

C34 October 2017 (MA)

Q1a) $f(x) = 0$: $x^5 + x^3 - 12x^2 - 8 = 0$

$$x^3 = 12x^2 + 8 - x^5$$

$$x^3 + x^5 = 4(3x^2 + 2)$$

$$x^3(1 + x^2) = 4(3x^2 + 2)$$

$$\therefore x^3 = \frac{4(3x^2 + 2)}{x^2 + 1}$$

$$\text{hence } x = \sqrt[3]{\frac{4(3x^2 + 2)}{x^2 + 1}}$$

b) $x_1 = \sqrt[3]{\frac{4(3(2)^2 + 2)}{(2)^2 + 1}} = \boxed{2.237}$

Similarly, $x_2 = \boxed{2.246}$

$$x_3 = \boxed{2.247}$$

c) $f(2.2465) = -0.0057 \dots$ } change of sign
 $f(2.2475) = +0.083 \dots$ } between $x = 2.2465$
 and $x = 2.2475$ //

hence $\alpha = 2.247$ to 3 dp

$$(Q2a) \quad \frac{d}{dx} (y^3 + x^2y - 6x) = 0$$

$$\Rightarrow 3y^2 \frac{dy}{dx} + x^2 \frac{dy}{dx} + 2xy - 6 = 0$$

$$\Rightarrow \frac{dy}{dx} (x^2 + 3y^2) = 6 - 2xy$$

$$\Rightarrow \frac{dy}{dx} = \boxed{\frac{6 - 2xy}{x^2 + 3y^2}}$$

$$b) \quad \frac{dy}{dx} = 0 : \quad \frac{6 - 2xy}{x^2 + 3y^2} = 0$$

$$\therefore 6 - 2xy = 0$$

$$xy = 3$$

$$\therefore y = \frac{3}{x} //$$

Sub into eqn for C: $\left(\frac{3}{x}\right)^3 + x^2\left(\frac{3}{x}\right) - 6x = 0$

$$\frac{27}{x^3} + 3x - 6x = 0$$

$$3x = \frac{27}{x^3}$$

$$\therefore x^4 = 9 // \therefore x = \pm\sqrt{3} //$$

$$\text{if } x = \sqrt{3}, y = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\text{if } x = -\sqrt{3}, y = -\frac{3}{\sqrt{3}} = -\sqrt{3}$$

$$\therefore \text{ points are: } (\sqrt{3}, \sqrt{3}) \text{ and } (-\sqrt{3}, -\sqrt{3})$$

$$(Q3a) \quad t=0: N = 3500 (1.035)^0 = \boxed{3500}$$

$$b) \quad 10000 = 3500 (1.035)^t$$

$$\frac{10000}{3500} = \frac{20}{7} = 1.035^t$$

$$\ln\left[\frac{20}{7}\right] = \ln[(1.035)^t]$$

$$\therefore t \ln(1.035) = \ln\left(\frac{20}{7}\right)$$

$$\Rightarrow t = \frac{\ln\frac{20}{7}}{\ln 1.035} = 30.517 \text{ hours}$$

$$= \boxed{30 \text{ hours } 31 \text{ min}}$$

$$c) \quad \frac{dN}{dt} = 3500 (1.035)^t \ln(1.035)$$

$$\text{at } t = 8: \frac{dN}{dt} = 3500 (1.035)^8 (\ln 1.035)$$

$$= \boxed{159} \text{ 3 s.f.}$$

Q4a) $\frac{1 - \cos 2x}{\sin 2x} = \tan x$

$$\text{LHS} = \frac{1 - (1 - 2\sin^2 x)}{2\sin x \cos x} = \frac{2\sin^2 x}{2\sin x \cos x} = \frac{\sin x}{\cos x} = \tan x //$$

b) $3\sec^2 \theta - 7 = \tan \theta$

$$\left(\begin{array}{l} \rightarrow \\ 1 + \tan^2 \theta = \sec^2 \theta \end{array} \right) 3(1 + \tan^2 \theta) - 7 = \tan \theta$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\Rightarrow 3 + 3\tan^2 \theta - 7 - \tan \theta = 0$$

$$\Rightarrow 3\tan^2 \theta - \tan \theta - 4 = 0$$

$$\Rightarrow (3\tan \theta - 4)(\tan \theta + 1) = 0$$

$$3\tan \theta - 4 = 0$$

$$\tan \theta = \frac{4}{3} //$$

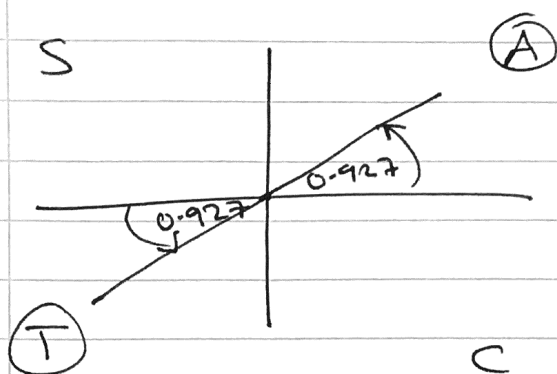
$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 0.927^\circ //$$

$$\tan \theta + 1 = 0$$

$$\tan \theta = -1 //$$

$$\theta = \tan^{-1}(-1) = -\frac{\pi}{4} //$$

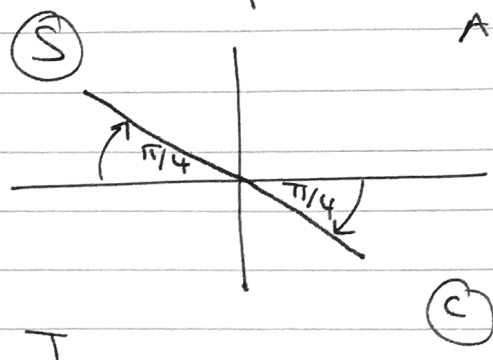
$$\theta = 0.927^\circ$$



$$\theta = 0.927, \pi + 0.927$$

$\hookrightarrow 4.069$

$$\theta = -\frac{\pi}{4}$$



$$\theta = \frac{3\pi}{4}, 2\pi - \frac{\pi}{4}$$

$$\theta = 0.927^\circ, 4.069^\circ, 2.356^\circ, 5.498^\circ$$

QSi) $\int ((3x+5)^9 + e^{5x}) dx$

$$= \frac{[3x+5]^{10}}{3 \times 10} + \frac{1}{5} e^{5x} + c$$

$$= \frac{1}{30} (3x+5)^{10} + \frac{e^{5x}}{5} + c$$

ii) $\int_2^b \left[\frac{x}{x^2+5} \right] dx = \left[\frac{1}{2} \ln(x^2+5) \right]_2^b$

By pattern!

$$\frac{d}{dx} (x^2+5) = 2x = 2 \times \text{numerator}$$

$$\therefore \left[\frac{1}{2} \ln(x^2 + 5) \right]_2^b = \ln \sqrt{6}$$

$$\frac{1}{2} \ln(b^2 + 5) - \frac{1}{2} \ln(9) = \ln \sqrt{6}$$

$$\frac{1}{2} \ln(b^2 + 5) - \ln 3 = \ln \sqrt{6}$$

$$\ln(b^2 + 5) = 2\ln 3 + 2\ln \sqrt{6}$$

$$\ln(b^2 + 5) = 2\ln(3\sqrt{6})$$

$$\therefore \ln(b^2 + 5) = \ln(54) //$$

$$\Rightarrow b^2 + 5 = 54$$

$$\Rightarrow b^2 = 49$$

$$\Rightarrow b = \pm 7 //$$

$$\text{but } b > 2 \text{ so } \boxed{b = 7}$$

(Q6a)

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	0.76679	a	0.15940	0

$$a = 2e^{-\frac{\pi}{2}} \sqrt{\sin \frac{\pi}{2}} = \boxed{0.41576}$$

$$b) \quad h = \frac{b-a}{n} = \frac{\pi - 0}{4} = \frac{\pi}{4} //$$

$$\therefore \text{Area} \approx \frac{1}{2} \times \frac{\pi}{4} \left[2(0.76679 + 0.41576 + 0.15940) \right]$$

$$\approx \boxed{1.0540}$$

$$c) \quad y = 2e^{-x} (\sin x)^{\frac{1}{2}} \quad \text{PRODUCT RULE}$$

$$\frac{dy}{dx} = -2e^{-x} (\sin x)^{\frac{1}{2}} + e^{-x} (\sin x)^{-\frac{1}{2}} (\cos x)$$

$$= \boxed{e^{-x} \left(\frac{\cos x}{\sqrt{\sin x}} - 2\sqrt{\sin x} \right)} \quad \text{o.e.}$$

$$d) \quad \text{at turning point, } \frac{dy}{dx} = 0 \dots$$

$$\Rightarrow e^{-x} \left(\frac{\cos x}{\sqrt{\sin x}} - 2\sqrt{\sin x} \right) = 0$$

$$\Rightarrow \frac{\cos x}{\sqrt{\sin x}} - 2\sqrt{\sin x} = 0$$

$$\Rightarrow \cos x - 2\sin x = 0$$

$$\div \cos x$$

$$\Rightarrow 1 - 2\tan x = 0$$

$$\Rightarrow \tan x = \frac{1}{2} \quad \therefore x = \tan^{-1}\left(\frac{1}{2}\right)$$

$$= \boxed{0.464}$$

$$\begin{aligned} \text{Q7a)} \quad (2-3x)^{-3} &= 2^{-3} \left(1 - \frac{3}{2}x\right)^{-3} \\ &= \frac{1}{8} \left(1 - \frac{3}{2}x\right)^{-3} \quad \left[\begin{array}{l} x = -\frac{3}{2}x \\ n = -3 \end{array} \right] \end{aligned}$$

$$\frac{1}{8} \left(1 - \frac{3}{2}x\right)^{-3} \approx \frac{1}{8} \left[1 + \frac{9}{2}x + \frac{-3(-4)}{2} \left(-\frac{3}{2}x\right)^2 \right]$$

$$\approx \frac{1}{8} \left[1 + \frac{9}{2}x + \frac{27}{2}x^2 \right]$$

$$\approx \boxed{\frac{1}{8} + \frac{9}{16}x + \frac{27}{16}x^2}$$

$$\text{b)} \quad (4+4x) \left(\frac{1}{8} + \frac{9}{16}x + \frac{27}{16}x^2 \right) = \frac{1}{2} + Ax + \frac{81}{16}x^2$$

$$\begin{aligned} \frac{1}{2} + \frac{9}{4}x + \frac{27}{4}x^2 + \frac{4x}{8} + \frac{94x^2}{16} + \frac{274x^3}{16} \\ = \frac{1}{2} + Ax + \frac{81}{16}x^2 \end{aligned}$$

$$\Rightarrow \frac{1}{2} + \frac{9}{4}x + \frac{27}{4}x^2 + \frac{4x}{8} + \frac{94x^2}{16} = \frac{1}{2} + Ax + \frac{81}{16}x^2$$

Comparing Coefficients

$$\underline{x} : \quad \frac{9}{4} + \frac{4}{8} = A \quad \sim (1)$$

$$\underline{x^2} : \quad \frac{27}{4} + \frac{94}{16} = \frac{81}{16}$$

$$\therefore 4 = \frac{\frac{81}{16} - \frac{27}{4}}{\frac{9}{16}} = -3 //$$

c) so from ①, $A = \frac{9}{4} + \frac{(-3)}{8} = \boxed{\frac{15}{8}}$

(Q8) $\frac{2x^2-3}{x(x-1)} = A + \frac{B}{x} + \frac{C}{x-1}$ (partial fractions)

finding A: $x^2 - x + 0 \overline{) 2x^2 + 0x - 3}$
 $\underline{2x^2 - 2x + 0}$
 $0 + 2x - 3 \leftarrow \text{remainder}$

so $\frac{2x^2-3}{x(x-1)} = 2 + \frac{2x-3}{x(x-1)}$

$\frac{2x-3}{x(x-1)} = \frac{B}{x} + \frac{C}{x-1}$

$\therefore 2x-3 = B(x-1) + C(x)$

let $x=0$: $-3 = -B \therefore B=3$

$x=1$: $-1 = C$

so $\frac{2x-3}{x(x-1)} = \frac{-1}{x-1} + \frac{3}{x}$

$$\text{So } \frac{2x^2-3}{x(x-1)} = \frac{3}{x} - \frac{1}{x-1} + 2$$

$$\therefore \int_3^4 \left[\frac{2x^2-3}{x(x-1)} \right] dx = \int_3^4 \left[2 + \frac{3}{x} - \frac{1}{x-1} \right] dx$$

$$= \left[2x + 3 \ln|x| - \ln|x-1| \right]_3^4$$

$$= \left[8 + 3 \ln 4 - \ln 3 \right] - \left[6 + 3 \ln 3 - \ln 2 \right]$$

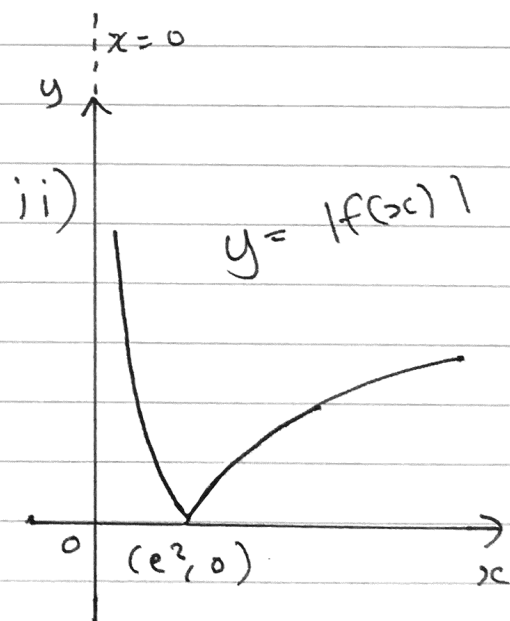
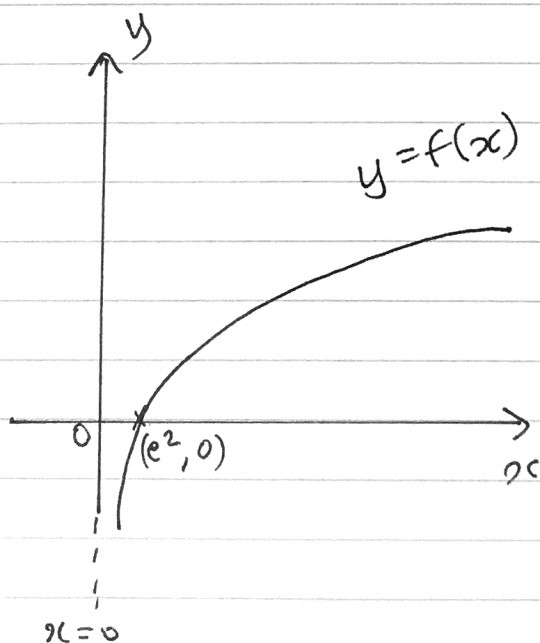
$$= 2 + 3 \ln 4 - 3 \ln 3 + \ln 2 - \ln 3$$

$$= 2 + 3 \ln 4 + \ln 2 - 4 \ln 3$$

$$= 2 + \ln(4^3) + \ln(2) - (\ln 3^4)$$

$$= \boxed{2 + \ln \frac{128}{81}}$$

Q9ai)



b) $|f(x)| = 4$

$$2\ln(x) - 4 = 4 \quad \text{and} \quad -2\ln(x) + 4 = 4$$

$$\ln(x) = 4$$

$$\boxed{e^4 = x}$$

$$\ln(x) = 0$$

$$e^0 = \boxed{x = 1}$$

c) $f(x) = 2\ln(x) - 4$

$$g(x) = e^{x+5} - 2$$

$$g(f(x)) = g[f(x)] = e^{2\ln(x)-4+5} - 2$$

$$= e^{2\ln(x)+1} - 2$$

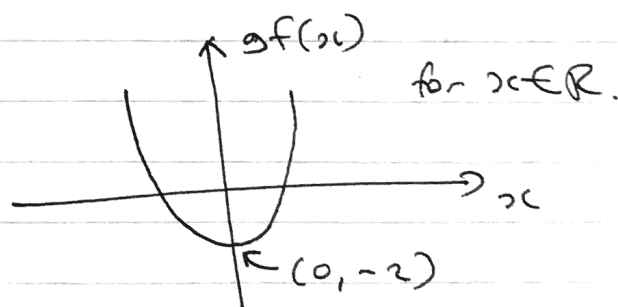
$$= e^{\ln(x^2)+1} - 2$$

$$= x^2 \times e^1 - 2$$

$$= \boxed{ex^2 - 2}$$

d) $\boxed{g(f(x)) > -2}$

note that the sign
is not " $\geq$ " as $f(x)$
is only defined for $x > 0$.



● Q10a) $y = 0 : t(t-4) = 0$

$$t > 0, \boxed{t=4} \therefore x = \frac{20(4)}{2(4)-1} = \boxed{\frac{80}{9}}$$

b) $x = 20t(2t+1)^{-1}$ PRODUCT RULE

$$\frac{dx}{dt} = 20(2t+1)^{-1} + 20t(-1)(2t+1)^{-2}(2)$$

$$\frac{dx}{dt} = \frac{20}{2t+1} - \frac{40t}{(2t+1)^2}$$

$$= \frac{20(2t+1) - 40t}{(2t+1)^2} = \frac{40t - 40t + 20}{(2t+1)^2}$$

$$= \frac{20}{(2t+1)^2} // \quad \text{and} \quad y = t^2 - 4t$$

$$\frac{dy}{dt} = 2t - 4 //$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t-4}{\frac{20}{(2t+1)^2}} = \frac{(2t-4)(2t+1)^2}{20} //$$

$$= \frac{2(t-2)(2t+1)^2}{20} = \boxed{\frac{(t-2)(2t+1)^2}{10}}$$

$$c) \quad x = \frac{20t}{2t+1}$$

$$2tx + x = 20t$$

$$t(2x - 20) + x = 0$$

$$t = \frac{-x}{2x-20} = \boxed{\frac{x}{20-2x} = t}$$

$$ii) \quad y = t(t-4)$$

$$y = \left(\frac{x}{20-2x} \right) \left(\frac{x}{20-2x} - 4 \right)$$

$$y = \left(\frac{x}{20-2x} \right) \left(\frac{x}{20-2x} - \frac{4(20-2x)}{20-2x} \right)$$

$$y = \frac{x}{20-2x} \left(\frac{x-80+8x}{20-2x} \right)$$

$$y = \frac{x(9x-80)}{(20-2x)^2}$$

denominator $\neq 0$. (anything $\div$ by 0 is undefined)
 $x > 0 \quad \therefore 20-2x > 0$

$$20-2x > 0$$

$$x < 10 //$$

$$\therefore \boxed{u = 10}$$

● Q11a)

$$\int \left[\frac{1}{5 - \sqrt{h}} \right] dh$$

$$u = 5 - h^{\frac{1}{2}}$$

$$\frac{du}{dh} = -\frac{1}{2} h^{-\frac{1}{2}}$$

$$dh = -2\sqrt{h} du$$

$$\Rightarrow \int \left[\frac{1}{u} \times -2\sqrt{h} \right] du$$

$$\Rightarrow -2 \int \left[\frac{\sqrt{h}}{u} \right] du = -2 \int \left[\frac{5-u}{u} \right] du$$

$$\Rightarrow -2 \int \left[\frac{5}{u} - 1 \right] du = -2 \left[5 \ln|u| - [u] \right] + c$$

$$\Rightarrow -10 \ln u + 2u + c //$$

$$\Rightarrow 2(5 - \sqrt{h}) - 10 \ln(5 - \sqrt{h}) + c$$

$$\Rightarrow 10 - 2\sqrt{h} + c - 10 \ln(5 - \sqrt{h})$$

$10 + c$ is just a constant, we can call it k .

$$\Rightarrow -2\sqrt{h} - 10 \ln(5 - \sqrt{h}) + k = \text{result.}$$

$$b) \quad \frac{dh}{dt} = \frac{t^{0.2} (5 - \sqrt{h})}{5}$$

$$\left(\frac{5}{5 - \sqrt{h}} \right) \frac{dh}{dt} = t^{0.2}$$

$$5 \int \left[\frac{1}{5 - \sqrt{h}} \right] dh = \int (t^{0.2}) dt$$

$$5 \left[-10 \ln(5 - \sqrt{h}) - 2\sqrt{h} \right] = \frac{t^{1.2}}{1.2} + k'$$

$$\underline{t=0, h=2} : 5 \left[-10 \ln(5 - \sqrt{2}) - 2\sqrt{2} \right] = k'$$

$$\therefore k' = \left[-50 \ln(5 - \sqrt{2}) - 10\sqrt{2} \right]$$

$$\therefore \left[-50 \ln(5 - \sqrt{h}) - 10\sqrt{h} \right] = \frac{t^{1.2}}{1.2} - 50 \ln(5 - \sqrt{2}) - 10\sqrt{2}$$

$$\underline{h=15} : \left[-50 \ln(5 - \sqrt{15}) - 10\sqrt{15} + 50 \ln(5 - \sqrt{2}) + 10\sqrt{2} \right] \times 1.2 = t^{1.2}$$

$$\therefore t = \sqrt[1.2]{1.2 \left(-50 \ln(5 - \sqrt{15}) - 10\sqrt{15} + 50 \ln(5 - \sqrt{2}) + 10\sqrt{2} \right)}$$

$$t = \sqrt[1.2]{39.939...} = \boxed{21.6} \text{ years.}$$

c) $h = 15, \quad t = 21.6$

$$\therefore \frac{dh}{dt} = \frac{(21.6)^{0.2} (5 - \sqrt{15})}{5} = 0.42$$

$$= \boxed{42 \text{ cm per year}}$$

Q12a) $\begin{pmatrix} 2 \\ 0 \\ 7 \end{pmatrix}$ is part of the equations l_1 and l_2 .

Hence they meet at

$$\boxed{\begin{pmatrix} 2 \\ 0 \\ 7 \end{pmatrix}}$$

b) Using dir. vectors : $\begin{pmatrix} 8 \\ 4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = 16 - 8 + 1 = 9$

$$\left| \begin{pmatrix} 8 \\ 4 \\ 1 \end{pmatrix} \right| = \sqrt{8^2 + 4^2 + 1^2} = 9$$

$$\left| \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \right| = \sqrt{2^2 + 2^2 + 1^2} = 3$$

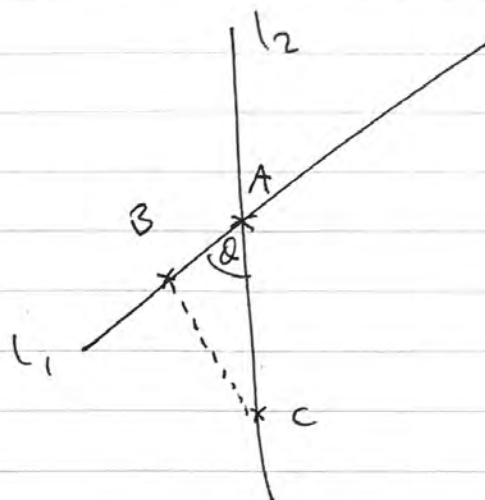
$$\therefore \cos \theta = \frac{9}{3 \times 9} = \frac{1}{3}$$

$$\therefore \sin^2 \theta = 1 - \cos^2 \theta = 1 - \left(\frac{1}{3}\right)^2 = \frac{8}{9}$$

$$\text{so } \sin \theta = \sqrt{\frac{8}{9}} = \left| \frac{2}{3} \sqrt{2} \right|$$

$$c) \quad \vec{OB} = \begin{pmatrix} 2 + 2(4) \\ 0 - 2(4) \\ 7 + 1(4) \end{pmatrix} = \begin{pmatrix} 10 \\ -8 \\ 11 \end{pmatrix}$$

$$\begin{aligned} \vec{AB} &= \vec{OB} - \vec{OA} \\ &= \begin{pmatrix} 10 \\ -8 \\ 11 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 7 \end{pmatrix} \\ &= \begin{pmatrix} 8 \\ -8 \\ 4 \end{pmatrix} \end{aligned}$$

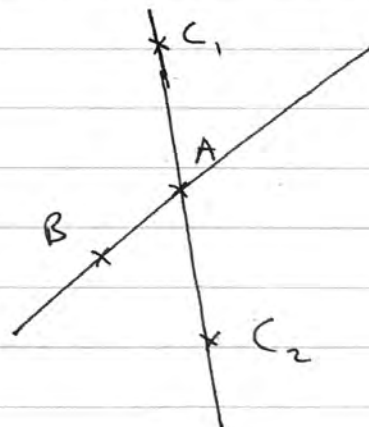


$$\therefore |\vec{AB}| = \sqrt{8^2 + 8^2 + 4^2} = 12$$

$$\begin{aligned} \text{Area} &= \frac{1}{2} ab \sin C = \frac{1}{2} \times AB \times AC \times \sin \theta \\ &= \frac{1}{2} \times 12 \times 24 \times \sin \theta \\ &= 144 \times \frac{2}{3} \sqrt{2} = \boxed{96\sqrt{2}} \end{aligned}$$

$$d) \quad \vec{AC} = \begin{pmatrix} 8\mu \\ 4\mu \\ \mu \end{pmatrix}$$

$$|\vec{AC}| = 24$$



$$\therefore \sqrt{(8\mu)^2 + (4\mu)^2 + (\mu)^2} = 24$$

$$\begin{aligned} \therefore \sqrt{81\mu^2} &= 24 & \therefore 81\mu^2 &= 576 \\ & & \therefore \mu^2 &= \frac{64}{9} \end{aligned}$$

$$\text{hence } \mu = \sqrt{\frac{64}{9}} = \pm \frac{8}{3}$$

$$\text{so } \vec{OC} = \vec{OA} \pm \frac{8}{3} \begin{pmatrix} 8 \\ 4 \\ 1 \end{pmatrix}$$

$$\vec{OC}_1 = \begin{pmatrix} 2 \\ 0 \\ 7 \end{pmatrix} + \begin{pmatrix} 64/3 \\ 32/3 \\ 8/3 \end{pmatrix} = \boxed{\begin{pmatrix} 70/3 \\ 32/3 \\ 29/3 \end{pmatrix}}$$

$$\vec{OC}_2 = \begin{pmatrix} 2 \\ 0 \\ 7 \end{pmatrix} - \begin{pmatrix} 64/3 \\ 32/3 \\ 8/3 \end{pmatrix} = \boxed{\begin{pmatrix} -58/3 \\ -32/3 \\ 13/3 \end{pmatrix}}$$