
|---|---|---|
| **9(a) Alt3 By cross multiplying** | $\dfrac{\cos^2\theta}{\cos 2\theta-\sin 3\theta}=\dfrac{1+\sin\theta}{1-2\sin\theta-4\sin^2\theta}\Leftrightarrow$<br>$\cos^2\theta\left(1-2\sin\theta-4\sin^2\theta\right)=\left(1+\sin\theta\right)\left(\cos 2\theta-\sin 3\theta\right)\Leftrightarrow$<br>$\cos^2\theta\left(1-2\sin\theta-4\sin^2\theta\right)=\left(1+\sin\theta\right)\left(\cos 2\theta-\underline{\sin 2\theta\cos\theta-\cos 2\theta\sin\theta}\right)$ | **M1** |
| | $\Leftrightarrow\cos^2\theta\left(1-2\sin\theta-4\sin^2\theta\right)$<br>$=\left(1+\sin\theta\right)\left(\underline{1-2\sin^2\theta-2\sin\theta\cos^2\theta-\left(1-2\sin^2\theta\right)\sin\theta}\right)$ | **M1** |
| | $\Leftrightarrow\cos^2\theta-\cancel{2\sin\theta\cos^2\theta}-4\sin^2\theta\cos^2\theta=1-2\sin^2\theta-\cancel{2\sin\theta\cos^2\theta}$<br>$-\cancel{\sin\theta}+\cancel{2\sin^3\theta}+\cancel{\sin\theta}\cancel{-2\sin^3\theta}-2\sin^2\theta\cos^2\theta-\sin^2\theta+2\sin^4\theta$<br>$\Leftrightarrow\cos^2\theta-2\sin^2\theta\cos^2\theta=1-3\sin^2\theta+2\sin^2\theta\underline{\left(1-\cos^2\theta\right)}$ | **M1** |
| | $\Leftrightarrow\cos^2\theta-\cancel{2\sin^2\theta\cos^2\theta}=1-3\sin^2\theta+2\sin^2\theta-\cancel{2\sin^2\theta\cos^2\theta}$<br>$\Leftrightarrow\cos^2\theta=1-\sin^2\theta$<br>which is true, hence given result is true.* | **A1*** |
| | | **(4)** |

**Notes:**

**(a)**

**M1:** Applies the compound angle formula to $\sin 3\theta$ at some stage in the working. Accept $\pm\sin\theta\cos 2\theta\pm\sin 2\theta\cos\theta$ Allow for students who apply $\sin 3\theta=3\sin\theta-4\sin^3\theta$ directly. May be seen anywhere (e.g. in separate working).

**M1:** Uses correct double angle formula for $\cos 2\theta$ **and** $\sin 2\theta$ (or $\sin 3\theta$) to achieve all terms in single angle. (May be before or after cross multiplying.)

**M1:** Cross multiplies and uses Pythagorean identity somewhere in the working in an attempt to achieve a true statement by reaching the same expression on both sides, or reducing to a known true trigonometric identity (as shown in scheme). Award at the stage the identity is applied to an appropriate term in an expression where the angles have been reduced to just θ.

**A1:** Reaches a correct true statement with no errors seen **and** makes conclusion that the original result is true.

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