



S15 S2

1. In a survey it is found that barn owls occur randomly at a rate of 9 per 1000 km².
- (a) Find the probability that in a randomly selected area of 1000 km² there are at least 10 barn owls. (2)
- (b) Find the probability that in a randomly selected area of 200 km² there are exactly 2 barn owls. (3)
- (c) Using a suitable approximation, find the probability that in a randomly selected area of 50000 km² there are at least 470 barn owls. (6)

$$a) x = \# \text{ barn owls per } 1000 \text{ km}^2 \approx x \sim P_0(9)$$

$$P(x \geq 10) = 1 - P(x \leq 9) = 1 - 0.5874 = 0.4126$$

$$b) y = \# \text{ barn owls per } 200 \text{ km}^2 \approx y \sim P_0(1.8)$$

$$P(y=2) = \frac{e^{-1.8} \times 1.8^2}{2!} = 0.2678$$

$$c) t = \# \text{ barn owls per } 50000 \text{ km}^2 \approx t \sim P_0(450)$$

$$\approx t \sim N(450, 450) \quad P(t \geq 470) \approx P(t > 469.5)$$

$$\approx P\left(z > \frac{469.5 - 450}{\sqrt{450}}\right) = P(z > 0.92) \\ = 1 - \Phi(0.92) = 0.1788$$

2. The proportion of houses in Radville which are unable to receive digital radio is 25%.
In a survey of a random sample of 30 houses taken from Radville, the number, X , of houses which are unable to receive digital radio is recorded.

(a) Find $P(5 \leq X < 11)$

(3)

A radio company claims that a new transmitter set up in Radville will reduce the proportion of houses which are unable to receive digital radio. After the new transmitter has been set up, a random sample of 15 houses is taken, of which 1 house is unable to receive digital radio.

(b) Test, at the 10% level of significance, the radio company's claim. State your hypotheses clearly.

(5)

$$\begin{aligned} \text{a) } X &\sim B(30, 0.25) & P(5 \leq X < 11) &= P(X \leq 10) - P(X \leq 4) \\ & & &= 0.8943 - 0.0979 = \underline{0.7964} \end{aligned}$$

b) $y = \#$ houses that cannot receive signal

$$\begin{aligned} y &\sim B(15, 0.25) & H_0: p &= 0.25 \\ & & H_1: p &< 0.25 \end{aligned}$$

$$P(y \leq 1) = 0.0802 < 10\%$$

\therefore result is ~~not~~ statistically significant
 \therefore enough evidence to reject null hypothesis
 \therefore Company's claim is supported

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3. A random variable X has probability density function given by
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$$f(x) = \begin{cases} kx^2 & 0 \leq x \leq 2 \\ k\left(1 - \frac{x}{6}\right) & 2 < x \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

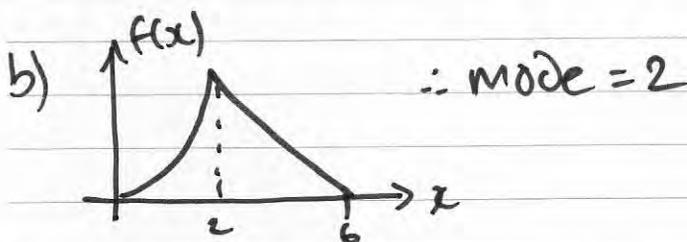
(a) Show that $k = \frac{1}{4}$ (4)

(b) Write down the mode of X . (1)

(c) Specify fully the cumulative distribution function $F(x)$. (5)

(d) Find the upper quartile of X . (4)

a) $\int f(x) dx = 1 \Rightarrow k \int_0^2 x^2 dx + k \int_2^6 \left(1 - \frac{x}{6}\right) dx = 1$
 $\Rightarrow k \left[\left[\frac{1}{3}x^3\right]_0^2 + \left[x - \frac{x^2}{12}\right]_2^6 \right] = k \left[\left(\frac{8}{3} - 0\right) + (6-3) - \left(2 - \frac{4}{12}\right) \right] = 1$
 $\Rightarrow k \left(\frac{8}{3} + 3 - \frac{5}{3} \right) = 1 \Rightarrow 4k = 1 \therefore k = \frac{1}{4}$



c) $0 \leq x \leq 2$ $F(x) = \frac{1}{4} \int_0^x t^2 dt = \frac{1}{4} \left[\frac{1}{3} t^3 \right]_0^x = \frac{1}{12} x^3$
 $2 < x \leq 6$ $F(x) = F(2) + \int_2^x \frac{1}{4} \left(1 - \frac{t}{6}\right) dt = \frac{1}{4} \left[t - \frac{t^2}{12} \right]_2^x + \frac{2}{3}$
 $= \frac{1}{4} \left[\left(x - \frac{x^2}{12}\right) - \left(2 - \frac{4}{12}\right) \right] + \frac{2}{3} = \frac{1}{4} x - \frac{x^2}{48} + \frac{1}{4}$

$$f(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}x^3 & 0 \leq x \leq 2 \\ \frac{1}{48}(12 + 12x - x^2) & 2 < x \leq 6 \\ 1 & \text{otherwise} \end{cases}$$

$$d) f(Q_3) = 0.75$$

$$\Rightarrow \frac{1}{48}(12 + 12x - x^2) = \frac{3}{4} \Rightarrow x^2 - 12x + 24 = 0$$

$$\Rightarrow (x-6)^2 = 36 - 24 \Rightarrow x-6 = \pm\sqrt{12} = \pm 2\sqrt{3}$$

$$\therefore x = 6 - 2\sqrt{3} \quad Q_3 = \frac{2 \cdot 54}{2}$$

4. The continuous random variable L represents the error, in metres, made when a machine cuts poles to a target length. The distribution of L is a continuous uniform distribution over the interval $[0, 0.5]$

(a) Find $P(L < 0.4)$. (1)

(b) Write down $E(L)$. (1)

(c) Calculate $\text{Var}(L)$. (2)

A random sample of 30 poles cut by this machine is taken.

(d) Find the probability that fewer than 4 poles have an error of more than 0.4 metres from the target length. (3)

When a new machine cuts poles to a target length, the error, X metres, is modelled by the cumulative distribution function $F(x)$ where

$$F(x) = \begin{cases} 0 & x < 0 \\ 4x - 4x^2 & 0 \leq x \leq 0.5 \\ 1 & \text{otherwise} \end{cases}$$

(e) Using this model, find $P(X > 0.4)$ (2)

A random sample of 100 poles cut by this new machine is taken.

(f) Using a suitable approximation, find the probability that at least 8 of these poles have an error of more than 0.4 metres. (3)

a) $P(L < 0.4) = \frac{4}{5}$

b) $E(L) = 0.25$

c) $V(L) = \frac{(b-a)^2}{12} = \frac{0.5^2}{12} = \frac{1}{48}$

d) $X \sim B(30, \frac{1}{5})$ $X = \text{Pole with an error} > 0.4m$

$P(X < 4) = P(X \leq 3) = 0.1227$

e) $F(0.4) = 4(0.4) - 4(0.4)^2 = 0.96$

$\therefore P(X > 0.4) = 0.04$

f) $y = \# \text{Poles with an error} > 0.4m$

~~$y \sim B(100, 0.04)$~~

~~$\approx y \sim N(4, (\sqrt{3.84})^2)$~~

~~$\mu = np = 4$~~

~~$\sigma^2 = np(1-p) = 4(0.96) = 3.84$~~

~~$P(y \geq 8)$~~

~~$\approx P(y \geq 7.5)$~~

~~$P(y \geq 7)$~~

~~$\approx P(Z \geq \frac{7.5 - 4}{\sqrt{3.84}})$~~

~~$P(Z > 1.78) = 1 - \Phi(1.78) = 0.0375$~~

alt $y \sim B(100, 0.04) \approx Po(4)$

$P(y \geq 8) = 1 - P(y \leq 7) = 0.0511$

5. *Liftsforall* claims that the lift they maintain in a block of flats breaks down at random at a mean rate of 4 times per month. To test this, the number of times the lift breaks down in a month is recorded.

- (a) Using a 5% level of significance, find the critical region for a two-tailed test of the null hypothesis that ‘the mean rate at which the lift breaks down is 4 times per month’. The probability of rejection in each of the tails should be as close to 2.5% as possible. (3)

Over a randomly selected 1 month period the lift broke down 3 times.

- (b) Test, at the 5% level of significance, whether *Liftsforall*'s claim is correct. State your hypotheses clearly. (2)
- (c) State the actual significance level of this test. (1)

The residents in the block of flats have a maintenance contract with *Liftsforall*. The residents pay *Liftsforall* £500 for every quarter (3 months) in which there are at most 3 breakdowns. If there are 4 or more breakdowns in a quarter then the residents do not pay for that quarter.

Liftsforall installs a new lift in the block of flats.

Given that the new lift breaks down at a mean rate of 2 times per month,

- (d) find the probability that the residents do not pay more than £500 to *Liftsforall* in the next year. (6)

a) $X = \text{breakdowns per month}$ $X \sim P_0(4)$

$$P(X \leq L) \approx 0.025$$

$$P(X \geq U) \approx 0.025$$

$$P(X \leq 0) = 0.0183^*$$

$$1 - P(X \leq U-1) \approx 0.025$$

$$P(X \leq 1) = 0.0916$$

$$P(X \leq U-1) \approx 0.025$$

$$\therefore L = 0$$

$$P(X \leq 7) = 0.9489 \quad \therefore U-1 = 8$$

$$P(X \leq 8) = 0.9786^* \quad \therefore U = 9$$

CR $(X=0) \cup (X \geq 9)$

$$b) H_0: \lambda = 4$$

$X=3$ IS NOT in the critical region

$$H_1: \lambda \neq 4$$

\therefore result is NOT statistically significant

\therefore not enough evidence to reject null

\therefore Claim IS supported

$$c) A.S.L = 0.0183 + 0.0214 = \underline{0.0397} \quad 3.97\% \text{ C.L.}$$

d) $Y = \# \text{ breakdowns per 3 months}$ $Y \sim P_0(6)$

$$\text{do not pay if } Y \geq 4 = 1 - P(Y \leq 3) = 0.8488$$

do not pay more than £500 in a year if

$Y \geq 4$ occurs zero or once in 4 quarters

$$\text{zero} \Rightarrow 0.8488^4$$

$$= 0.5191$$

$$\text{once} \Rightarrow 4 \times 0.8488^3 \times 0.1512$$

$$= 0.3699^{\dagger}$$

$$\underline{0.8890}$$

$\underline{\underline{2}}$

7. A bag contains a large number of 10p, 20p and 50p coins in the ratio 1:2:2
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A random sample of 3 coins is taken from the bag.

Find the sampling distribution of the median of these samples.

(7)

$$\text{Median} = 10p \quad 10, 10, 10 = \left(\frac{1}{5}\right)^3 = \frac{1}{125}$$

$$(x3) \quad 10, 10, 20 = \left(\frac{1}{5}\right)^2 \times \frac{2}{5} \times 3 = \frac{6}{125}$$

$$(x3) \quad 10, 10, 50 = \left(\frac{1}{5}\right)^2 \times \frac{2}{5} \times 3 = \frac{6}{125}$$

$$\text{Median} = 20p \quad (x3) \quad 10, 20, 20 = \left(\frac{1}{5}\right) \times \left(\frac{2}{5}\right)^2 \times 3 = \frac{12}{125}$$

$$(x6) \quad 10, 20, 50 = \left(\frac{1}{5}\right) \times \left(\frac{2}{5}\right)^2 \times 6 = \frac{24}{125}$$

$$20, 20, 20 = \left(\frac{2}{5}\right)^3 = \frac{8}{125}$$

$$(x3) \quad 20, 20, 50 = \left(\frac{2}{5}\right)^2 \times \frac{1}{5} \times 3 = \frac{24}{125} \Rightarrow \frac{68}{125}$$

$$\text{Median} = 50p \quad (x3) \quad 10, 50, 50 = \frac{1}{5} \times \left(\frac{2}{5}\right)^2 \times 3 = \frac{12}{125}$$

$$(x3) \quad 20, 50, 50 = \left(\frac{2}{5}\right)^2 \times \frac{1}{5} \times 3 = \frac{24}{125} \quad \frac{44}{125}$$

$$50, 50, 50 = \left(\frac{1}{5}\right)^3 = \frac{8}{125}$$

Median, m	10p	20p	50p
$P(M=m)$	$\frac{13}{125}$	$\frac{68}{125}$	$\frac{44}{125}$