

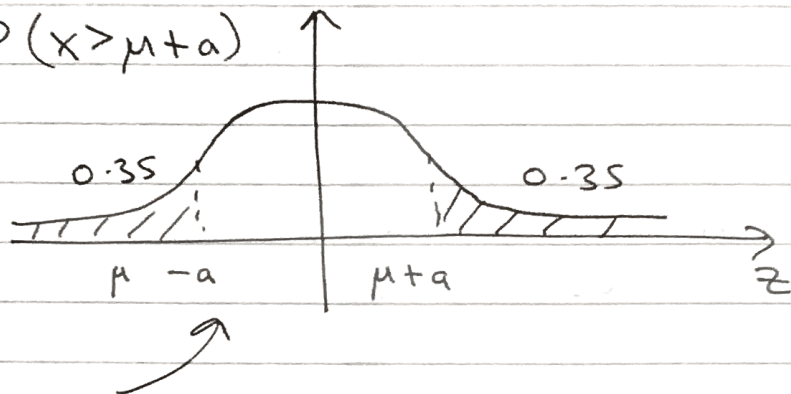
S1 October 2016 (MA)

Q1a) $X \sim N(\mu, \sigma^2)$ $P(X > \mu + a) = 0.35$

$$P(X > \mu - a) = 1 - P(X > \mu + a)$$

$$= 1 - 0.35$$

$$= \boxed{0.65}$$



b) $P(\text{required}) = \text{unshaded area} = 1 - 0.35 - 0.35$
 $= \boxed{0.30}$

c) $P(X < \mu + a \mid X > \mu - a) = \frac{P(\mu - a < X < \mu + a)}{P(X > \mu - a)}$

$$= \frac{0.30}{0.65} = \boxed{\frac{6}{13}}$$

Q2a) $b + a + a + b + \frac{1}{5} = 1$

$$\boxed{2b + 2a = 0.8}$$

b) $E(X) = \sum xP(X=x) = -2b - a + a + 2b + \frac{3}{5}$
 $= \boxed{\frac{3}{5}}$

$$c) \quad E(X^2) = \sum x^2 P(X=x) = (-2)^2 b + (1)^2 a + (1)^2 a + (2)^2 b + (3)^2 \left(\frac{1}{5}\right)$$

$$E(X^2) = 4b + a + a + 4b + \frac{9}{5} = 3.5$$

$$\therefore 8b + 2a = 1.7$$

$$ii) \quad \left[\text{from (a)} \right] - \left[2b + 2a = 0.8 \right]$$

$$6b + 0 = 0.9$$

$$\therefore b = \frac{0.9}{6} = \boxed{0.15}$$

$$\text{so } 2a + 2(0.15) = 0.8$$

$$\therefore a = \frac{0.8 - 2(0.15)}{2} = \boxed{0.25}$$

$$d) \quad \text{Var}(X) = E(X^2) - [E(X)]^2$$

$$E(X) = 0.6, \quad E(X^2) = 3.5$$

$$\text{so } \text{Var}(X) = 3.5 - (0.6)^2 = \boxed{3.14}$$

$$e) \quad P(Y > 0) = P(5 - 3X > 0) = P\left(\frac{5}{3} > X\right)$$

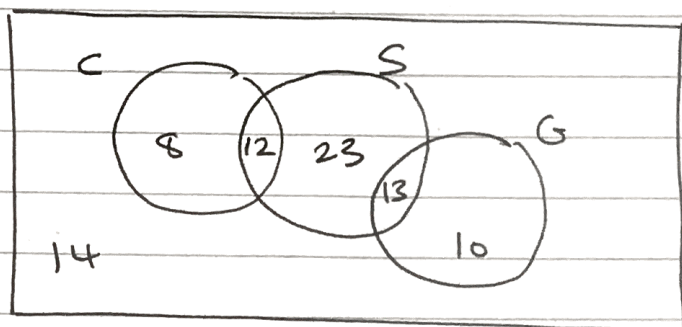
$$= P\left(X < \frac{5}{3}\right) = P(X = -2) + P(X = -1) + P(X = 1)$$

$$= b + a + a = 0.15 + 0.50 = \boxed{0.65}$$

$$\begin{aligned}
 \text{fi) } E(Y) &= E(S - 3X) = -3E(X) + S \\
 &= -3(0.6) + 5 = \boxed{3.2}
 \end{aligned}$$

$$\begin{aligned}
 \text{ii) } \text{Var}(Y) &= \text{Var}(S - 3X) = 9 \text{Var}(X) = 9 \times 3.14 \\
 &= \boxed{28.3}
 \end{aligned}$$

Q3a)



$$\text{bi) } P(S) = \frac{12 + 23 + 13}{80} = \boxed{0.6}$$

$$\text{ii) } P(S | \text{coat}) = \frac{P(S \cap C)}{P(C)} = \frac{12}{8+12} = \boxed{0.6}$$

$$\text{iii) } P(S) = P(S | C) \quad \text{so } S \text{ and } C \text{ are independent} \\
 0.6 = 0.6$$

$$\text{c) } P(S | G) = \frac{P(S \cap G)}{P(G)} = \frac{13}{13+10} = \frac{13}{23} = 0.57 //$$

$$P(S | C) = 0.6 //$$

$$0.57 < 0.60$$

so the assistant selling coats is better //

$$\begin{array}{l}
 \bullet \text{ Q4a) } \left. \begin{array}{l} \sum f = 456 \\ \sum f^2 = 26072 \\ n = 8 \end{array} \right\} S_{ff} = 26072 - \frac{(456)^2}{8} \\
 \hspace{15em} = \boxed{80}
 \end{array}$$

$$\text{b) } S_{hh} = 610463 - \frac{(2209)^2}{8} = \boxed{503} \quad (35f)$$

$$\text{c) } PMCC = \frac{S_{fh}}{\sqrt{S_{ff} \times S_{hh}}} = \frac{182}{\sqrt{80 \times 503}} = \boxed{0.907}$$

d) $r \approx 1$ so strong +ve correlation between f and h . Hence belief is supported.

$$\text{e) } b = \frac{S_{fh}}{S_{ff}} = \frac{182}{80} = 2.275$$

$$\begin{aligned}
 a = \bar{y} - b\bar{x} &= \bar{h} - b\bar{f} = \frac{2209}{8} - 2.275 \left(\frac{456}{8} \right) \\
 &= 146.45
 \end{aligned}$$

$$\text{so } \boxed{h = 146 + 2.28f} \quad (h = a + bf)$$

$$\begin{aligned}
 \text{f) } P(\text{required}) &= P(-3 < E < 3) \\
 &= P\left(\frac{-3-0}{4} < Z < \frac{3-0}{4}\right) = P(-0.75 < Z < 0.75) \\
 &= P(Z < 0.75) - [1 - P(Z < 0.75)] = \\
 &= 0.7734 - [1 - 0.7734] = \boxed{0.547}
 \end{aligned}$$

$$(Q5a) \quad A \sim N(\mu, \sigma^2)$$

$$P(A < 388) = 0.001$$

$$P\left(Z < \frac{388 - \mu}{\sigma}\right) = 0.001$$

$$\therefore P\left(Z > \frac{\mu - 388}{\sigma}\right) = 0.001$$

$$\text{but } P(Z > 3.0902) = 0.001$$

$$\text{hence } \frac{\mu - 388}{\sigma} = 3.0902$$

$$\textcircled{1} \sim \mu - 388 = 3.0902 \sigma$$

$$\text{also, } P(A > 410) = 0.0197$$

$$P\left(Z > \frac{410 - \mu}{\sigma}\right) = 0.0197$$

$$P\left(Z < \frac{410 - \mu}{\sigma}\right) = 1 - 0.0197 = 0.9803$$

$$\text{but } P(Z < 2.06) = 0.9803$$

$$\text{so } \frac{410 - \mu}{\sigma} = 2.06$$

$$\text{so } 410 - \mu = 2.06 \sigma \sim \textcircled{2}$$

$$\textcircled{1} + \textcircled{2} : 410 - 388 = (2.06 + 3.0902) \sigma$$

$$\text{so } \sigma = \frac{22}{5.1502} = \boxed{4.27}$$

$$\text{and so } \mu = 388 + 3.0902(4.27)$$

$$\boxed{\mu = 401} \quad (3 \text{ s.f.})$$

$$\text{b) } P(A < 388) = 0.001$$

$$P(A > 410) = 0.0197$$

$$P(388 < A < 410) = 1 - 0.001 - 0.0197 = 0.9793$$

so... Expected profit will be

$$\begin{aligned} & 0.001 \times (-100) \\ & + 0.0197 \times (-0.3) \\ & + 0.9793 \times (0.25) \end{aligned}$$

$$\underline{\underline{0.14}}$$

so expected profit will be 14p.

$$\Rightarrow \boxed{\pounds 0.14}$$

$$\text{(Q6a) } \frac{19}{4} = 4.75 \rightarrow 5^{\text{th}} \text{ term} = \boxed{Q_1 = 117}$$

$$\frac{19}{2} = 9.5 \rightarrow 10^{\text{th}} \text{ term} = \boxed{\text{median} = 122}$$

$$\frac{3 \times 19}{4} = 14.25 \rightarrow 15^{\text{th}} \text{ term} = \boxed{Q_3 = 125}$$

$$\text{b) } 125 - 117 = \boxed{8} = 10 \text{ p.}$$

$$\begin{aligned} \text{c) Lower limit} &= Q_1 - 1.5(Q_3 - Q_1) \\ &= 117 - 1.5(8) = 105 // \end{aligned}$$

$$\begin{aligned} \text{Upper limit} &= Q_3 + 1.5(Q_3 - Q_1) \\ &= 125 + 1.5(8) = 137 // \end{aligned}$$

so outliers are:

101
102
139

d) see mark scheme.

$$\text{e) mean} = \frac{\sum x}{n} = \frac{2299}{19} = \boxed{121}$$

$$\text{variance} = \frac{\sum x^2}{n} - \left[\frac{(\sum x)^2}{(\sum n)} \right] = \frac{279709}{19} - (121)^2$$

$$\text{so s.d.} = \sigma = \sqrt{\frac{279709}{19} - (121)^2}$$

$$= \boxed{8.97}$$

$$\text{f) } \bar{x} \pm 2.7\sigma$$

$$\begin{aligned} \bar{x} + 2.7\sigma &= \boxed{145} \\ \bar{x} - 2.7\sigma &= \boxed{96.8} \end{aligned}$$

g) from a, $Q_3 - Q_2 < Q_2 - Q_1$,
3 < 5

so data is skewed.
 \therefore N.D not suitable