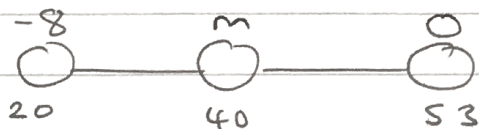


S1 October 2017 (MA)

Q1a) $25 - - 38 = \boxed{63}$ (outlier counts)

b) IQR = $10 - - 8 = \boxed{18}$

c) $\frac{n}{2} = 40$. 40th data point lies in interval $-8 \leq e < 0$

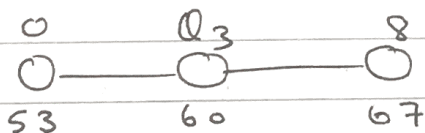


$$\frac{m - - 8}{0 - - 8} = \frac{40 - 20}{53 - 20}$$

$$m + 8 = 8 \left(\frac{20}{33} \right)$$

$$\therefore m = \boxed{-3.15}$$

d) $\frac{3n}{4} = \frac{3}{4} \times 80 = 60$, 60th data point lies in interval $0 \leq e < 8$



$$\frac{Q_3}{8} = \frac{60 - 53}{67 - 53}$$

$$Q_3 = 8 \left(\frac{60 - 53}{67 - 53} \right) = \boxed{4}$$

$$e) \text{ IQR} = Q_3 - Q_1 = 4 - (-8) = 12 //$$

$$\text{upper limit} : 4 + 1.5(12) = 22 //$$

$$\text{lower limit} : -8 - 1.5(12) = -26 //$$

So the only outliers are 23 and 28

ii) see mark scheme.

f) The IQR is lower and the median is closer to 0. This indicates their ability has improved.

Q2a) B and C ($P(B \cap C) = 0$)

b) $P(A) \times P(C) = P(A \cap C)$ if independent.

$$0.6 \times 0.35 = P(A \cap C) = w = \boxed{0.21}$$

$$c) P(A) = 0.60 \rightarrow 0.15 + y + w = 0.60$$

$$y = 0.6 - 0.15 - 0.21 = \boxed{0.24}$$

$$P(C) = w + z = 0.35 \rightarrow z = 0.35 - 0.21 = \boxed{0.14}$$

$$z = 1 - 0.15 - 0.24 - 0.21 - 0.14 = \boxed{0.26}$$

$$d) P(A \text{ or } B \text{ or } C) = x + y = \boxed{0.38}$$

(if you buy B then you must be buying A too.
so $P(\text{required}) = P(A \cap B' \cap C') + P(C \cap B' \cap A')$)

$$e) P(B \cup C) = P(B) + P(C) - P(B \cap C)$$

$$= 0.15 + 0.35$$

$$= \boxed{0.50}$$

$$f) P(A | B \cup C) = \frac{P(A \cap B \cup C)}{P(B \cup C)} = \frac{0.15 + w}{0.50}$$

$$= \frac{0.15 + 0.21}{0.50}$$

$$= \boxed{0.72}$$

$P(A \cap B \cup C) = P(A \cap B) + P(A \cap C)$

$$(Q3a) L \sim N(\mu, \sigma^2)$$

$$\text{We are told } P(L < 45) = 0.4$$

$$\Rightarrow P\left(Z < \frac{45 - \mu}{\sigma}\right) = 0.4$$

$$\Rightarrow P\left(Z > \frac{\mu - 45}{\sigma}\right) = 0.4$$

$$\text{but } P(Z > 0.2533) = 0.4$$

$$\therefore 0.2533 = \frac{\mu - 45}{\sigma} //$$

$$\text{hence } 45 + 0.2533\sigma = \mu \quad \text{---} \quad \textcircled{1}$$

$$\text{b) also, } P(L < 35) = 0.15$$

$$P\left(Z < \frac{35 - \mu}{\sigma}\right) = 0.15$$

$$P\left(Z > \frac{\mu - 35}{\sigma}\right) = 0.15$$

$$\text{but } P(Z > 1.0364) = 0.15$$

$$\text{hence } \frac{\mu - 35}{\sigma} = 1.0364$$

$$\Rightarrow \mu - 35 = 1.0364\sigma$$

$$\Rightarrow \mu = 35 + 1.0364\sigma \quad \text{---} \quad \textcircled{2}$$

$$\text{c) } \textcircled{1} = \textcircled{2} : 35 + 1.0364\sigma = 45 + 0.2533\sigma$$

$$\sigma(1.0364 - 0.2533) = 10$$

$$\text{so } \sigma = \frac{10}{1.0364 - 0.2533} = \boxed{12.8}$$

$$\text{and } \mu = 35 + 1.0364(12.8)$$

$$\boxed{\mu = 48.2}$$

$$\begin{aligned}
 \text{d) } P(L > 35 \mid L < 45) &= \frac{P(35 < L < 45)}{P(L < 45)} \\
 &= \frac{P(L < 45) - P(L < 35)}{P(L < 45)} \\
 &= \frac{(1 - 0.60) - (0.15)}{(1 - 0.60)} \\
 &= \boxed{\frac{5}{8}}
 \end{aligned}$$

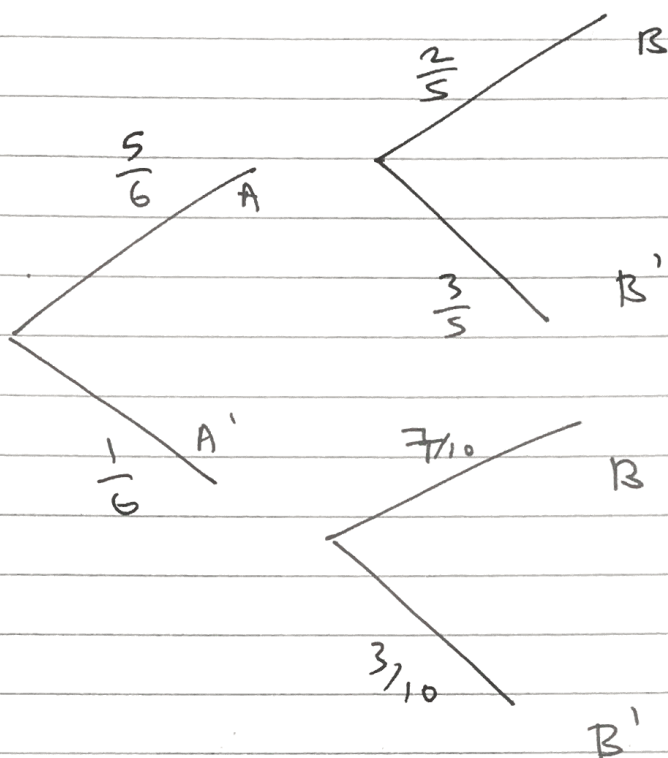
recall that $P(L > 45) = 0.60$
 $P(L < 35) = 0.15$

$$\begin{aligned}
 \text{ii) } P(L < 45 \mid L > 35) &= \frac{P(L < 45 \cap L > 35)}{P(L > 35)} = \frac{P(35 < L < 45)}{P(L > 35)} \\
 &= \frac{(1 - 0.60) - (0.15)}{(1 - 0.15)} \\
 &= \boxed{\frac{5}{17}}
 \end{aligned}$$

$$\begin{aligned}
 \text{e) } 0.60 - \text{red} \quad P(L > 35 \mid L < 45) &= P(\text{stick is yellow} \mid \text{its blue or yellow}) \\
 0.15 - \text{blue} &= \frac{5}{8} // \\
 0.25 - \text{yellow} & \\
 P(L < 45 \mid L > 35) &= P(\text{stick is yellow} \mid \text{its red or yellow}) \\
 &= \frac{5}{17} //
 \end{aligned}$$

$\frac{8}{5} > \frac{5}{17}$ hence the stick is more likely to be from Heil.

Q4a)



$$P(B) = P(A) \times \frac{2}{5} + P(A') \times \frac{7}{10}$$

$$\frac{9}{20} = \frac{2}{5} P(A) + (1 - P(A)) \times \frac{7}{10}$$

$$\frac{9}{20} = \frac{2}{5} P(A) + \frac{7}{10} - \frac{7}{10} P(A)$$

$$\therefore P(A) \times \frac{3}{10} = \frac{1}{4}$$

$$P(A) = \frac{\frac{1}{4}}{\frac{3}{10}} = \boxed{\frac{5}{6}}$$

$$b) P(A' | B') = \frac{P(A' \cap B')}{P(B')}$$

$$= \frac{\frac{1}{6} \times \frac{3}{10}}{\frac{1}{6} \left(\frac{3}{10}\right) + \frac{5}{6} \left(\frac{3}{5}\right)} = \boxed{\frac{1}{11}}$$

$$\frac{1}{6} \left(\frac{3}{10}\right) + \frac{5}{6} \left(\frac{3}{5}\right)$$

$$\text{Q5a) } \left. \begin{array}{l} \sum x = 283 \\ \sum x^2 = 9011 \\ n = 10 \end{array} \right\} \begin{array}{l} \bar{x} = \frac{283}{10} = \boxed{28.3} \\ \sigma^2 = \frac{9011}{10} - (28.3)^2 \end{array}$$

$$= 100.21 = \boxed{100}$$

$$b) \bar{y} = \frac{306.1}{10} = \boxed{30.61} \text{ or } 30.6 \text{ to } 3 \text{ s.f.}$$

$$S_{yy} = \sum y^2 - \frac{(\sum y)^2}{n}$$

$$\text{variance} = \frac{\sum y^2}{n} - \frac{(\sum y)^2}{(n)^2}$$

$$\text{hence Variance} = \frac{S_{yy}}{n} = \frac{546.3}{10} = \boxed{54.63}$$

$$c) b = 0.659 = \frac{S_{xy}}{S_{xx}} //$$

$$S_{xx} = 10 \times \text{Var}(x) = 1002.1 //$$

$$\text{So } S_{xy} = 0.659 \times 1002.1 = 660.4$$

$$\text{hence PMCC} = \frac{660.4}{\sqrt{1002.1 \times 546.3}} = \boxed{0.892}$$

d) $r \approx 1$ so strong +ve correlation suggested.
So yes the PMCC supports the use of a regression line.

$$\text{e) } y = 12 + 0.659(35) = 35.065$$

So salary will be $\boxed{\pounds 35,065}$

Q6a) Discrete Uniform.

$$\begin{aligned} \text{b) } P(3 \text{ points}) &= P(D=3) + [P(D=1) \times P(D=2)] \\ &= \frac{1}{4} + \left(\frac{1}{4}\right)\left(\frac{1}{4}\right) = \boxed{\frac{5}{16}} \end{aligned}$$

$$\text{c) } P(0) = [P(D=1)]^2 = \boxed{\frac{1}{16}}$$

x	0	2	3	4	5
$P(X=x)$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{5}{16}$	$\frac{1}{16}$

$$\begin{aligned} E(X) &= \sum_{x} x P(X=x) = 2\left(\frac{1}{4}\right) + 3\left(\frac{5}{16}\right) \\ &\quad + 4\left(\frac{5}{16}\right) + 5\left(\frac{1}{16}\right) \\ &= \boxed{3} \end{aligned}$$

$$\checkmark = \sum x^2 P(X=x)$$

$$e) E(X^2) = 2^2 \left(\frac{1}{4}\right) + 3^2 \left(\frac{5}{16}\right) + 4^2 \left(\frac{5}{16}\right) + 5^2 \left(\frac{1}{16}\right)$$

$$= \frac{83}{8} //$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$= \frac{83}{8} - (3)^2 = \boxed{\frac{11}{8}}$$

f) r can only be 1 or 2.

$$P(r=1) = P(d=2, 3 \text{ or } 4) = \frac{3}{4} //$$

$$\therefore P(r=2) = 1 - \frac{3}{4} = \frac{1}{4} //$$

r	1	2
P(R=r)	$\frac{3}{4}$	$\frac{1}{4}$

//

$$g) Y = 2R + 0.5$$

$$E(Y) = 2E(R) + 0.5$$

$$E(R) = 1 \left(\frac{3}{4}\right) + 2 \left(\frac{1}{4}\right) = \frac{5}{4}$$

$$\text{so } E(Y) = 2 \left(\frac{5}{4}\right) + 0.5 = 2.5 + 0.5$$

$$= 3 // = E(X)$$

$$b) \quad y = 2.5 \text{ if } r = 1$$

$$y = 4.5 \text{ if } r = 2$$

$$P(X > Y) = \frac{P(X=3 \cap R=1) + P(X=4 \cap R=1) + P(X=5)}{\frac{1}{4} + \frac{1}{4} + \frac{1}{16}}$$

$$= \boxed{\frac{9}{16}}$$

note : if $r=1$ then x can be 3, 4, 5 for $X > y$ to be true.

if $r=2$ then x can only be 5 for $X > y$ to be true.

$$\text{so } P(X > Y) = \frac{1}{4} + \frac{1}{4} + \frac{1}{16}$$

$$\begin{array}{ccc}
 \uparrow & \uparrow & \uparrow \\
 P(X=3 \cap R=1) & P(X=4 \cap R=1) & P(X=5 \cap R=1) \\
 = P(D=3) & = P(D=4) & + P(X=5 \cap R=2) \\
 & & = P(X=5) \\
 & & (= \frac{1}{16})
 \end{array}$$