

Leave
blank1. The curve C has equation

$$y = \frac{3x-2}{(x-2)^2}, \quad x \neq 2$$

The point P on C has x coordinate 3Find an equation of the normal to C at the point P in the form $ax + by + c = 0$, where a , b and c are integers.

(6)

$$\text{When } x=3, \quad y = \frac{3(3)-2}{(3-2)^2} = 7$$

$$\frac{dy}{dx} = \frac{(x-2)^2 \cdot 3 - 2(x-2)(3x-2)}{(x-2)^4}$$

$$\text{When } x=3, \quad \frac{dy}{dx} = \frac{(3-2)^2 \cdot 3 - 2(3-2)(9-2)}{(3-2)^4}$$

$$= -11$$

$$\therefore m = \frac{1}{11}$$

$$y - y_1 = m(x - x_1)$$

$$y - 7 = \frac{1}{11}(x - 3)$$

$$11y - 77 = x - 3$$

$$x - 11y + 74 = 0$$

Leave
blank2. Solve, for $0 \leq \theta < 2\pi$,

$$2\cos 2\theta = 5 - 13\sin \theta$$

Give your answers in radians to 3 decimal places.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

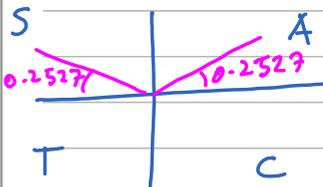
$$2(1 - 2\sin^2 \theta) = 5 - 13\sin \theta \quad (5)$$

$$2 - 4\sin^2 \theta = 5 - 13\sin \theta$$

$$4\sin^2 \theta - 13\sin \theta + 3 = 0$$

$$(4\sin \theta - 1)(\sin \theta - 3) = 0$$

$$\sin \theta = \frac{1}{4} \quad \text{or} \quad \sin \theta = 3$$



No soln.

$$\theta = 0.253, 2.889 \quad (3 \text{ d.p.})$$

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3. The function g is defined by

$$g : x \mapsto |8 - 2x|, \quad x \in \mathbb{R}, \quad x \geq 0$$

(a) Sketch the graph with equation $y = g(x)$, showing the coordinates of the points where the graph cuts or meets the axes. (3)

(b) Solve the equation

$$|8 - 2x| = x + 5$$

(3)

The function f is defined by

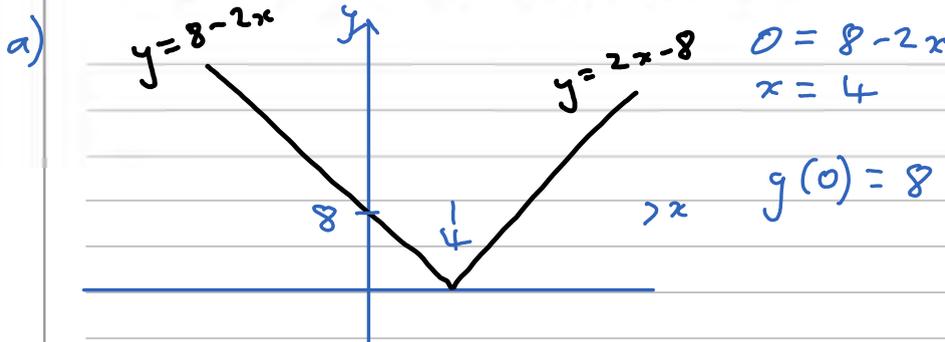
$$f : x \mapsto x^2 - 3x + 1, \quad x \in \mathbb{R}, \quad 0 \leq x \leq 4$$

(c) Find $fg(5)$.

(2)

(d) Find the range of f . You must make your method clear.

(4)



b)

$$8 - 2x = x + 5 \quad \text{OR} \quad -(8 - 2x) = x + 5$$

$$3 = 3x \quad \quad \quad x = 13$$

$$x = 1$$

c)

$$f(x) = x^2 - 3x + 1, \quad g(x) = |8 - 2x|$$

$$fg(5) = f[|8 - 2(5)|] = f(2)$$

$$= (2)^2 - 3(2) + 1$$

$$= -1$$

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Question 3 continued

d)

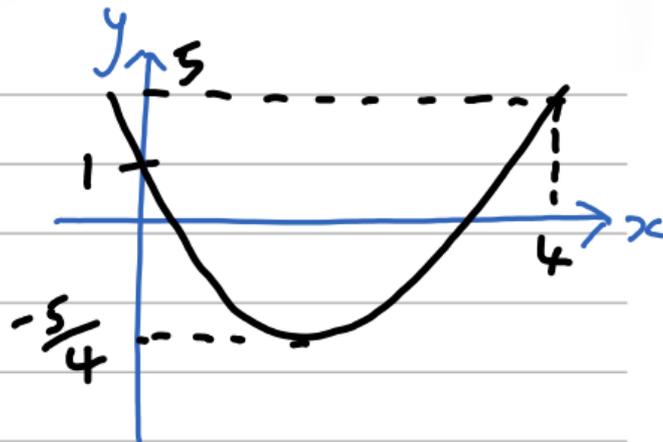
$$f(x) = x^2 - 3x + 1$$

$$= \left(x - \frac{3}{2}\right)^2 - \frac{5}{4}$$

$$f(0) = (0)^2 - 3(0) + 1 = 1$$

$$f(4) = (4)^2 - 3(4) + 1 = 5$$

$$\Rightarrow -\frac{5}{4} \leq f(x) \leq 5$$



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4. Use the substitution $x = 2 \sin \theta$ to find the exact value of

$$\int_0^{\sqrt{3}} \frac{1}{(4-x^2)^{3/2}} dx$$

(7)

$$x = 2 \sin \theta$$

$$dx = 2 \cos \theta d\theta$$

When $x = 0$, $0 = 2 \sin \theta$
 $\theta = 0$

$$x = \sqrt{3}, \sqrt{3} = 2 \sin \theta$$

$$\theta = \pi/3$$

$$\int_0^{\sqrt{3}} \frac{1}{(4-x^2)^{3/2}} dx = \int_0^{\pi/3} \frac{2 \cos \theta d\theta}{(4-4\sin^2 \theta)^{3/2}}$$

$$= \int_0^{\pi/3} \frac{2 \cos \theta}{(4)^{3/2} (1-\sin^2 \theta)^{3/2}} d\theta$$

$$= \int_0^{\pi/3} \frac{2 \cos \theta}{8 \cos^3 \theta} d\theta$$

$$= \int_0^{\pi/3} \frac{1}{4 \cos^2 \theta} d\theta$$

$$= \int_0^{\pi/3} \frac{1}{4} \sec^2 \theta d\theta$$

$$= \left[\frac{1}{4} \tan \theta \right]_0^{\pi/3}$$

$$= \frac{1}{4} (\sqrt{3} - 0)$$

$$= \frac{\sqrt{3}}{4}$$

5. (a) Use the binomial expansion, in ascending powers of x , of $\frac{1}{\sqrt{1-2x}}$ to show that

$$\frac{2+3x}{\sqrt{1-2x}} \approx 2 + 5x + 6x^2, \quad |x| < 0.5 \quad (4)$$

- (b) Substitute $x = \frac{1}{20}$ into

$$\frac{2+3x}{\sqrt{1-2x}} = 2 + 5x + 6x^2$$

to obtain an approximation to $\sqrt{10}$

Give your answer as a fraction in its simplest form. (3)

a)
$$\frac{1}{\sqrt{1-2x}} = (1-2x)^{-1/2}$$

$$\approx 1 + \left(-\frac{1}{2}\right)(-2x) + \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(-2x)^2 \cdot \frac{1}{2}$$

$$= 1 + x + \frac{3x^2}{2}$$

$$\frac{2+3x}{\sqrt{1-2x}} \approx (2+3x)\left(1+x+\frac{3x^2}{2}\right)$$

$$= 2 + 5x + 6x^2$$

b)
$$\frac{2+3\left(\frac{1}{20}\right)}{\sqrt{1-2\left(\frac{1}{20}\right)}} = 2 + 5\left(\frac{1}{20}\right) + 6\left(\frac{1}{20}\right)^2$$

$$\frac{43}{20} \times \sqrt{\frac{10}{9}} = 2 + \frac{5}{20} + \frac{3}{200}$$

$$\frac{43}{60} \sqrt{10} = \frac{453}{200}$$

$$\sqrt{10} = \frac{1359}{430}$$

6. (i) Given $x = \tan^2 4y$, $0 < y < \frac{\pi}{8}$, find $\frac{dy}{dx}$ as a function of x .

Write your answer in the form $\frac{1}{A(x^p + x^q)}$, where A , p and q are constants to be found.

(5)

- (ii) The volume V of a cube is increasing at a constant rate of $2 \text{ cm}^3 \text{ s}^{-1}$. Find the rate at which the length of the edge of the cube is increasing when the volume of the cube is 64 cm^3 .

(5)

i) $x = \tan^2 4y$

$$\begin{aligned} \frac{dx}{dy} &= 2 \tan 4y (\sec^2 4y) \cdot 4 \\ &= 8 \tan 4y (1 + \tan^2 4y) \\ &= 8(\tan 4y + \tan^3 4y) \\ &= 8(x^{1/2} + x^{3/2}) \end{aligned}$$

$$\frac{dy}{dx} = \frac{1}{8(x^{1/2} + x^{3/2})}$$

ii) $V = x^3$, $\frac{dV}{dt} = 2 \text{ cm}^3 \text{ s}^{-1}$, find $\frac{dx}{dt}$

$$\frac{dV}{dx} = 3x^2$$

$$\frac{dx}{dt} = \frac{dx}{dV} \cdot \frac{dV}{dt} = \frac{2}{3x^2}$$

When $V = 64$, $x = 4$

$$\frac{dx}{dt} = \frac{2}{3(4)^2} = \frac{1}{24} \text{ cm}^3 \text{ s}^{-1}$$

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7. (a) Given that

$$2 \cos(x + 30)^\circ = \sin(x - 30)^\circ$$

without using a calculator, show that

$$\tan x^\circ = 3\sqrt{3} - 4$$

(5)

(b) Hence or otherwise solve, for $0 \leq \theta < 180$,

$$2 \cos(2\theta + 40)^\circ = \sin(2\theta - 20)^\circ$$

Give your answers to one decimal place.

(4)

a) $2 \cos(x+30) = \sin(x-30)$

$$2 \cos x \cos 30 - 2 \sin x \sin 30 = \sin x \cos 30 - \cos x \sin 30$$

$$\sqrt{3} \cos x - \sin x = \frac{\sqrt{3}}{2} \sin x - \frac{1}{2} \cos x$$

$$2\sqrt{3} - 2 \tan x = \sqrt{3} \tan x - 1$$

$$(\sqrt{3} + 2) \tan x = 2\sqrt{3} + 1$$

$$\tan x = \frac{2\sqrt{3} + 1}{\sqrt{3} + 2}$$

$$= \frac{(2\sqrt{3} + 1)(\sqrt{3} - 2)}{3 - 4}$$

$$= \frac{6 - 4\sqrt{3} + \sqrt{3} - 2}{-1}$$

$$= 3\sqrt{3} - 4$$

b) $x + 30 = 2\theta + 40$

$$x = 2\theta + 10$$

$$\theta = \frac{x - 10}{2}$$

$$0 \leq \theta < 180$$

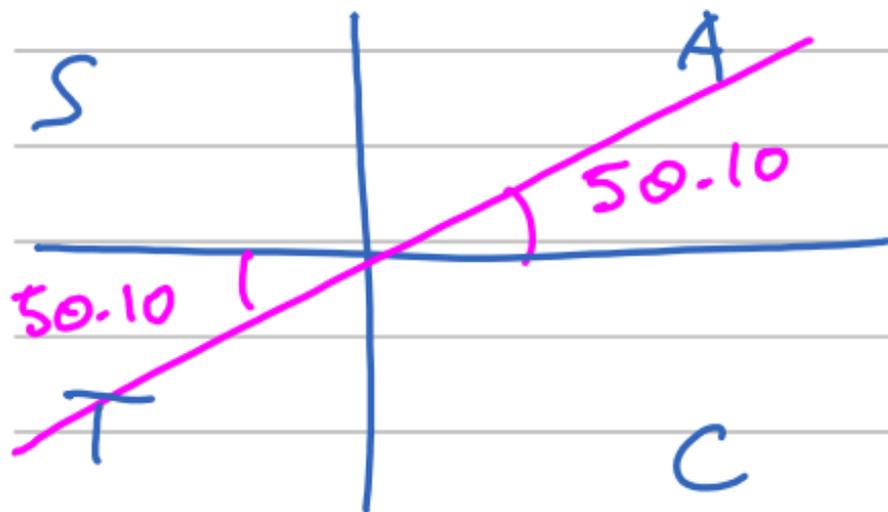
$$10 \leq x < 370$$

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Question 7 continued

$$\tan x = 3\sqrt{3} - 4$$

$$x = 50.1, 230.1$$



$$\theta = \frac{x - 10}{2}$$

$$= 20.1, 110.1$$

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8.

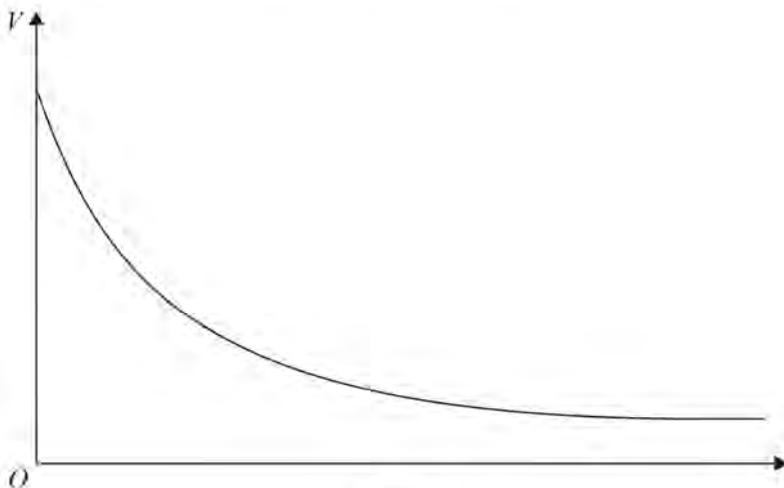


Figure 1

The value of Lin's car is modelled by the formula

$$V = 18000e^{-0.2t} + 4000e^{-0.1t} + 1000, \quad t \geq 0$$

where the value of the car is V pounds when the age of the car is t years.

A sketch of t against V is shown in Figure 1.

(a) State the range of V .

(2)

According to this model,

(b) find the rate at which the value of the car is decreasing when $t = 10$
Give your answer in pounds per year.

(3)

(c) Calculate the exact value of t when $V = 15000$

(4)

$$a) \text{ At } t=0, \quad V = 18000 + 4000 + 1000 = 23000$$

$$\text{As } t \rightarrow \infty, \quad V \rightarrow 1000$$

$$\therefore 1000 < V < 23000$$

$$b) \left. \frac{dV}{dt} \right|_{t=10} = -0.2(18000)e^{-0.2(10)} - 0.1(4000)e^{-0.1(10)}$$

$$= -£634/\text{year}$$

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Question 8 continued

$$c) \quad 15000 = 18000e^{-0.2t} + 4000e^{-0.1t} + 1000$$

$$0 = 18(e^{-0.1t})^2 + 4e^{-0.1t} - 14$$

$$= 9(e^{-0.1t})^2 + 2(e^{-0.1t}) - 7$$

$$= (9e^{-0.1t} - 7)(e^{-0.1t} + 1)$$

$$e^{-0.1t} = \frac{7}{9} \quad \text{or} \quad e^{-0.1t} = -1$$

No soln.

$$-0.1t = \ln\left(\frac{7}{9}\right)$$

$$t = 10 \ln\left(\frac{9}{7}\right)$$

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9.

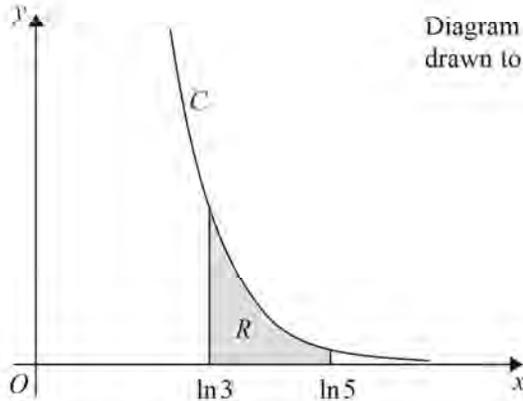


Diagram not drawn to scale

Figure 2

The curve C has parametric equations

$$x = \ln(t + 2), \quad y = \frac{4}{t^2} \quad t > 0$$

The finite region R , shown shaded in Figure 2, is bounded by the curve C , the x -axis and the lines with equations $x = \ln 3$ and $x = \ln 5$

(a) Show that the area of R is given by the integral

$$\int_1^3 \frac{4}{t^2(t+2)} dt \tag{3}$$

(b) Hence find an exact value for the area of R .

Write your answer in the form $(a + \ln b)$, where a and b are rational numbers. (7)

(c) Find a cartesian equation of the curve C in the form $y = f(x)$. (2)

a) When $x = \ln 3$, $\ln 3 = \ln(t+2)$
 $t = 1$

$x = \ln 5$, $\ln 5 = \ln(t+2)$
 $t = 3$

$$\frac{dx}{dt} = \frac{1}{t+2}$$

$$R = \int_{\ln 3}^{\ln 5} y dx = \int_1^3 y \frac{dx}{dt} dt = \int_1^3 \frac{4}{t^2} \cdot \frac{1}{t+2} dt$$

Question 9 continued

b)

$$\frac{4}{t^2(t+2)} = \frac{A}{t} + \frac{B}{t^2} + \frac{C}{t+2}$$

$$At(t+2) + B(t+2) + Ct^2 = 4$$

$$t = -2: 4C = 4 \quad t = 0: 2B = 4$$

$$C = 1 \quad B = 2$$

$$t = 1: 1(3)A + 3B + C = 4$$

$$A = \frac{4 - 3(2) - 1}{3} = -1$$

$$\therefore R = \int_1^3 \left(\frac{1}{t+2} + \frac{2}{t^2} - \frac{1}{t} \right) dt$$

$$= \left[\ln|t+2| - \frac{2}{t} - \ln|t| \right]_1^3$$

$$= \ln 5 - \frac{2}{3} - \ln 3 - \ln 3 + 2 + \ln 1$$

$$= \frac{4}{3} + \ln\left(\frac{5}{9}\right)$$

$$c) \quad x = \ln(t+2), \quad y = \frac{4}{t^2}$$

$$e^x - 2 = t$$

$$\Rightarrow y = \frac{4}{(e^x - 2)^2}$$

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10.

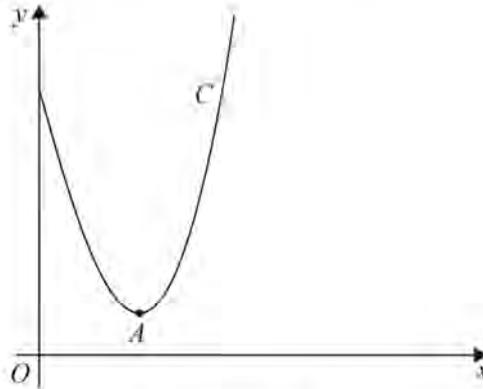


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 4, \quad x > 0$$

Point A is the minimum turning point on the curve.

(a) Show, by using calculus, that the x coordinate of point A is a solution of

$$x = \frac{6}{1 + \ln(x^2)} \tag{5}$$

(b) Starting with $x_0 = 2.27$, use the iteration

$$x_{n+1} = \frac{6}{1 + \ln(x_n^2)}$$

to calculate the values of x_1 , x_2 and x_3 , giving your answers to 3 decimal places. (3)

(c) Use your answer to part (b) to deduce the coordinates of point A to one decimal place. (2)

a)
$$y = \frac{x^2 \ln x}{3} - 2x + 4$$

$$\frac{dy}{dx} = \frac{x^2}{3} \cdot \frac{1}{x} + \frac{2x}{3} \cdot \ln x - 2 = 0$$

$$x + 2x \ln x = 6$$

$$x(1 + 2 \ln x) = 6$$

$$x = \frac{6}{1 + \ln(x^2)}$$

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Question 10 continued

$$b) \quad x_{n+1} = \frac{6}{1 + \ln(x_n^2)}, \quad x_0 = 2.27$$

$$x_1 = \frac{6}{1 + \ln(2.27^2)} = 2.273$$

$$x_2 = \frac{6}{1 + \ln(2.273^2)} = 2.271$$

$$x_3 = \frac{6}{1 + \ln(2.271^2)} = 2.273$$

c) When $x = 2.27$,

$$y = \frac{(2.27)^2 \ln(2.27)}{3} - 2(2.27) + 4$$

$$= 0.87$$

$$\therefore A(2.3, 0.9)$$

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11. With respect to a fixed origin O the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 14 \\ -6 \\ -13 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} \quad l_2: \mathbf{r} = \begin{pmatrix} p \\ -7 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 1 \end{pmatrix}$$

where λ and μ are scalar parameters and p and q are constants.

Given that l_1 and l_2 are perpendicular,

(a) show that $q = 3$

(2)

Given further that l_1 and l_2 intersect at point X ,

find

(b) the value of p ,

(5)

(c) the coordinates of X .

(2)

The point A lies on l_1 and has position vector $\begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$

Given that point B also lies on l_1 and that $AB = 2AX$

(d) find the two possible position vectors of B .

(3)

a)
$$\begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} q \\ 2 \\ 1 \end{pmatrix} = -2q + 2 + 4 = 0$$

$$q = \frac{-6}{-2} = 3$$

b) At X , $14 - 2\lambda = p + 3\mu$ (1)

$$-6 + \lambda = -7 + 2\mu$$
 (2)
$$-13 + 4\lambda = 4 + \mu$$
 (3)

$$2 \times (3) - (2): -35 + 7\lambda = 0$$

$$\lambda = 5$$

$$1 \times (3): \mu = 4(5) - 17 = 3 \quad 1 \times (1): p = 14 - 2(5) + 3(3) = 12$$

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Question 11 continued

$$c) \text{ When } \lambda = 5, \underline{r} = \begin{pmatrix} 14 - 2(5) \\ -6 + 5 \\ -13 + 4(5) \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ 7 \end{pmatrix}$$

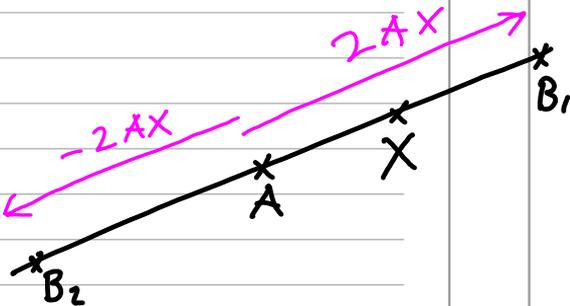
$$\therefore X(4, -1, 7)$$

$$d) A(6, -2, 3), AB = 2AX$$

$$\vec{AX} = \begin{pmatrix} 4 \\ -1 \\ 7 \end{pmatrix} - \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

$$\underline{r}_{B_1} = \underline{r}_A + 2\vec{AX} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} + 2 \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 11 \end{pmatrix}$$

$$\underline{r}_{B_2} = \underline{r}_A - 2\vec{AX} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} - 2 \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 10 \\ -4 \\ -5 \end{pmatrix}$$



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12.

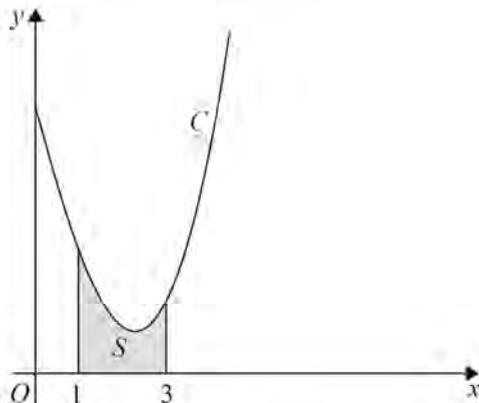


Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 4, \quad x > 0$$

The finite region S , shown shaded in Figure 4, is bounded by the curve C , the x -axis and the lines with equations $x = 1$ and $x = 3$

(a) Complete the table below with the value of y corresponding to $x = 2$. Give your answer to 4 decimal places.

x	1	1.5	2	2.5	3
y	2	1.3041	0.9242	0.9089	1.2958

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of S , giving your answer to 3 decimal places.

(3)

(c) Use calculus to find the exact area of S .

Give your answer in the form $\frac{a}{b} + \ln c$, where a , b and c are integers.

(6)

(d) Hence calculate the percentage error in using your answer to part (b) to estimate the area of S . Give your answer to one decimal place.

(2)

(e) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S .

(1)

Question 12 continued

a) When $x=2$, $y = \frac{(2)^2 \ln 2}{3} - 2(2) + 4$

$$= 0.9242 \text{ (4 dp)}$$

b) $S = \frac{1}{2}(0.5) [2 + 1.2958 + 2(1.3041 + 0.9242 + 0.9089)]$

$$= 2.393$$

c) $S = \int_1^3 \left(\frac{x^2 \ln x}{3} - 2x + 4 \right) dx$ Let $u = \ln x$ $v' = x^2$
 $u' = \frac{1}{x}$ $v = \frac{x^3}{3}$

$$S = \frac{1}{3} \left[\frac{x^3 \ln x}{3} \right]_1^3 - \frac{1}{3} \int_1^3 \frac{x^2}{3} dx + \left[-x^2 + 4x \right]_1^3$$

$$= \frac{3^3 \ln 3}{9} - \frac{(1) \ln(1)}{9} - \left[\frac{x^3}{27} \right]_1^3 - 3^2 + 4(3) + (1)^2 - 4(1)$$

$$= 3 \ln 3 - 1 + \frac{1}{27}$$

$$= \frac{-26}{27} + \ln 27$$

d) $\text{Error} = \frac{\ln 27 - \frac{26}{27} - 2.393}{\ln 27 - \frac{26}{27}} = 2.6\%$

e) Use smaller intervals between x -values

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13. (a) Express $10\cos\theta - 3\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$

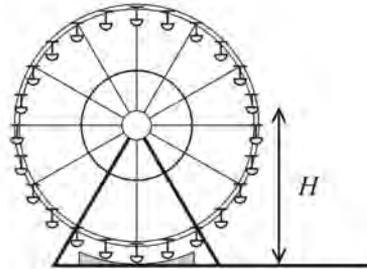
Give the exact value of R and give the value of α to 2 decimal places.

(3)

Alana models the height above the ground of a passenger on a Ferris wheel by the equation

$$H = 12 - 10\cos(30t)^\circ + 3\sin(30t)^\circ$$

where the height of the passenger above the ground is H metres at time t minutes after the wheel starts turning.



(b) Calculate

(i) the maximum value of H predicted by this model,

(ii) the value of t when this maximum first occurs.

Give each answer to 2 decimal places.

(4)

(c) Calculate the value of t when the passenger is 18m above the ground for the first time.

Give your answer to 2 decimal places.

(4)

(d) Determine the time taken for the Ferris wheel to complete two revolutions.

(2)

a) $R\cos(\theta + \alpha) \equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$

$$\equiv 10\cos\theta - 3\sin\theta$$

$$\Rightarrow R\sin\alpha = 3$$

$$R\cos\alpha = 10$$

$$R = \sqrt{(-3)^2 + 10^2} = \sqrt{109}$$

$$\tan\alpha = \frac{3}{10}$$

$$\alpha = 16.70^\circ$$

$$\therefore 10\cos\theta - 3\sin\theta \equiv \sqrt{109}\cos(\theta + 16.70)$$

Leave
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Question 13 continued

$$b) H = 12 - 10 \cos(30t) + 3 \sin(30t)$$

$$= 12 - \sqrt{109} \underbrace{\cos(30t + 16.70)}_{\text{min. } -1}$$

$$i) H_{\max} = 12 - \sqrt{109}(-1) = 22.44 \text{ m}$$

$$ii) \text{ For } H_{\max}, \cos(30t + 16.70) = -1$$

$$30t + 16.70 = 180$$

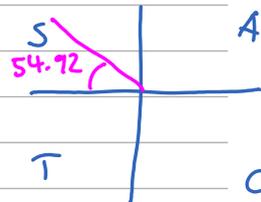
$$t = 5.44 \text{ min}$$

$$c) 18 = 12 - \sqrt{109} \cos(30t + 16.70)$$

$$\cos(30t + 16.70) = \frac{-6}{\sqrt{109}}$$

$$30t + 16.70 = 125.08$$

$$t = 3.61 \text{ min}$$



$$d) \text{ Period} = \frac{360}{30} = 12 \text{ min.}$$

\therefore It takes 24 mins to complete two revolutions