

S1 June 2018 (MA)

$$Q1a) F(3) = P(X=2) = a = \boxed{0.2}$$

$$F(6) = P(X=2) + P(X=4) = a + b = 0.8$$

$$\therefore \underline{\underline{b = 0.6}}$$

$$a + b + 0.1 + c = 1$$

$$c = 1 - 0.1 - 0.6 - 0.2 = \boxed{0.1} = c$$

$$b) F(7) = P(X \leq 7) = 0.1 + b + a = \boxed{0.9}$$

$$Q2a) (3-6) \rightarrow \text{width} = 2 = (4 \times \frac{1}{2})$$

$$(11-15) \rightarrow \text{width} = \underline{\underline{2.5}} = (5 \times \frac{1}{2})$$

$$\text{Area} = u \times f$$

$$(2 \times 9.5) = u (38) \quad \therefore u = \frac{1}{2} \quad \left. \vphantom{\frac{1}{2}} \right\} \text{for } (3-6)$$

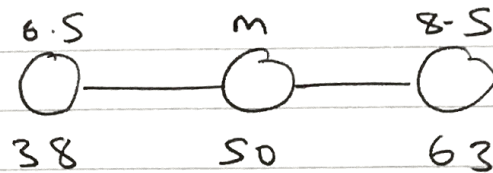
$$\text{so Area for } (11-15) = \frac{1}{2} \times 12 = 6 //$$

$$\rightarrow 2.5 \times h = 6$$

$$\therefore h = \frac{6}{2.5} = \boxed{2.4} \text{ cm}$$

$$b) \frac{n}{2} = \frac{100}{2} = 50 //$$

so median lies in 2nd interval.



$$\frac{m - 6.5}{8.5 - 6.5} = \frac{50 - 38}{63 - 38}$$

$$m - 6.5 = 2 \left(\frac{12}{25} \right)$$

$$m = 6.5 + \frac{24}{25} = \boxed{7.46}$$

$$c) \text{mean} = \frac{\sum fx}{\sum f} = \frac{38(4.5) + 25(7.5) + 18(9.5) + 12(13) + 7(18)}{100}$$

$$= \frac{811.5}{100} = \boxed{8.12} \text{ (3sf)}$$

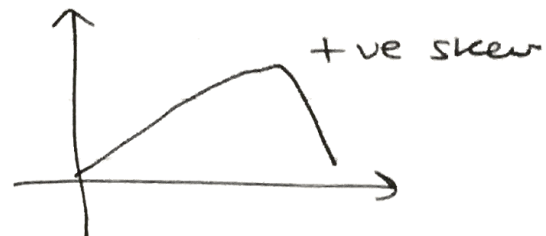
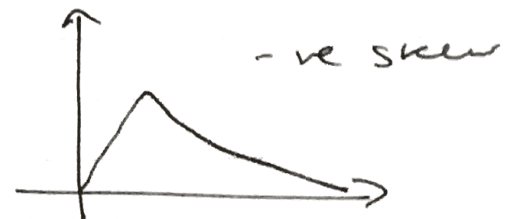
$$d) \text{s.d} = \sqrt{\frac{\sum fx^2}{\sum f} - (8.12)^2} = \sqrt{\frac{8096.25}{100} - (8.12)^2}$$

$$= \boxed{3.89}$$

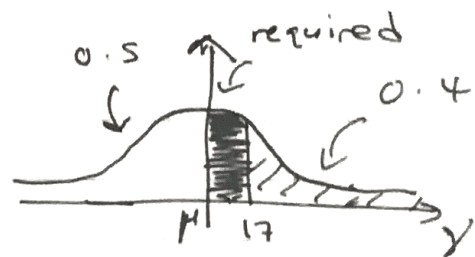
$$e) \frac{3(8.12 - 7.46)}{3.89} = 0.5055... = \boxed{0.51}$$

f) The skewness are different for Mondays and Fridays, $\left(\begin{array}{l} -0.22 < 0 \\ 0.51 > 0 \end{array} \right)$

So Friday will have longer delays since -ve skew



$$Q3a) P(\mu < Y < 17) = 0.6 - 0.5 = \boxed{0.1}$$



$$\begin{aligned} b) P(\mu - \sigma < Y < 17) &= P(Y < 17) - P(Y < \mu - \sigma) \\ &= 0.6 - P\left(Z < \frac{-\sigma}{\sigma}\right) \\ &= 0.6 - P(Z < -1) \\ &= 0.6 - [1 - P(Z < 0)] \\ &= \boxed{0.4413} \end{aligned}$$

$$r + y = 63$$

$$\begin{aligned}
 \bullet \text{ Q4a) } P(\text{green}) &= P(G) + P(RG) + P(YG) \\
 &= \frac{1}{64} + \left(\frac{63-y}{64}\right)\left(\frac{1}{63}\right) + \left(\frac{63-r}{64}\right)\left(\frac{1}{63}\right) \\
 &= \frac{63 + 63 + 63 - r - y}{64 \times 63} \\
 &= \frac{189 - (r+y)}{64 \times 63} = \frac{189 - 63}{64 \times 63} \\
 &= \boxed{\frac{1}{32}}
 \end{aligned}$$

$$b) P(RR) = \left(\frac{63-y}{64}\right)\left(\frac{62-y}{63}\right) = \frac{5}{84}$$

$$\underline{y=63-r} : \left(\frac{63-63+r}{64}\right)\left(\frac{62-63+r}{63}\right) = \frac{5}{84}$$

$$r(r-1) = \frac{5}{84} \times 64 \times 63$$

$$r^2 - r = 240$$

$$\text{hence } r^2 - r - 240 = 0$$

$$c) \left. \begin{array}{l} a=1 \\ b=-1 \\ c=-240 \end{array} \right\} r = \frac{1 \pm \sqrt{1 - 4(-240)}}{2}$$

$$r = \frac{1 \pm \sqrt{961}}{2}$$

$$r = \frac{1 \pm 31}{2}$$

$$\text{now } r > 0 \text{ so } r = \frac{1+31}{2} \\ = \boxed{16}$$

$$d) P(\text{both red} \mid \text{at least one red}) = \frac{P(\text{both red} \cap \text{one is red})}{P(\text{at least one red})}$$

$$= \frac{P(\text{both red})}{P(\text{at least one red})}$$

$$= \frac{P(RR)}{1 - P(\text{no red})}$$

$$= \frac{5}{84} \quad \leftarrow \text{we are given this.}$$

$$\frac{\frac{5}{84}}{37}$$

$$= \boxed{\frac{5}{37}}$$

$$P(\geq 1 \text{ red}) = P(R) + P(R'R)$$

$$= \frac{16}{64} + \frac{48}{64} \times \frac{16}{63} = \frac{37}{84} //$$

Q5a) $E(X) (= -1b + 0a + 2a + 4a + 5b)$

$= 2$ because the distribution is symmetric about $x = 2$

b) $-b + 2a + 4a + 5b = 2$

$$6a + 4b = 2$$

$$\boxed{3a + 2b = 1}$$

c) $E(X^2) = b + 4a + 16a + 25b$
 $= 26b + 20a$

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$\Rightarrow \text{Var}(X) + [E(X)]^2 = E(X^2) //$$

d) from (b), $b = \frac{1}{2} - \frac{3}{2}a$

$$20a + 26\left(\frac{1}{2} - \frac{3}{2}a\right) = 11.1$$

$$20a + 13 - 39a = 11.1$$

$$19a = 13 - 11.1 = 1.9$$

$$a = \frac{1.9}{19} = \boxed{0.1}$$

$$\therefore b = \frac{1}{2} - \frac{3}{2}(0.1) = \boxed{0.35}$$

ei) $E(Y) = E(10 - 3X) = -3E(X) + 10$
 $= -3(2) + 10 = \boxed{4}$

ii) $\text{Var}(Y) = \text{Var}(10 - 3X) = 3^2 \text{Var}(X) = 9(7.1) = \boxed{63.9}$

f) $P(Y > X) = P(10 - 3X > X) = P(10 > 4X)$

$$= P\left(\frac{5}{2} > X\right)$$

$$= P(X < 2.5)$$

$$= P(X=2) + P(X=0) + P(X=-1)$$

$$= 2a + b$$

$$= 2(0.1) + 0.35 = \boxed{0.55}$$

$$Q6a) S_{eh} = 31070 - \frac{(61)(6370)}{8} = \boxed{-17501.25}$$

$$S_{ee} = 693 - \frac{(61)^2}{8} = \boxed{227.875}$$

b) $r \approx -1$ \therefore strong -ve correlation suggested.
So data supports the linear model.

$$c) b = \frac{S_{eh}}{S_{hh}}$$

$$r = \frac{S_{eh}}{\sqrt{S_{hh} \times S_{ee}}} = -0.985$$

$$\therefore \sqrt{S_{hh} \times 227.875} = \frac{-17501.25}{-0.985}$$

$$S_{hh} \times 227.875 = \left(\frac{17501.25}{0.985} \right)^2$$

$$S_{hh} = \frac{\left(\frac{17501.25}{0.985} \right)^2}{227.875} = \underline{\underline{1385380.26}}$$

$$\therefore b = \frac{-17501.25}{1385380.26} = \underline{\underline{-0.013}}$$

$$a = \bar{t} - b\bar{h} = \frac{61}{8} - (-0.013)\left(\frac{6370}{8}\right)$$

$$= 17.68 \dots = 17.7 \underline{\underline{}}$$

so $t = 17.7 - 0.013h$

$$\hookrightarrow \boxed{t = 17.7 - 0.0126h}$$

↑
3sf.

d) temperature at sea level = 17.7°C

e) $150 \times b = \boxed{1.89^\circ}$

(Q7a) $W \sim N(140, 40^2)$

$$P(W < 92) = P\left(Z < \frac{92-140}{40}\right) = P(Z < -1.2)$$

$$= 1 - P(Z < 1.2) = 1 - 0.8849$$

$$= 0.1151 \underline{\underline{}}$$

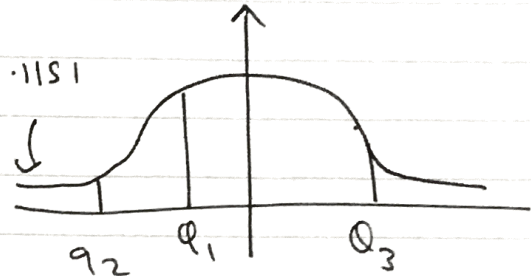
so % $\rightarrow \boxed{11.5\%}$

$$b) P(W > 92) = 1 - 0.1151 = 0.8849$$

$$0.25 \times 0.8849 = \boxed{0.2212}$$

$$c) P(W < Q_1) = 0.1151 + 0.2212$$

$$P(W < Q_1) = 0.3363$$



$$P\left(Z < \frac{Q_1 - 140}{40}\right) = 0.3363$$

$$P\left(Z < \frac{140 - Q_1}{40}\right) = 0.6637$$

$$\therefore \frac{140 - Q_1}{40} \approx 0.42$$

$$Q_1 \approx 140 - 40(0.42) \approx 123.2$$

$$\text{so } \boxed{Q_1 = 123 \text{ g}}$$

$$d) P(W < Q_1) = 0.2212$$

$$P(W > Q_3) = 0.2212$$

$$P(Q_1 < W < Q_3) = 1 - 2(0.2212) = 0.5576$$

$$\therefore P(\text{required}) = 6 \times (0.2212)(0.2212)(0.5576)$$

=

d) Betty only picks sold potatoes.

$$\therefore P(W < Q_1) = P(W > Q_3) = \frac{1}{4}$$

$$P(Q_1 < W < Q_3) = \frac{1}{2}$$

$$P(\text{required}) = 6 \times \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{2}\right) = \boxed{\frac{3}{16}}$$

There are 6 different ways these potatoes could be arranged.

(ie) the first could be $< Q_1$, and the second between Q_1 and Q_3 or maybe the first is $> Q_3$ and the second is $< Q_1$, etc