

S1 January 2018 (IAL) (MA)

$$\text{Q1a)} \quad \frac{\sum x}{15} = 61$$

$$\frac{t}{15} = 61 \quad \therefore t = 61 \times 15 = \boxed{915}$$

$$\begin{aligned} \text{b)} \quad \text{Var}(A) &= \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = \frac{59610}{10} - (77)^2 \\ &= \boxed{32} \end{aligned}$$

$$\begin{aligned} \text{Var}(B) &= \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = \frac{58035}{15} - (61)^2 \\ &= \boxed{148} \end{aligned}$$

e) \boxed{B} since $\text{Var}(B) > \text{Var}(A)$
(so larger spread of data)

$$\text{di)} \quad \mu = \frac{n_A \bar{x}_A + n_B \bar{x}_B}{n_A + n_B} = \frac{770 + 915}{25} = \boxed{67.4}$$

$$\begin{aligned} \text{ii)} \quad \sigma^2 &= \frac{\sum x^2}{n} - (\bar{x})^2 = \frac{(59610) + (58035)}{25} - (67.4)^2 \\ &= \boxed{163} \end{aligned}$$

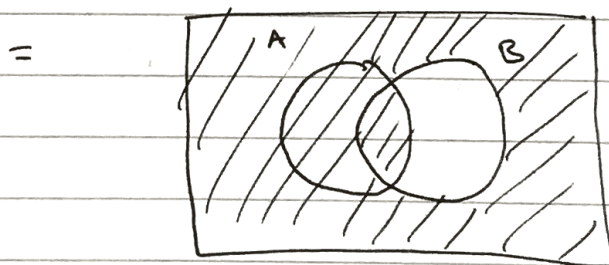
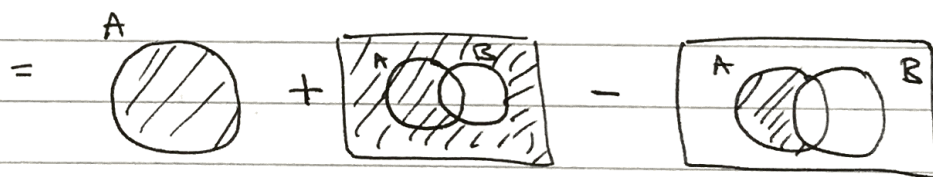
ei) No effect - addition / subtraction to all values won't affect variance.

$$\left[\text{ie } \text{Var}(X+a) = \text{Var}(X) \right]$$

ii) mean will increase since $\sum x$ has increased for all students.

iii) Total variance increases - mean of A was originally 16 more than the mean of B. This will now increase meaning greater overall variance since mean of A is further away from mean of B.

$$\text{Q2a) } P(A \cup B') = P(A) + P(B') - P(A \cap B')$$



$$\text{bi) } P(C \cup D) = P(C) + P(D) - P(C \cap D)$$

← 0 (mutually exclusive)

$$= \frac{1}{5} + \frac{3}{10} - 0 = \boxed{\frac{1}{2}}$$

$$\text{ii) } P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{0}{\frac{3}{10}} = \boxed{0}$$

$$\text{ci) } P(G)$$

$$P(F \cup G) = P(F) + P(G) - P(F \cap G)$$

$$\frac{3}{8} = \frac{1}{6} + P(G) - P(F) \times P(G)$$

$$P(G) \left[1 - \frac{1}{6} \right] = \frac{5}{24}$$

$$\therefore P(G) = \frac{\frac{5}{24}}{\frac{5}{6}} = \boxed{\frac{1}{4}}$$

$$\text{ii) } P(F|G') = \frac{P(F \cap G')}{P(G')} = \frac{\frac{1}{6} \times (1 - \frac{1}{4})}{(1 - \frac{1}{4})} = \boxed{\frac{1}{6}}$$

alt : F and G are independent

$$\text{so } P(F|G') = P(F) = \frac{1}{6}$$

and total probabilities = 1

$$\Rightarrow a + b + a + b + 0.2 = 1$$

$$\Rightarrow 2a + 2b = 0.8$$

$$\Rightarrow a + b = 0.4 \quad \text{--- (2)}$$

$$\begin{array}{r} \textcircled{1} - \textcircled{2} : \\ \underline{3a + b = 1} \\ - [a + b = 0.4] \\ \hline 2a + 0 = 0.6 \end{array}$$

$$\therefore \boxed{a = 0.3}$$

$$\hookrightarrow b = 0.4 - 0.3 = \boxed{0.1}$$

$$\begin{aligned} \text{c) } \text{Var}(1-3X) &= (-3)^2 \text{Var}(X) = 9 \text{Var}(X) \\ &= 9E(X^2) \end{aligned}$$

$$E(X^2) = \sum x^2 P(X=x)$$

$$= 16a + 9b + a + 4b + 25(0.2)$$

$$= 17(0.3) + 13(0.1) + 25(0.2) = 11.4 //$$

$$\text{hence } 9E(X^2) = 9 \times 11.4 = \boxed{103} = \text{Var}(1-3X)$$

$$\text{d) } P(Y < 0) = P(1-X < 0) = P(X > 1)$$

$$= P(X=2) + P(X=5) = 0.1 + 0.2 = \boxed{0.3}$$

Q3a) $r \approx -1 \therefore$ strong -ve correlation.

b) The dependent variable.

$$c) b = \frac{S_{ch}}{S_{cc}} = \frac{-3034.6}{303448} = -0.01$$

$$h = a + bc$$

$100x - 0.01 = -1$ (hence the data support this statement)
 \swarrow
 for 100mg of caffeine

$$d) a = \bar{h} - b\bar{c} = \frac{126}{20} - 0.01 \left(\frac{3660}{20} \right)$$

$$= 8.13 //$$

$$h = a + bc$$

$$\text{When } c = 0, h = a = \boxed{8.13}$$

$$Q4a) E(X) = \sum xP(X=x) = \boxed{-4a - 3b + a + 2b + 1}$$

$$b) \text{Var}(X) = E(X^2) - [E(X)]^2$$

$$\text{so if } \text{Var}(X) = E(X^2) \text{ then } \boxed{E(X) = 0}$$

$$ii) \Rightarrow -3a - b + 1 = 0$$

$$\Rightarrow 3a + b = 1 // \sim \textcircled{1}$$

$$\text{ii) } P(y < u) = 0.2$$

$$P(1-x < u) = 0.2$$

$$P(x > 1-u) = 0.2$$

$$P(x > 2) = 0.2 //$$

largest value of u is when x is smallest.

2 is the smallest value where this is true.

$$\text{So } 1-u = 2$$

$$\boxed{u = -1}$$

$$\text{(Q5a) } -2 = \frac{x - 20000}{1000}$$

$$\therefore x = -2000 + 20000 = \boxed{18000}$$

$$\text{b) } \text{PMCC} = \frac{S_{wy}}{\sqrt{S_{ww} \times S_{yy}}}$$

$$S_{wy} = \sum wy - \frac{(\sum w)(\sum y)}{n}$$

$$S_{wy} = 2490 - \frac{(81)(405)}{9} = -1155 //$$

$$\therefore r = \frac{-1155}{\sqrt{660 \times 2500}} = \boxed{-0.899}$$

c) -0.899 [as PMCC is unaffected by coding.]

d) $y = 60.75 - 1.75w$

$$y = 60.75 - 1.75 \left(\frac{x + 20000}{1000} \right)$$

$$y = 60.75 + 1.75(20) - \frac{1.75x}{1000}$$

$$y = 95.75 - 0.00175x$$

e) $x = 21000$: $y = 95.75 - 0.00175(21000)$
 $= \boxed{59}$ min

f) Data range is from $x = 18u$ to $x = 45u$.

$25u - 40u$ is within this range so any estimates are interpolations so they are reliable.

Q6a) 0 - can't get both blue socks in one selection.

$$b) P(S > 2) = 1 - P(S \leq 2) = 1 - [P(S = 2)]$$

$$= 1 - \left(\frac{2}{5}\right)\left(\frac{1}{4}\right) = \boxed{\frac{9}{10}}$$

$$c) P(S = 3) = \left(\frac{2}{5}\right)\left(\frac{3}{4}\right)\left(\frac{1}{3}\right) \times 2 = \boxed{\frac{1}{5}}$$

(2 possible ways for $S = 3$)

$$d) P(S = 3 \mid \text{second sock is blue}) = \frac{P(S = 3 \cap \text{2nd sock blue})}{P(\text{2nd sock blue})}$$

$$= \frac{\left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{1}{3}\right)}{\left(\frac{3}{5}\right)\left(\frac{2}{4}\right) + \left(\frac{2}{5}\right)\left(\frac{1}{4}\right)} = \boxed{\frac{1}{4}}$$

$P(BB) + P(B'B)$

if 2nd sock is blue and $S = 3$ then socks 2 and 3 have to be blue

$$e) P(S = 5) = \left. \begin{aligned} &P(BB'B'B'B) \\ &+ P(B'B'BB'B) \\ &+ P(B'B'BB'B) \\ &+ P(B'B'B'BB) \end{aligned} \right\} \begin{array}{l} \text{these will all be the} \\ \text{same since socks are} \\ \text{chosen at random} \end{array}$$

5th sock must be blue

$$= 4 \times \left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{1}{3}\right)\left(\frac{2}{2}\right)\left(\frac{1}{1}\right) = \boxed{\frac{2}{5}}$$

$$\bullet \text{ (Q7a)} \quad G \sim N(180, 15^2)$$

$$P(G > 174) = P\left(Z > \frac{174 - 180}{15}\right) = P(Z > -0.4) \\ = P(Z < 0.4) = \boxed{0.6554}$$

$$\bullet \text{ b) } P(k < G < 174) = P(G < 174) - P(G < k) \\ = (1 - 0.6554) - P(G < k)$$

$$\Rightarrow 0.3446 - P(G < k) = 0.3196$$

$$\Rightarrow P(G < k) = 0.025$$

$$\Rightarrow P\left(Z < \frac{k - 180}{15}\right) = 0.025$$

$$\Rightarrow P\left(Z > \frac{180 - k}{15}\right) = 0.025$$

$$\bullet \text{ but } P(Z > 1.96) = 0.025$$

$$\text{so } 1.96 = \frac{180 - k}{15}$$

$$\therefore k = 180 - 15(1.96) = \boxed{150.6}$$

$$\bullet \text{ ci) } B \sim N(216, 30^2)$$

$$P(G > w) = P(B < w)$$

$$P\left(Z > \frac{w - 180}{15}\right) = P\left(Z < \frac{w - 216}{30}\right)$$

$$\text{So } P\left(Z > \frac{w-180}{15}\right) = P\left(Z > \frac{216-w}{30}\right)$$

$$\text{hence } \frac{w-180}{15} = \frac{216-w}{30}$$

$$\times 30 : 2w - 360 = 216 - w$$

$$3w = 576$$

$$\therefore w = \frac{576}{3} = \boxed{192}$$

$$\text{ii) } P(G > w) = p = P\left(Z > \frac{192-180}{15}\right)$$

$$= P(Z > 0.8)$$

$$= 1 - P(Z < 0.8)$$

$$= 1 - 0.7881 = \boxed{0.2119}$$