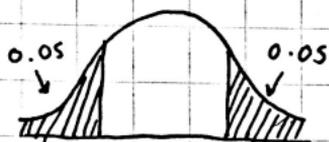JUNE 2017   S2 MA

1a) $X \sim B(25, 0.2)$

$P(X \leq 1) = 0.0274$

$P(X \leq 8) = 0.9532$
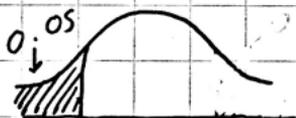
$1 - P(X \leq 8) = P(X \geq 9) = 0.04$

CR:

$\underline{X \leq 1}$ and $\underline{X \geq 9}$

b) $X \sim B(50, 0.2)$ $H_0 : p = 0.2$

$H_1 : p < 0.2$

$P(X \leq 5) = 0.0480$

CR: $X \leq 5$

6 is not within CR, $\therefore$ insufficient evidence to reject H_0, $\therefore$ accept H_0
$\therefore$ evidence that increasing number of pots does $\underline{NOT}$ reduce percentage of broken pots.

2 a i) $X \sim P_o (2.5)$

$P(X \geqslant 4) = 1 - P(X \leqslant 3)$

$\qquad\qquad 1 - 0.7576 \;\; = 0.2424$

ii) $Y \sim P_o (0.625)$

$2.5 = 60$

$0.625 = 15$

$\qquad\qquad P(Y = 3)$

$e^{-0.625} \times \dfrac{0.625^3}{3!} = 0.0218 \quad (3 \text{ s.f.})$

b)

$\qquad P(X = 0) > 0.8 \cdot$

$\qquad e^{-2.5t} > 0.8$

$\log_{(e^{-2.5})}(0.8) = t = 0.08925\ldots \text{ hours}$

$\qquad\qquad 60 \times 0.08925 = 5.36 \text{ mins}$

to nearest minute, $\underline{t = 5}$

2 c) $2 \cdot 5 \times 2 = 5$ $X \sim P_o(5)$

0.05

$H_o : \lambda = 5$
$H_1 : \lambda > 5$

$P(X \leq 9) = 0.9682$
$1 - P(X \leq 9) = 0.0318$
$1 - P(X \leq 9) = P(X \geq 10)$

CR: $X \geq 10$

$10 \geq 10$ ∴ reject H_o and accept
H_1. There is sufficient evidence
that the mean rate of telephone
calls has increased.

3)

$$f(x) \begin{cases} \frac{1}{9} x(4-x) & 1 \leq x \leq 4 \\ \\ 0 & \text{otherwise} \end{cases}$$

a) $E(X) = \frac{1}{9} \int_1^4 4x^2 - x^3 \, dx$

$\left(\int x f(x) \, dx \right)$

$$\frac{1}{9} \left[\frac{4}{3} x^3 - \frac{1}{4} x^4 \right]_1^4$$

$$\frac{1}{9} \left(\frac{64}{3} - \frac{13}{12} \right)$$

$$= \frac{9}{4} = \underline{2.25}$$

b) $P(X > 2.5)$

$$\int_{2.5}^{4} \frac{1}{9} x(4-x) \, dx$$

$$\frac{1}{9} \int_{2.5}^{4} 4x - x^2 \, dx$$

$$\frac{1}{9} \left[2 x^2 - \frac{1}{3} x^3 \right]_{2.5}^{4}$$

$$\frac{1}{9} \left(\frac{32}{3} - \frac{175}{24} \right)$$

$$\frac{1}{9} \times \frac{27}{8} = \frac{3}{8} = 0.375$$

c) life time X is in tens of hours

∴ 25 hours = 2.5 × ten

= $P(X > 2.5) = 0.375$

of 2 batteries = $(0.375)^2$

= 0.141

d) 16 hours = 1.6 × 10

$P(X > 1.6)$

$\frac{1}{9} \int_{1.6}^{4} 4x - x^2 \, dx$

$\frac{1}{9} \left[2x^2 - \frac{1}{3}x^3 \right]_{1.6}^{4} = \frac{1}{9} \left(\frac{32}{3} - \frac{1408}{375} \right)$

$= \frac{96}{125} = 0.768$

two batteries ∴ = $0.788^2 = (0.768)^2$

16 + 9 = 25 hours
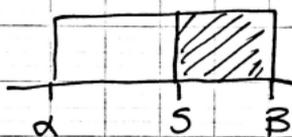
∴ $P\left(\dfrac{\text{working 25 hours}}{\text{working 16 hours}} \right)$

$= \dfrac{0.141\ldots}{(0.768)^2} = 0.2384\ldots$

4) $[\alpha, \beta]$

$E(X) = 3.5 \qquad P(X > 5) = \dfrac{2}{5}$

a) $E(X) = \dfrac{1}{2}(\alpha + \beta) = 3.5 \quad \cdots \text{①}$

$$P(X > 5) = \dfrac{\beta - 5}{\beta - \alpha} = \dfrac{2}{5}$$

$\cdots \text{③}$

from ① $\alpha + \beta = 7$

$\qquad \alpha = 7 - \beta \quad \cdots \text{②}$

② sub in ③

$$\dfrac{\beta - 5}{\beta - (7 - \beta)} = \dfrac{2}{5}$$

$$\dfrac{\beta - 5}{2\beta - 7} = \dfrac{2}{5} \qquad \beta - 5 = \dfrac{2}{5}(2\beta - 7)$$

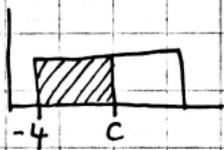$$\beta - \dfrac{4}{5}\beta = -\dfrac{14}{5} + 5$$

$\alpha = 7 - 11 \qquad\qquad \dfrac{1}{5}\beta = \dfrac{11}{5}$

$\underline{\alpha = -4} \qquad\qquad\qquad \underline{\beta = 11}$

b) $P(X < c) = \dfrac{2}{3}$

$$\frac{c - -4}{11 - -4} = \frac{2}{3}$$

$$\frac{c+4}{15} = \frac{2}{3}$$

$$c = \frac{2}{3} \times 15 - 4$$

$$\underline{c = 6}$$
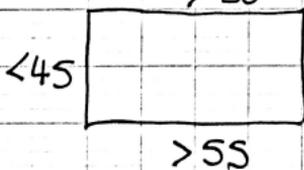
ii) $P(6 < X < 9)$

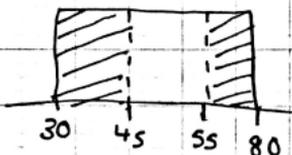$$\frac{9-6}{15} = \frac{3}{15} = \frac{1}{5} = \underline{0.2}$$

c) $[30, 80]$ perimeter $= 200$

if one side is <45 the other must be >55 so that the perimeter $= 200$

$\therefore P(s<45) + P(s>55)$

$$\frac{45-30}{50} + \frac{80-55}{50} = \frac{3}{10} + \frac{1}{2} = \frac{4}{5}$$

5 a)

$$P(M < 12) = 0.1$$

$$P\left(z < \frac{12-14}{6}\right) = 0.1$$

0.1 not in tables $\therefore$ $1 - 0.1 = 0.9$

$$1 - P\left(z < \frac{12-14}{6}\right) = 0.9$$

$$P\left(z < -\frac{12-14}{6}\right) = 0.9$$

closest to 0.9 in
tables = 0.8997
= 1.28

$$\therefore \frac{-12+14}{6} = 1.28$$

$$6 = \frac{2}{1.28} = \frac{25}{16} = 1.5625$$

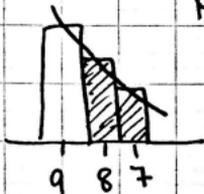b) $x \sim B(15, 0.1)$

$$P(X < 2) = P(X \leq 1) = 0.5490$$

c $X \sim B(n, 0.1)$

   $P(X < 9) = P(X \leq 8)$

   $\qquad\qquad\qquad 0.1 \times 0.9 = 0.09$

   $X \sim N(0.1n, 0.09n)$

continuity
correction

   $P(X < 8.5) = 0.3085$

9 8 7

   $P\left(z < \dfrac{8.5 - 0.1n}{\sqrt{0.09n}}\right) = 0.3085$

   $1 - P\left(z < \dfrac{8.5 - 0.1n}{\sqrt{0.09n}}\right) = 1 - 0.3085$

   $\qquad\qquad\qquad\qquad\qquad\qquad = 0.6915$

   $P\left(z - \dfrac{8.5 - 0.1n}{\sqrt{0.09n}}\right) = 0.6915$

   $\therefore \quad \dfrac{-8.5 + 0.1n}{\sqrt{0.09n}} = 0.5$

   $-8.5 + 0.1n = 0.5 \times 0.3 \times n^{1/2}$

   $0.1n - 0.15n^{1/2} - 8.5 = 0$

let $n^{1/2} = x$

   $0.1x^2 - 0.15x - 8.5 = 0$

   $x = 10 \text{ or } x = -8.5$

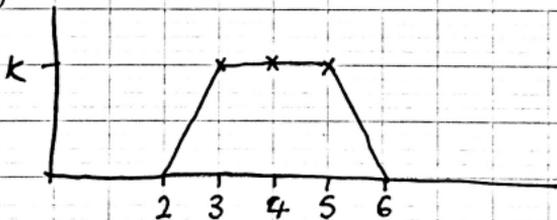continued c) accept $x = 10$ as time cannot be negative

$$\therefore n^{1/2} = 10$$
$$n = 10^2 = 100$$
$$\underline{\underline{n = 100}}$$

6)

$$f(x) \begin{cases} k(x-2) & 2 \le x \le 3 \\ k & 3 < x < 5 \\ k(6-x) & 5 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$$

a)

b) area under graph $= 1$

$$\therefore \frac{1}{2}k + 2k + \frac{1}{2}k = 1$$

$$3k = 1$$
$$k = \frac{1}{3}$$

c)

$$f(x) \begin{cases} 0 & x < 2 \\ \frac{1}{6}x^2 - \frac{2}{3}x + c_1 & 2 \leq x \leq 3 \\ \frac{1}{3}x + C_2 & 3 < x < 5 \\ 2x - \frac{1}{6}x^2 + C_3 & 5 \leq x \leq 6 \\ 1 & x > 6 \end{cases}$$

$$\frac{1}{6}x^2 - \frac{2}{3}x + c_1 = 0 \quad \text{when } x = 2$$

$$\therefore c_1 = \frac{2}{3}$$

$$\frac{1}{6}x^2 - \frac{2}{3}x + \frac{2}{3} = \frac{1}{3}x + C_2 \quad \text{when } x = 3$$

$$\therefore C_2 = -\frac{5}{6}$$

$$2x - \frac{1}{6}x^2 + C_3 = 1 \quad \text{when } x = 6$$

$$\therefore C_3 = -5$$

$$F(x) \begin{cases} 0 & x < 2 \\ \frac{1}{6}x^2 - \frac{2}{3}x + \frac{2}{3} & 2 \leq x \leq 3 \\ \frac{1}{3}x - \frac{5}{6} & 3 < x < 5 \\ 2x - \frac{1}{6}x^2 - 5 & 5 \leq x \leq 6 \\ 1 & x > 6 \end{cases}$$

d) 90th percentile between
$5 \leq x \leq 6$

$\therefore 2x - \frac{1}{6}x^2 - 5 = 0.9$

$\frac{1}{6}x^2 - 2x + 5.9 = 0$

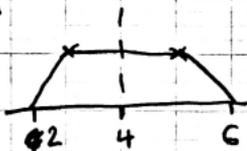$$\frac{2 \pm \sqrt{2^2 - 4 \times \frac{1}{6} \times 5.9}}{(2 \times \frac{1}{6})}$$

$x = \frac{30 \pm \sqrt{15}}{5} = 6.77$ or 5.23

$\underline{x = 5.23}$ as $6.77 > 6$ disgard

e) $P\left[E(x) < X < 5.5\right]$

$E(x) = 4$

$P(X < 5.5) - P(X < 4)$
$F(5.5) - F(4)$
$\left(2(5.5) - \frac{1}{6}(5.5)^2 - 5\right) - \left(\frac{1}{3}(4) - \frac{5}{6}\right)$
$= \frac{11}{24}$