Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Statistics 3
(6691/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75

2. The Edexcel Mathematics mark schemes use the following types of marks:

-   **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.

-   **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.

-   **B** marks are unconditional accuracy marks (independent of M marks)

-   Marks should not be subdivided.

3. Abbreviations

    These are some of the traditional marking abbreviations that will appear in the mark schemes.

    -   bod – benefit of doubt
    -   ft – follow through
    -   the symbol $\sqrt{}$ will be used for correct ft
    -   cao – correct answer only
    -   cso   - correct solution only. There must be no errors in this part of the question to obtain this mark
    -   isw – ignore subsequent working
    -   awrt – answers which round to
    -   SC: special case
    -   oe – or equivalent (and appropriate)
    -   d… or dep – dependent
    -   indep – independent
    -   dp decimal places
    -   sf significant figures
    -   ✱ The answer is printed on the paper or ag- answer given
    -   ⌐ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks
1(a)	(This is a sample where) **every** (possible) **sample** (of size *n*) has an **equal chance** of being chosen.	B1
		(1)
(b)	'When it is impossible to provide a **sampling frame**' or a correct example with an indication of sampling frame being impossible.	B1
		(1)
(c)(i)	A **list/register** of **all** the students.	B1
(ii)	Number the students (from 0 to 74, 1 to 75 etc.)	B1
	Using the **random no. table** read off the nos. and **identify or select the students allocated those nos**.	B1
		(3)
		Total 5
Notes		
(a)	Require **all / each / every** etc **sample** and **same/equal** etc **chance / probability** etc for B1	
(b)	Require **impossible / no / doesn't exist** etc and **sampling frame** for B1	
(c)(i)	Require **list/register etc** and **all/every/75 etc students** for B1	
	List of 8 students is B0	
(ii)	First B1 accept **'in the corresponding position' o.e.** if numbering omitted	
	Second B1 require both for mark.	

Question Number	Scheme	Marks
2a(i)	Only contains **known** data / function of **data only / no population parameters**	B1
	therefore it **is a statistic**	B1d
(ii)(iii)	(ii) and (iii) contain **unknown** parameters / **population parameters** / μ **and / or** σ	B1
	therefore it is **not a statistic**.	B1d
		(4)
(b)	$(E(\frac{3X_1 - X_{20}}{2}) = \frac{3\mu - \mu}{2} =) \ \mu$	B1
	$Var(\frac{3X_1 - X_{20}}{2}) = \frac{9\sigma^2 + \sigma^2}{2^2}$	M1
	$= \frac{5\sigma^2}{2}$	A1
		(3)
		Total 7
Notes		
(a)(i)	First B1 for known / no unknowns o.e. in (i)	
	Second B1 dependent on first B1 for 'Yes' / is a statistic o.e. in (i)	
	Third B1 for unknowns o.e. in **both** (ii) **and** (iii)	
	Fourth B1 dependent on third B1 for 'No' / not a statistic o.e. in **both** (ii) **and** (ii)	
(b)	B1 for μ	
	M1 for some squaring on numerator or denominator **and** must **add** on numerator	
	A1 for $\frac{5\sigma^2}{2}$ o.e.	

Question Number	Scheme	Marks
3		

		Happiness				
		Not happy	Fairly happy	Very happy		M1
Gender	Female	13.51	41.77	30.71		A1
	Male	8.49	26.23	19.29		

H_0 : Happiness and gender are independent/ not associated

H_1 : Happiness and gender are not independent/ associated

B1

dM1

O	E	$\dfrac{(O-E)^2}{E}$	$\dfrac{O^2}{E}$
9	13.51	1.508	5.996
43	41.77	0.0361	44.264
34	30.71	0.351	37.637
13	8.49	2.402	19.915
25	26.23	0.0575	23.829
16	19.29	0.560	13.274

A1

A1

$$\sum \frac{(O-E)^2}{E} = 4.91 \quad \text{or} \quad \sum \frac{O^2}{E} - N = 144.91 - 140 = 4.91$$

$$\nu = (3-2)(2-1) = 2$$

B1

B1ft

$$\sum \frac{(O-E)^2}{E} < 5.991$$

4.91 < 5.991 so 'insufficient evidence to reject H0 ' or 'Accept H0'

M1

No association between gender and happiness.

A1

(10)

Total 10

Notes

1st M1 for some use of $\frac{Row\ Total \times Column\ Total}{Grand\ Total}$. May be implied by at least 1 correct Ei

1st A1 awrt 13.5, 41.8, 30.7, 8.5, 26.2 and 19.3 Allow M1A0 for Ei rounded to integers

1st B1 for both hypotheses. Must mention "happiness" and "gender" at least once.

Use of "relationship" or "correlation" or "connection" is B0

2nd dM1 for at least 2 correct terms (in 3rd or 4th columns) or correct expressions with their Ei

Dependent on 1st M1. Accept 2sf accuracy for the M mark.

2nd A1 for all correct terms (2sf or better). May be implied by a correct ans

Allow truncation e.g. 44.2…

3rd A1 awrt 4.91 . Condone 4.915

2nd B1 for correct degrees of freedom (may be implied by a cv of 5.991)

3rd B1ft for cv that follows from their degrees of freedom

3rd M1 for a correct statement linking their test statistic and their cv

Contradictory statements score M0 e.g. "significant, do not reject H0"

Condone "reject H1 "

4th A1 for a correct comment in context - must mention "gender" and "happiness"

Condone "relationship" or "connection" here but **not** "correlation".

e.g. "There is no evidence of a relationship between gender and happiness"

No follow through. If e.g hypotheses are the wrong way around A0 here.

SC Use of calculator with **no working** may get M0A0B1M1A0A1B1B1M1A1

Question Number	Scheme	Marks
4	$E(A) = E(B) + 4E(C) - 3E(D)$	M1
	$\quad = 22$	A1
	$Var(A) = Var(B) + 16Var(C) + 9Var(D)$	M1
	$\quad = 168.25$	A1
	$P(A < 45) = P\left(Z < \dfrac{45 - 22}{\sqrt{168.25}} \right)$	M1
	$\quad = P(Z < 1.773)$	
	$\quad = 0.9616 \qquad\qquad\qquad\qquad$ awrt 0.962	A1
		(6)
		Total 6
	Notes	
	1st M1 for $E(4C) = 4E(C)$ **and** $-E(3D) = -3E(D)$	
	1st A1 for 22 cao	
	2nd M1 for use of $Var(aX) = a^2 Var X$ and $+$ their '9Var(D)'	
	2nd A1 for 168.25 cao	
	3rd M1 for standardising using their mean and their sd	
	3rd A1 for awrt 0.962. NB Calculator gives 0.961899….	

Question Number	Scheme	Marks
5(a)	The seeds are **independent /** There are a **fixed number** of seeds in a row / There are only **two outcomes** to the seed germinating – either it germinates or it does not / The **probability** of a seed germinating is **constant**	B1 B1 (2)
(b)	$$\frac{(0\times 2)+(1\times 6)+(2\times 11)+(3\times 19)+(4\times 25)+(5\times 32)+(6\times 16)+(7\times 9)}{120\times 7} = \frac{504}{840}$$ $$= 0.6 **$$	M1 A1cso (2)
(c)	$p = 0.6 \quad q = 0.4$ $s = 120\times 21q^5 p^2 = 120\times 21 \text{ x } 0.4^5 \text{ x } 0.6^2 = 9.29$ $t = 120\times 35q^3 p^4 = 120\times 35 \text{ x } 0.4^3 \text{ x } 0.6^4 = 34.84$	B1 B1 (2)
(d)	H_0: A binomial distribution is a suitable model. H_1: A binomial distribution is not a suitable model.	B1 M1

Observed number of rows	19	19	25	32	25
Expected number of rows	11.55	23.22	34.84	31.35	19.04
$\frac{(O-E)^2}{E}$	4.81	0.77	2.78	0.013	1.87
$\frac{O^2}{E}$	31.26	15.55	17.94	32.66	32.83

$\mathfrak{v} = 5 - 2 = 3$ Critical value for $\chi^2 = 11.345$ $$\sum \frac{(O-E)^2}{E} = 10.23 \quad \text{or} \quad \sum \frac{O^2}{E} - N = 130.23 - 120 = 10.23$$ $10.23 < 11.345$ therefore do not reject H_0 A binomial is a suitable model.	B1ft B1ft M1A1 A1	(7) **Total 13**

Notes

(a)	Any two and at least one must have context. 2 correct, no context B1B0. Do not award B0B1.
(b)	M1 require at least two correct terms in numerator **and** /(120x7) or /120 then /7
	A1 cso as given answer
(c)	Cao for each B1
(d)	1st B1 for both hypotheses. B0 if they include 0.6 Condone $X\sim$B(n,p) etc
	1st M1 for using some combined columns (<8)
	2nd B1ft follows from 'their no of columns' -2
	3rd B1ft follows from the degrees of freedom
	2nd M1 for attempting $\frac{(O-E)^2}{E}$ **or** $\frac{O^2}{E}$ with at least 2nd (3 seeds) and 4th (5 seeds) accurate to 2sf
	Contradictory statements score M0 e.g. "significant" do not reject H0
	1st A1 for awrt 10.2
	2nd A1 dependent on 2nd M for a correct comment suggesting that binomial model is suitable. **No follow through** .
	Condone mention of 0.6 here. Hypotheses wrong way round scores A0

Question Number	Scheme	Marks
6(a)	$\bar{X} = \frac{1}{n}(X_1 + \ldots + X_n)$ $\begin{aligned} \mathrm{E}(\bar{X}) \quad &= \frac{1}{n}\mathrm{E}(X_1 + \ldots + X_n) \\ &= \frac{1}{n}(\mathrm{E}(X_1) + \ldots + \mathrm{E}(X_n)) \\ &= \frac{1}{n}(\mu + \ldots + \mu) \\ &= \frac{n\mu}{n} = \mu \end{aligned}$	B1cso (1)
(b)	$\bar{x} = \frac{1}{5}(197 + 203 + 205 + 201 + 195)$ $= 200.2(g)$ $s^2 = \frac{1}{n-1}(\sum x^2 - n\bar{x}^2) \qquad \text{or} \quad \frac{n}{n-1}\mathrm{Var}\,x$ $= \frac{1}{5-1}(200469 - 5(200.2^2))$ $= 17.2$	B1 M1 A1 (3)
(c)	We require $2 \times 1.25 \geq$ Width of confidence interval $2.5 \geq \frac{2 \times 1.96 \times 4.8}{\sqrt{n}}$ or $1.25 \geq \frac{1.96 \times 4.8}{\sqrt{n}}$ or $\frac{1.25}{\frac{4.8}{\sqrt{n}}} \geq 1.96$ $\sqrt{n} \geq \frac{2 \times 1.96 \times 4.8}{2.5} = 7.5264$ $n \geq 56.6(5)$ Minimum sample size is 57	M1B1 A1 A1 (4) **Total 8**
	Notes	
(a) (b)	B1 cso: require $\mathrm{E}(\bar{X}) = \mu$ with at least 1 correct intermediate step and no incorrect working. B1 for 200.2 or $\dfrac{1001}{5}$ M1 for use of correct formula. Accept $\dfrac{1}{4}S_{xx} = \dfrac{1}{4} \times 68.8$ A1 for awrt 17.2	
(c)	M1 for use of any equivalent expression. Accept equality. Accept their s instead of 4.8 B1 for 1.96 seen with s.e. 1st A1 for 56.6(5) 2nd A1 for 57. Must follow from correct working e.g. $\sqrt{n} \leq 7.5264$ resulting in $n = 57$ award A0	

Question Number	Scheme	Marks
7(a)	$z = \pm\ 3.2905$	B1
	$\sigma = \frac{30}{3.2905}$	M1
	$\sigma = 9.117$ **	A1cso
		(3)
(b)	$H_0 :\ \mu = 1000\ \ H_1 :\ \mu < 1000$	B1
	mean weight = 999.54	B1
	$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{(999.54 - 1000)}{\frac{9.117}{\sqrt{10}}} = -0.160$ or $\frac{c - 1000}{\sqrt{83.12/10}} = -2.3263 \therefore \text{CR}\ c < 993.29$	M1A1
	1% critical value $= -\ 2.3263$	B1
	$-\ 2.3263 < -0.160$	
	Accept H_0 / not in critical region	dM1
	There is no evidence that that the machine is delivering packets of mean weight less than 1 kg	A1ft
		(7)
		Total 10
Notes		
(a)	M1 for 30/'their $|z|$ ' >1	
	A1 cso as given answer	
(b)	1st B1 both hypotheses correct.	
	Accept 1kg in hypotheses if consistent units used in working usually either kg or g.	
	2nd B1 999.54 (g) or 0.99954 (kg)	
	1st M1 for standardising using their mean allow $\pm$, 1000 and $\frac{9.117}{\sqrt{10}}$ o.e. in kg	
	1st A1 awrt -0.160 unless clearly using $|z|$ (stated) then accept 0.160 or CR awrt 993	
	Condone -0.16 if fully correct expression seen.	
	3rd B1 $\pm$ 2.3263 sign consistent with test statistic or $p = 0.4364 > 0.01$ NB $p = 0.5636 < 0.99$	
	2nd dM1 dependent upon 1st M for a correct statement linking their test statistic and their cv	
	Contradictory statements score M0 e.g. "significant, do not reject H_0"	
	2nd A1 for correct conclusion in context. Must mention 'machine' and 'packets'.	

Question Number	Scheme	Marks
8(a)	$r = \dfrac{9.3433}{\sqrt{0.0632 \times 1957.5556}}$ $= 0.840$ 	M1 A1 (2)
(b)	$H_0 : \rho = 0$ $H_1 : \rho > 0$ Critical value $= 0.5822$ $0.840 > 0.5822$ There is evidence to reject H_0. There is evidence of a positive correlation between a man's height and his weight. 	B1 B1 M1 A1ft (4)
(c)	<table><tr><td>Man</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td><td>I</td></tr><tr><td>Actual weight</td><td>1</td><td>2</td><td>7</td><td>3</td><td>4</td><td>5</td><td>8</td><td>6</td><td>9</td></tr><tr><td>Peter's order</td><td>1</td><td>4</td><td>2</td><td>6</td><td>3</td><td>8</td><td>5</td><td>9</td><td>7</td></tr><tr><td>d^2</td><td>0</td><td>4</td><td>25</td><td>9</td><td>1</td><td>9</td><td>9</td><td>9</td><td>4</td></tr></table> $\sum d^2 = 70$ $r_s = 1 - \dfrac{6\sum d^2}{n(n^2-1)}$ $= 1 - \dfrac{6 \times 70}{9(81-1)}$ $= 0.417$ 	B1 B1 M1A1 dM1 A1 (6)
(d)	$H_0 : \rho = 0$ $H_1 : \rho > 0$ Critical value 0.600 $(0.417 < 0.600)$ There is insufficient evidence to reject H_0. Peter does not have the ability to correctly order men, by weight, from their photograph. 	B1 B1 M1 A1 (4) **Total 16**

		Notes	
(a)		M1 Clear use of $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$ A1 0.840 cao	
(b)		1st B1 for both hypotheses in terms of ρ , one tail H$_1$ must be compatible with their r Hypotheses just in words e.g. "no correlation" score B0 2nd B1 for 0.5822 cao M1 for a statement comparing 'their r' with 'their cv' A1 for a correct contextualised comment. Must mention positive correlation, be carrying out a 1-tailed test and mention height and weight. Follow through their r and their cv (provided their $|cv| <1$ and their $|r| <1$)	
(c)		1st B1 for attempt to rank actual weight / Peter's order with at least 4 correct 2nd B1 for correct rankings for both (one or both may be reversed) 1st M1 for use of $\sum d^2$ with at least 4 values correct and attempt to add 1st A1 for 70 or 170 with reversed rankings 2nd dM1 for use of the correct formula, follow through their $\sum d^2$.Dependent on 1st M1 If answer is not correct, a correct expression is required. 2nd A1 for awrt 0.417 or $\frac{5}{12}$	
(d)		1st B1 for both hypotheses in terms of ρ or ρ_s One tail H$_1$ must be compatible with their ranking Hypotheses just in words e.g. "no correlation" score B0 2nd B1 for cv of 0.6(00) cao Their cv must be compatible with their H$_1$ which may be in words M1 for statement comparing 'their r' with 'their cv' A1 for a correct contextualised comment. Must mention Peter and Men. Follow through their r and their cv (provided their $|cv| <1$and their $\left|r_s\right| <1$)	