

## S2 October 2016 IAL (MA)

Q1a)  $H_0: p = 0.05 \quad X \sim B[30, 0.05]$

$H_1: p > 0.05 \quad P(X \geq 4) = 1 - P(X \leq 3)$

$$= 1 - 0.9392 = 0.0608 //$$

$$0.0608 < 0.10$$

$\therefore$  Result is significant.

Reject  $H_0$ .

Evidence suggests that the claim is true.

b) For new sample,  $Y \sim B[90, 0.05]$

$n$  large,  $p$  small.  $np = 90 \times 0.05$   
 $= 4.5 //$

so  $Y \approx \sim P_0(4.5)$

$$\left. \begin{array}{l} P(Y \geq 7) = 0.1689 \\ P(Y \geq 8) = 0.0866 \end{array} \right\} \boxed{Y \geq 8} \text{ is the C.R.} //$$

$$Q2a) \quad F(t) = \begin{cases} 0, & t < 8 \\ \frac{1}{96} (74t - \frac{5}{2}t^2 + u), & 8 \leq t \leq 12 \\ 1, & t > 12 \end{cases}$$

$$F(12) = 1 \quad \text{or} \quad F(8) = 0$$

$$\text{using } \underline{F(8) = 0} : \quad \frac{1}{96} (74(8) - \frac{5}{2}(8)^2 + u) = 0$$

$$\therefore K = \frac{5}{2}(8)^2 - 74(8) = \boxed{-432}$$

$$b) \quad \frac{d}{dt} \left( \frac{1}{96} (74t - \frac{5}{2}t^2 + -432) \right)$$

$$= \frac{1}{96} (74 - 5t)$$

$$\therefore f(t) = \begin{cases} \frac{1}{96} (74 - 5t), & 8 \leq t \leq 12 \\ 0, & \text{otherwise.} \end{cases}$$

$$c) \quad \text{highest point is at } t = 8. \quad \therefore \boxed{\text{mode} = 8}$$

$$d) \quad F(m) = 0.5$$

$$F(m) = \frac{1}{96} (74m - \frac{5}{2}m^2 - 432) = 0.5$$

$$74m - 2.5m^2 - 432 = 48$$

$$2.5m^2 - 74m + 480 = 0$$

By Quadratic Formula...

$$m = 20, \quad m = 9.6$$

$m$  has to be in the range  
 $8 \leq m \leq 12$

So  $m = 9.6$

$$\begin{aligned} \text{e) } P(\text{required}) &= F(9) = \frac{1}{96} (74(9) - \frac{5}{2}(9)^2 - 432) \\ &= \boxed{0.328} \end{aligned}$$

$$\text{f) } P(\text{lifetime} < 11 \text{ hrs} \mid \text{lifetime} > 9 \text{ hrs}) = \frac{P(9 < t < 11)}{P(t > 9)}$$

$$= \frac{F(11) - F(9)}{1 - F(9)}$$

$$F(11) = \frac{1}{96} (74(11) - \frac{5}{2}(11)^2 - 432) = \frac{53}{64}$$

$$\therefore P(\text{required}) = \frac{\frac{53}{64} - 0.328}{1 - 0.328} = \boxed{0.744}$$

Q3a)  $X \sim B[20, 0.4]$

$$\begin{aligned} \text{b) } P(4 \leq X < 9) &= P(X < 9) - P(X \leq 3) \\ &= P(X \leq 8) - P(X \leq 3) \\ &= \boxed{0.5796} \end{aligned}$$

c)  $P(\text{total} > 0)$

$$\begin{aligned} \text{Total no. of points} &= 7X - 3(20 - X) \\ &= 7X - 60 + 3X \\ &= 10X - 60 \end{aligned}$$

$$\begin{aligned} \Rightarrow P(\text{required}) &= P(10X - 60 > 0) \\ &= P(X > 6) = 1 - P(X \leq 6) \\ &= 1 - 0.2500 \\ &= \boxed{0.7500} \end{aligned}$$

● d) Total points =  $10X - 60$

$$\text{Variance of total points} = \text{Var}(10X - 60)$$

$$= 10^2 \text{Var}(X)$$

$$= 100 \text{Var}(X) //$$

$$\text{Var}(X) = np(1-p) = 20(0.4)(1-0.4)$$

$$= 4.8 //$$

$$\therefore \text{Var}(\text{total points}) = 100 \times 4.8$$

$$= \boxed{480}$$

Q4ai) total area = 1.

$$\text{Area } \triangle = \frac{1}{2} \times 5 \times u = \frac{5u}{2}$$

$$\text{Area } \square = (10.5 - 5) \times u = 5.5u // = \frac{11}{2}u$$

$$\therefore \left( \frac{5}{2} + \frac{11}{2} \right) u = 1$$

$$8u = 1 \quad \therefore u = \frac{1}{8}$$

hence area  $\triangle = \frac{5}{16}$

$$\therefore \int_0^5 (M \text{ or } dsc) = \frac{5}{16} //$$

$$\left[ \frac{mx^2}{2} \right]_0^5 = \frac{5}{16}$$

$$\frac{m}{2} (25) = \frac{5}{16}$$

$$m = \frac{5}{16} \times \frac{2}{25} = \boxed{\frac{1}{40}}$$

$$b) E(X) = \int_0^5 (mx^2) dx + \int_5^{10.5} (4x) dx$$

$$= \frac{1}{40} \left[ \frac{x^3}{3} \right]_0^5 + \frac{1}{8} \left[ \frac{x^2}{2} \right]_5^{10.5}$$

$$= \frac{1}{40} \left[ \frac{125}{3} \right] + \frac{1}{8} \left[ \frac{441}{8} - \frac{25}{2} \right]$$

$$= \boxed{\frac{1223}{192}} = 6.37.$$

$$c) \frac{1}{40} \int_0^y [x] dx = 0.25$$

where  $y$  is the lower Quartile  
Area  $\Delta = \frac{5}{16} > 0.25$   
so LQ lies in first interval.

$$\frac{1}{40} \left[ \frac{x^2}{2} \right]_0^y = 0.25$$

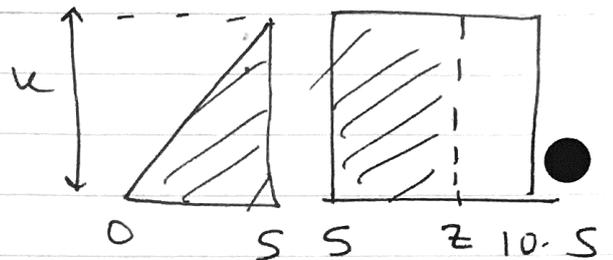
$$\frac{1}{40} \left[ \frac{y^2}{2} \right] = 0.25$$

$$\therefore y^2 = 80 \times 0.25 = 20$$

$$\therefore y = \sqrt{20} = 4.47$$

for UQ:

Let  $z$  be upper Quartile



Then shaded area = 0.75.

$$\Rightarrow \left( \frac{5}{2} \times \frac{1}{8} \right) + (z-5) \times \frac{1}{8} = 0.75$$

} Area  $\Delta$ 
} Area  $\square$

$$\frac{(z-5)}{8} = 0.75 - \frac{5}{16} = \frac{7}{16}$$

$$z-5 = \frac{7}{2}$$

$$z = 5 + \frac{7}{2} = 8.5 = \text{UQ}$$

$$\therefore \text{IQR} = 8.5 - \sqrt{20} = \boxed{4.03}$$

$$\text{Q5a)} \quad L \sim U[20, 40]$$

$$\begin{aligned} \text{b)} \quad P(27.5 < L < 28.5) &= \frac{28.5 - 27.5}{20} \\ &= \boxed{\frac{1}{20}} \end{aligned}$$

$$\text{c)} \quad P(\text{required}) = P\left(\left(\frac{L}{4}\right)^2 < 64\right)$$

if a string length  $L$  is used to form a square then each side will have length  $\frac{L}{4}$   $\therefore$  Area =  $\left(\frac{L}{4}\right)^2$

$$\begin{aligned} \Rightarrow \quad P\left(\frac{L^2}{16} < 64\right) &= P(L^2 < 1024) \\ &= P(L < 32) = \frac{(32 - 20)}{20} \\ &= \boxed{0.60} \end{aligned}$$

$$\text{d)} \quad \text{Area (bigger) square} = \frac{L^2}{16} \quad \parallel$$

$$\text{Area (smaller) square} = \left(\frac{40-L}{4}\right)^2 \quad \parallel$$

[ the shorter part will have length  $40-L$   
 $\therefore$  the square made will have side length of  $\frac{40-L}{4}$ . hence area =  $\left(\frac{40-L}{4}\right)^2$  ]

$$\text{Difference in area} = \frac{L^2}{16} - \left(\frac{(40-L)}{4}\right)^2$$

$$= \frac{L^2}{16} - \frac{(40-L)^2}{16}$$

$$= \frac{L^2 - (1600 - 80L + L^2)}{16}$$

$$= \frac{80L - 1600}{16} = 5L - 100 //$$

$$P(\text{required}) = P(5L - 100 > 81)$$

$$= P(5L > 181)$$

$$= P\left(L > \frac{181}{5}\right) = P(L > 36.2) //$$

$$= \frac{40 - 36.2}{20} = \boxed{0.19}$$

● Q6a)  $X \sim P_0(\lambda)$

$$\begin{aligned} P(X \leq 1) &= P(X=0) + P(X=1) \\ &= e^{-\lambda} + (e^{-\lambda})(\lambda) \\ &= e^{-\lambda}(\lambda+1) // \end{aligned}$$

● b) for 5 weeks...  $X_2 \sim P_0(\frac{\lambda}{2})$

$$P(X_2=1) = \boxed{(e^{-\frac{\lambda}{2}})(\frac{\lambda}{2})} //$$

ci) For 100 weeks, mean will be large ( $\lambda > 10$ ).  
So a normal approximation is valid.

ii)  $Y \sim P_0(10\lambda) \rightarrow Y \approx N(10\lambda, 10\lambda) //$

(applying c.c)  $P(Y < 15) = P(Y < 14.5) = 0.0179$

$$= P(Z < \frac{14.5 - 10\lambda}{\sqrt{10\lambda}}) = 0.0179 //$$

$$\therefore P(Z < \frac{10\lambda - 14.5}{\sqrt{10\lambda}}) = 0.9821$$

but  $P(Z < 2.10) = 0.9821$

$$\therefore 2 \cdot 10 = \frac{10\lambda - 14.5}{\sqrt{10\lambda}}$$

$$10\lambda - (2 \cdot 10\sqrt{10})\lambda^{\frac{1}{2}} - 14.5 = 0$$

$$10\lambda - 6.64\lambda^{\frac{1}{2}} - 14.5 = 0 //$$

$$\lambda^{\frac{1}{2}} = 1.58... //, \quad \lambda^{\frac{1}{2}} = -0.917..$$

$$\therefore \lambda = 1.58^2 //$$

reject,  $\lambda > 0$ .

$$\boxed{= 2.5}$$

Q7a) S could be 2, 3, 4, 5, 6

$$P(S=2) = \frac{5}{8} \times \frac{5}{8} = \frac{25}{64} //$$

$$P(S=3) = 2 \times \left[ \frac{5}{8} \right] \left[ \frac{2}{8} \right] = \frac{5}{16} //$$

$$P(S=4) = \left( \frac{2}{8} \right) \left( \frac{2}{8} \right) + 2 \left( \frac{5}{8} \right) \left( \frac{1}{8} \right) = \frac{7}{32} //$$

$$P(S=5) = 2 \left( \frac{1}{8} \right) \left( \frac{2}{8} \right) = \frac{1}{16} //$$

$$P(S=6) = \left( \frac{1}{8} \right) \left( \frac{1}{8} \right) = \frac{1}{64} //$$

$$\begin{array}{c|c|c|c|c|c} \therefore S & 2 & 3 & 4 & 5 & 6 \\ \hline P(S=S) & \frac{25}{64} & \frac{5}{16} & \frac{7}{32} & \frac{3}{32} & \frac{1}{64} \end{array}$$

b) let  $X = \#$  of scoops ordered by  $n$  customers.

$$P(X > n) > 0.99$$

$n$  is the min no. of scoops that can be ordered by  $n$  customers.

$$\text{So } P(X > n) = 1 - P(X = n)$$

$$P(X = n) = \left(\frac{5}{8}\right)^n \quad \left\{ \begin{array}{l} \text{all } n \text{ customers} \\ \text{ordering 1 scoop.} \end{array} \right.$$

$$\therefore P(X > n) = 1 - \left(\frac{5}{8}\right)^n$$

$$\Rightarrow 1 - \left(\frac{5}{8}\right)^n > 0.99$$

$$\left(\frac{5}{8}\right)^n < 0.01$$

$$n \ln\left(\frac{5}{8}\right) < \ln(0.01)$$

(signs change when  $\div$  by a negative number)

$$n > \frac{\ln(0.01)}{\ln\left(\frac{5}{8}\right)}$$

$$\therefore n > 9.798 \dots \quad \therefore \boxed{n_{\min} = 10}$$