

S1 Jan 17 (IAL) MA

Q1a) Area of 2<sup>nd</sup> bar =  $(1) \times (5) = u \times \text{freq.}$

$$5 = 5u \quad \therefore u = 1 // \therefore (\text{Area} = \text{frequency})$$

for first bar: Area =  $2 \times 2 = \boxed{4 = \text{frequency}}$

$$(\text{Area} = u \times \text{freq.})$$

b)  $P(T > 3) = \frac{\text{no. of tomatoes with weight} > 3\text{g}}{\text{total no. of tomatoes.}}$

$$= \frac{100 - 5 - 4}{100} = \boxed{0.91}$$

c) Area to the right of  $(x = 6.25)$  =  $[(7 - 6.25) \times 16]$

$$+ [25 \times 1] + [10 \times 1] \\ + [8 \times 1]$$

$$= 55 //$$

$$\therefore \text{proportion} = \boxed{\frac{55}{100}}$$

d)  $0.55 > 0.50 \therefore \text{median} > 6.25.$

e) median  $>$  mean for negative skew.  
 median  $>$  6.25  $\therefore$   $\boxed{\text{negative skew}}$

f)  $P(5.5 < \text{weight} < 7) = \frac{(7 - 5.5) \times 16}{100} = \frac{24}{100} //$

$$\therefore P(\text{one tomato is within } 0.75\text{g of mean}) = \frac{24}{100} //$$

but there is another tomato selected:

$$P(\text{2nd tomato is within } 0.75\text{g of mean}) = \frac{23}{99}$$

$$\therefore P(\text{both are within } 0.75\text{g}) = \frac{23}{99} \times \frac{24}{100} = \boxed{0.056}$$

2a) The event that the integer is both a prime and ends in 3.

$$\text{b) } \frac{6 + 2 + 2 + 5}{50} = \boxed{\frac{3}{10}}$$

$$\text{c) } \frac{12}{50} = \boxed{\frac{6}{25}} \quad (A, B, C \text{ can't happen})$$

$$\text{d) } P(\text{prime} | > 20) = P(A|C) = \frac{P(A \cap C)}{P(C)}$$

$$\left. \begin{array}{l} P(A \cap C) = \frac{5+2}{50} = \frac{7}{50} \\ P(C) = \frac{5+2+1+22}{50} = \frac{30}{50} \end{array} \right\} P(A|C) = \frac{\frac{7}{50}}{\frac{30}{50}} = \boxed{\frac{7}{30}}$$

$$\text{e) from (d), } P(A|C) = 7/30$$

$$\text{from (b), } P(A) = 3/10$$

these probabilities are different which means that whether or not C occurs has an effect on whether or not A occurs.

$\therefore$  they aren't independent.

$$f) P(B|A \cap C) = \frac{P(A \cap B \cap C)}{P(A \cap C)}$$

$$= \frac{2/50}{7/50} = \boxed{\frac{2}{7}}$$

$\Rightarrow$

$$3a) \text{ mean} = \frac{\sum y}{12} = \frac{-27}{12} = \boxed{-2.25}$$

$$\text{variance} = \frac{\sum y^2}{n} - \left(\frac{\sum y}{n}\right)^2$$

$$= \frac{62.98}{12} - (-2.25)^2 = \boxed{0.186}$$

$$b) S_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n}$$

$$= -1190.7 - \frac{(504)(-27)}{12} = -56.7 //$$

$$\therefore \text{PMCC} = \frac{-56.7}{\sqrt{1674 \times 2.23}} = \boxed{-0.928}$$

ii)  $\text{PMCC} \approx -1$   $\therefore$  a negative correlation is suggested between  $x$  and  $y$ .  
 $\therefore$  Priya's belief is incorrect.

$$c) \quad b = \frac{S_{xy}}{S_{xx}} = \frac{-56.7}{1674} = \underline{\underline{-0.03387}} \dots (-0.03387)$$

$$a = \bar{y} - b\bar{x} = \frac{-27}{12} + 0.03387 \left( \frac{564}{12} \right) = \underline{\underline{-0.827}}$$

$$\therefore y = \underline{\underline{-0.827 - 0.0339x}}$$

$$d) \quad \underline{\underline{x=32}} : y = -0.827 - 0.0339(32) = \underline{\underline{-1.91^\circ C}}$$

$$e) \quad \frac{w-32}{1.8} = y : \quad \frac{w-32}{1.8} = -0.827 - 0.0339x$$

$$w - 32 = -1.4886 - 0.06102x$$

$$w = \underline{\underline{30.5 - 0.061x}}$$

$$fi) \quad \text{Var}(w) = \text{Var}(1.8y + 32) = 1.8^2 \text{Var}(y)$$

$$= 1.8^2 \times 0.186 \approx \boxed{0.602}$$

ii) PMCC not affected by coding

$$\therefore r_{\text{new}} = \boxed{-0.928}$$

$$4a) E(X) = 5(0.13) + 6(0.21) + 7(0.29) + 8(0.37) \\ = \boxed{6.9}$$

$$b) E(X^2) = 25(0.13) + 36(0.21) + 49(0.29) + 64(0.37) \\ = 48.7$$

$$\text{Var}(X) = 48.7 - (6.9^2) = \boxed{1.09}$$

$$c) \text{Var}(3-2X) = \text{Var}(-2X+3) = 4 \text{Var}(X) \\ = 4 \times 1.09 = \boxed{4.36}$$

$$d) E(Y) = \frac{1}{4}(5) + \frac{1}{4}(6) + \frac{1}{4}(7) + \frac{1}{4}(8) = \boxed{\frac{13}{2}}$$

alt: uniform distribution  $\therefore$  mean =  $\frac{5+8}{2}$

$$e) P(X=Y) = P(X=5 \cap Y=5) + P(X=6 \cap Y=6) + \dots$$

$$\uparrow = \frac{1}{4}(0.13) + \frac{1}{4}(0.21) + \frac{1}{4}(0.29) + \frac{1}{4}(0.37)$$

X and Y are independent!

$$= \boxed{\frac{1}{4}}$$

f)  $P(X > Y)$ : possible ways this could happen:

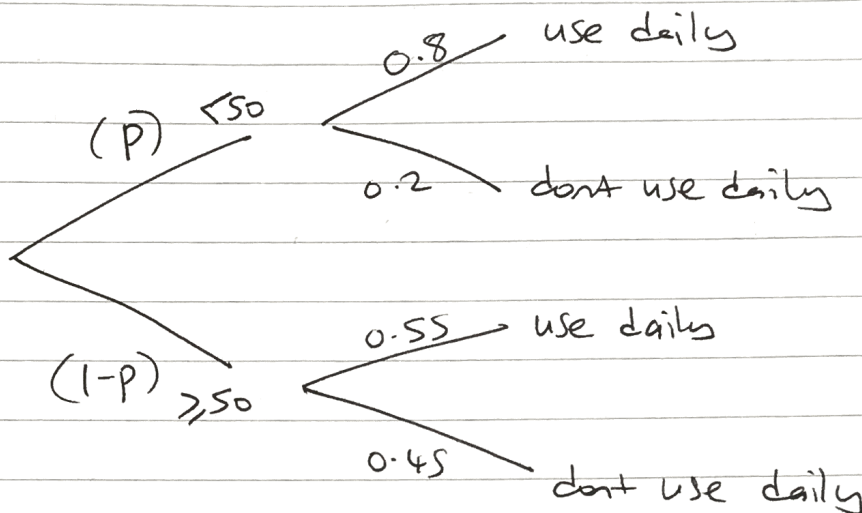
X	Y
5	no possibilities.
6	5
7	5, 6
8	5, 6, 7

$$\therefore P(X > Y) = P(X=6 \cap Y=5) + P(X=7 \cap Y=5 \text{ or } 6) \\ + P(X=8 \cap Y=5 \text{ or } 6 \text{ or } 7)$$

$$= \frac{1}{4}(0.21) + 0.29\left(\frac{1}{4} + \frac{1}{4}\right) + 0.37\left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right)$$

$$= \frac{21}{400} + \frac{29}{200} + \frac{111}{400} = \boxed{\frac{19}{40}}$$

5a)



$$b) P(\text{a random person uses computer daily}) = 0.7$$

← this is what we are told.

$$P(\text{a random person uses comp. daily}) = p(0.80) + (1-p)(0.55)$$

$$\therefore p(0.8) + (1-p)(0.55) = 0.70$$

$$0.8p + 0.55 - 0.55p = 0.70$$

$$0.25p = 0.15 \quad \therefore \boxed{p = 0.6}$$

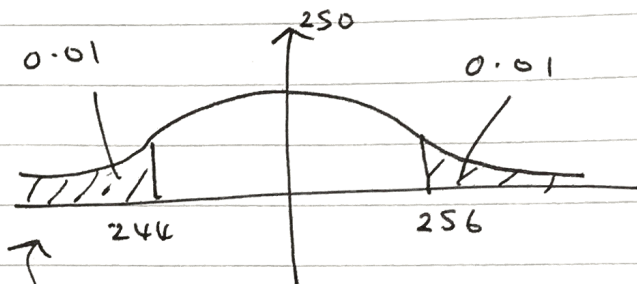
$$c) P(\leq 50 \mid \text{daily use}) = \frac{P(\text{under } 50 \cap \text{uses comp. daily})}{P(\text{uses comp. daily})}$$

$$= \frac{0.8 \times 0.6}{0.70} = \boxed{\frac{48}{70}}$$

6a)  $W \sim N(250, \sigma^2)$

$$100 - 0.01 - 0.01$$

$$= \boxed{0.98} = 98\%$$



(By symmetry)

b)

$$P(W > 256) = 0.01$$

$$P\left(Z > \frac{256 - 250}{\sigma}\right) = 0.01$$

$$\rightarrow \text{from tables, } 2.3263 = \frac{6}{\sigma}$$

$$\therefore \sigma = \frac{6}{2.3263} = \boxed{2.58}$$

c) For the machine to shut down we need 2 rice bags selected to both be more than 4g from mean.

$$\begin{aligned} P(\text{one bag} > 4g \text{ from mean}) &= P(W > 254) + P(W < 246) \\ &= 2P(W > 254) \quad [\text{due to symmetry}] \end{aligned}$$

$$= 2P\left(Z > \frac{254 - 250}{2.58}\right)$$

$$= 2P(Z > 1.55)$$

$$= 2[1 - P(Z < 1.55)]$$

$$= 2[1 - 0.9394] = 0.1212 //$$

$$\therefore P(\text{both bags} > 4g \text{ from mean}) = (0.1212)^2 = \boxed{0.0147}$$

$x$	1	2	3	4
$P(X=x)$	$\frac{a+b}{60}$	$\frac{2a+b}{60}$	$\frac{3a+b}{60}$	$\frac{4a+b}{60}$

$$\sum_{x=1}^4 P(X=x) = 1.$$

$$\Rightarrow \frac{a+b + 2a+b + 3a+b + 4a+b}{60} = 1$$

$$\Rightarrow 10a + 4b = 60$$

$$\textcircled{2} : \underline{5a + 2b = 30} \quad \sim \textcircled{1}$$

$$b) F(3) = P(X=1) + P(X=2) + P(X=3) = \frac{13}{20}$$

$$\Rightarrow \frac{a+b + 2a+b + 3a+b}{60} = \frac{13}{20}$$

$$\Rightarrow \underline{6a + 3b = 39} \quad \sim \textcircled{2}$$

solving  $\textcircled{1}$  and  $\textcircled{2}$  simultaneously :

$$\textcircled{2} \div 3 : \underline{2a + b = 13}$$

$$\dots b = 13 - 2a$$

$$\hookrightarrow \textcircled{1} : 5a + 2(13 - 2a) = 30$$

$$5a + 26 - 4a = 30$$

$$a + 26 = 30$$

$$\boxed{a = 4} \quad \therefore b = 13 - 8 = \boxed{5} = b$$

c)	y	1	4	9	16
	$P(Y=y)$	$\frac{9}{60}$	$\frac{13}{60}$	$\frac{17}{60}$	$\frac{21}{60}$

So for cumulative ...

	y	1	4	9	16
	$F(y)$	$\frac{9}{60}$	$\frac{22}{60}$	$\frac{39}{60}$	1

$$F(1) = P(y=1)$$

$$F(4) = P(y=1) + P(y=4) = \frac{9}{60} + \frac{22}{60}$$

$$F(9) = P(y=1) + P(y=4) + P(y=9) = \frac{9 + 13 + 17}{60}$$