

S2 January 2017 (MA) (IAL)

Q(a)  $W \sim N(32, 4^2)$

$$P(W=0) = \boxed{0}$$

b)  $X \sim B[20, 0.45]$

$$P(X=8) = \binom{20}{8} (0.45)^8 (0.55)^{12} = \boxed{0.162}$$

c)  $np(1-p) = \text{Variance} = 20(0.45)(0.55) = 4.95$

$$\therefore \sigma = \sqrt{4.95} = 2.22 \dots, \text{mean} = np = 9 \dots$$

so  $P(\text{required}) = P(9-2.22 < X < 9+2.22)$

$$= P(6.78 < X < 11.22)$$

$$= P(X=7) + P(X=8) + P(X=9) \\ + P(X=10) + P(X=11)$$

$$= P(7 \leq X \leq 11)$$

$$= P(X \leq 11) - P(X \leq 6)$$

$$= 0.8692 - 0.1299$$

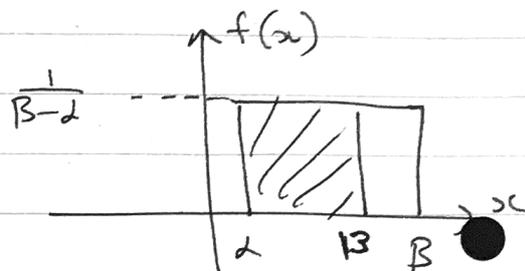
$$= \boxed{0.7393}$$

$$(Q2a) \quad X \sim U[\alpha, \beta]$$

$$E(X) = 8 \quad : \quad \frac{\alpha + \beta}{2} = 8$$

$$\therefore \boxed{\alpha + \beta = 16} \quad \sim \quad (1)$$

$$b) \quad (13 - \alpha) \times \frac{1}{\beta - \alpha} = 0.7$$



$$\frac{13 - \alpha}{\beta - \alpha} = 0.7$$

$$13 - \alpha = 0.7\beta - 0.7\alpha$$

$$0.7\beta + 0.3\alpha = 13$$

$$\times 10 : 7\beta + 3\alpha = 130 \quad //$$

$$\textcircled{1} \times 3 : 3\alpha + 3\beta = 48 \quad //$$

$$\begin{array}{r} 7\beta + 3\alpha = 130 \\ - (3\beta + 3\alpha = 48) \\ \hline \end{array}$$

$$4\beta + 0 = 82$$

$$\therefore \beta = \frac{82}{4} = \boxed{20.5}$$

$$\text{and } \alpha = 16 - \beta = \boxed{-4.5}$$

$$c) \text{Var}(X) = \frac{(b-a)^2}{12} = \frac{(20.5 - -4.5)^2}{12} = \boxed{\frac{625}{12}}$$

$$d) P(5 \leq X \leq 35) = P(5 \leq X \leq 20.5) \\ = \frac{(20.5 - 5)}{(20.5 + 4.5)} = \boxed{0.62}$$

Q3a)  $\lambda$  is large ( $> 10$ ).

$$b) X \sim P_0(3.5)$$

$$\left. \begin{aligned} P(X=3) &= 0.2158 \\ P(X=4) &= 0.1888 \end{aligned} \right\} \begin{aligned} &\text{mean is } 3.5 \text{ so consider} \\ &X=3 \text{ and } X=4. \\ &P(X=3) > P(X=4) \text{ so} \\ &\text{mode is } \boxed{3}. \end{aligned}$$

c) for 2 weeks,  $X_2 \sim P_0(7)$ .

$$P(X_2 > 5) = 1 - P(X_2 \leq 5) = 1 - 0.3007 \\ = \boxed{0.6993}$$

$$d) P(4 \text{ in } 1^{\text{st}} \text{ week} \mid 6 \text{ in } 2 \text{ weeks}) = \frac{P(4 \text{ in } 1^{\text{st}} \text{ week and } 6 \text{ in } 2^{\text{nd}} \text{ week})}{P(6 \text{ in } 2 \text{ weeks})} \\ = \frac{P(X=4) \cdot P(X=2)}{P(X_2=6)} = 0.1888 \left[ \frac{(e^{-3.5})(3.5)^2}{2} \right] \\ \frac{(e^{-7})(7^6)}{6!} \\ = \boxed{0.234}$$

e) For 40 wms,  $X_3 \sim P_0(140)$  //  $(40 \times 3.5 = 140)$  ●

$\lambda$  is large,  $X_3 \approx \sim N(140, 140)$

$$P(X_3 \geq 120) = P(X_3 > 119)$$

$$\text{(applying c.c)} = P(X_3 > 119.5)$$

$$= P\left(Z > \frac{119.5 - 140}{\sqrt{140}}\right)$$

$$= P(Z > -1.73)$$

$$= P(Z < 1.73) = \boxed{0.9582}$$

$$\text{Q4a)} E(X) = \int_0^4 x f(x) dx = \frac{3}{64} \int_0^4 [4x^3 - x^4] dx$$

$$= \frac{3}{64} \left[ \frac{4x^4}{4} - \frac{x^5}{5} \right]_0^4$$

$$= \frac{3}{64} \left[ 256 - \frac{1024}{5} \right] = 2.4 //$$

So mean # of hours =  $\boxed{2400}$

$$\text{b)} E(X^2) = \int_0^4 x^2 f(x) dx = \frac{3}{64} \int_0^4 [4x^4 - x^5] dx$$

$$= \frac{3}{64} \left[ \frac{4x^5}{5} - \frac{x^6}{6} \right]_0^4$$

$$= \frac{3}{64} \left[ \frac{4096}{5} - \frac{2048}{3} \right]$$

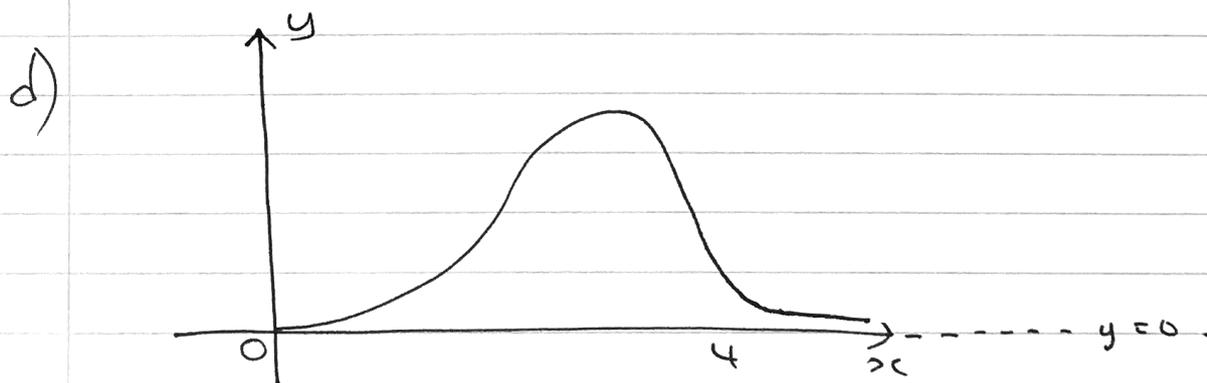
$$= 6.4 = E(x^2)$$

$$\text{Var}(x) = E(x^2) - [E(x)]^2$$

$$= 6.4 - (2.4)^2 = 0.64$$

$$\sigma_x = \sqrt{0.64} = \boxed{0.8}$$

- c) The model suggests it's impossible for a component to last longer than 4000 hours which may not necessarily be true.



- Q5a)  $X \sim \text{Po}(6)$ , where  $X$  = no. of defects in  $1\text{m}^2$  of cloth.

$$P(X \leq 7) = 0.7440 //$$

so for 12 pieces: ...  $Y \sim B[12, 0.7440]$

where  $Y$  = no. of pieces of cloth with at most 7 defects.

$$\begin{aligned} P(\text{required}) &= P(Y=6) = \binom{12}{6} (0.744)^6 (1-0.744)^6 \\ &= \boxed{0.044} \end{aligned}$$

bi)  $H_0: \lambda = 10$   
 $H_1: \lambda \neq 10$

ii)  $X_2 \sim \text{Po}(16)$   
 where  $X_2$  = no. of defects in  $2\text{m}^2$ .

iii) the range of values for the number of defects that would result in the null hypothesis  $H_0$  being rejected.

c)  $\left. \begin{aligned} P(X \leq 3) &= 0.0103 \\ P(X \leq 4) &= 0.0293 \end{aligned} \right\} 0.0103 \text{ is less than } 0.025.$

●  $0.0143$  is  $\left\{ \begin{aligned} P(X \geq 17) &= 0.0270 \\ P(X \geq 18) &= 0.0143 \end{aligned} \right.$  less than  $0.025$ .

$$d) 0.0103 + 0.0143 = \boxed{0.0246}$$

$$(2.46\%)$$

Q6)  $X \sim B[75, 0.96]$  where  $X =$  no. of seeds germinating.

$Y \sim B[75, 0.04]$  where  $Y =$  no. of seeds that don't germinate.

we use  $Y$  as  $p$  is small here, allowing us to use a poisson approximation.

9 beans didn't germinate.

$$np = 75 \times 0.04 = 3$$

$$\therefore Y \sim P_0(3)$$

$$P(Y \geq 9) = 1 - P(Y \leq 8) = 1 - 0.9962$$

$$= \boxed{0.0038}$$

$$0.0038 < 0.01$$

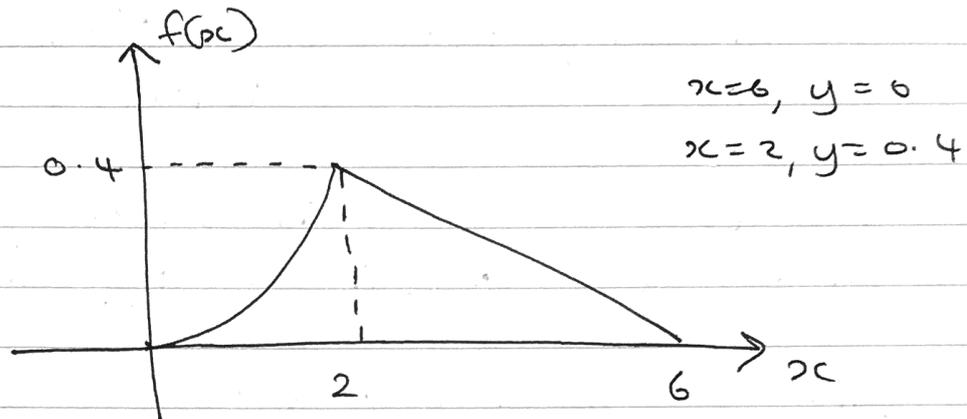
$\therefore$  Result is significant.

Reject  $H_0$ .

Evidence suggests the producer is overstating the probability.

Q7a)

$$f(x) = \begin{cases} \frac{1}{20} x^3, & 0 \leq x \leq 2 \\ \frac{1}{10} (6-x), & 2 < x \leq 6 \\ 0, & \text{otherwise} \end{cases}$$



$\frac{1}{10} (6-x)$  is a straight line.

b) mode  $\Rightarrow$  highest point.

so  $(x=2) = \text{mode}$

$$c) P(X > 2) = \frac{1}{10} \int_2^6 [6-x] dx = \frac{1}{10} \left[ 6x - \frac{x^2}{2} \right]_2^6$$

$$= \frac{1}{10} \left[ 36 - \frac{36}{2} \right] - \frac{1}{10} [12 - 2]$$

$$= \frac{1}{10} [8] = \boxed{0.8} //$$

alt: use area of  $\Delta$ :  $\frac{1}{2} \times (6-2) \times 0.4 = \boxed{0.8}$

for  $0 \leq x \leq 2$  :

$$d) \quad \frac{1}{20} \int_0^x (x^3) dx = \frac{1}{20} \left[ \frac{x^4}{4} \right] = \frac{1}{80} x^4 //$$

for  $2 < x \leq 6$  :

$$F(2) + \frac{1}{10} \int_2^x (6-x) dx = \frac{2^4}{80} + \frac{1}{10} \left[ 6x - \frac{x^2}{2} \right]_2^x$$

$$= 0.2 + \frac{1}{10} \left[ 6x - \frac{x^2}{2} \right] - \frac{1}{10} [10]$$

$$= \frac{1}{5} + \frac{6x}{10} - \frac{x^2}{20} - 1$$

$$= \frac{3x}{5} - \frac{x^2}{20} - \frac{4}{5} = \frac{1}{20} [12x - x^2 - 16] //$$

$$\text{so... } F(x) = \begin{cases} 0, & x < 0 \\ \frac{x^4}{80}, & 0 \leq x \leq 2 \\ \frac{1}{20} [12x - x^2 - 16], & 2 < x \leq 6 \\ 1, & x > 6 \end{cases}$$

$$e) P(X < a \mid X > 2) = \frac{5}{8}$$

$$P(X < a \mid X > 2) = \frac{P(X < a \cap X > 2)}{P(X > 2)}$$

$$\therefore \frac{P(2 < X < a)}{P(X > 2)} = \frac{5}{8}$$

$$\text{but } P(X > 2) = 0.8 \quad (\text{from c})$$

$$\therefore P(2 < X < a) = \frac{5}{8} (0.8) = \frac{1}{2}$$

$$P(X < a) - P(X < 2) = \frac{1}{2}$$

$$P(X < a) = \frac{1}{2} + P(X < 2)$$

$$P(X < a) = \frac{1}{2} + 0.2 = 0.7 //$$

$$F(a) = \boxed{0.7} = P(X < a)$$

$$f) F(a) = \frac{1}{20} (12a - a^2 - 16) = \frac{7}{10} \quad (\underline{a > 2})$$

$$\underline{\times 20} : 12a - a^2 - 16 = 14$$

By Quadratic  
Formula

$$a^2 - 12a + 30 = 0 \quad \therefore \boxed{a = 3.55}$$

reject  $a = 8.45$ , ( $a \leq 6$ )