



Mark Scheme (Final)

Summer 2015

Pearson Edexcel International A Level in  
Statistics 3 (WST03/01)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## PEARSON EDEXCEL IAL MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 75
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
  - ft – follow through
  - the symbol  $\surd$  will be used for correct ft
  - cao – correct answer only
  - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
  - isw – ignore subsequent working
  - awrt – answers which round to
  - SC: special case
  - oe – or equivalent (and appropriate)
  - d... or dep – dependent
  - indep – independent
  - dp decimal places
  - sf significant figures
  - \* The answer is printed on the paper or ag- answer given
  - $\square$  or d... The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
  - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

### June 2015 WST03 Statistics 3 Mark Scheme

Question Number	Scheme	Marks
1. (a)	$\{w\} = 018$ or 18	<b>018 or 18</b>
(b)	$\{x\} = 18$	<b>18</b>
(c)	$\{\text{prob} =\} 0$	<b>0</b>
(d)	<p><b>Advantage:</b> Any one of:</p> <ul style="list-style-type: none"> <li>• <u>Simple or easy</u> to use also allow “quick” or “efficient” (o.e.)</li> <li>• It is suitable for large samples ( or populations)</li> <li>• Gives a good spread of the data</li> </ul> <p><b>Disadvantage:</b> Any one of:</p> <ul style="list-style-type: none"> <li>• The alphabetical list is (probably) <u>not random</u></li> <li>• <u>Biased</u> since the list is not (truly) random</li> <li>• <u>Some combinations</u> of names are <u>not possible</u></li> </ul>	<p>B1</p> <p>[1]</p> <p>B1</p> <p>[1]</p> <p>B1</p> <p>[1]</p> <p>B1</p> <p>B1</p> <p>[2]</p> <p><b>(Total 5)</b></p>
<b>Notes</b>		
(d)	<p>If no labels are given treat the 1<sup>st</sup> reason as an advantage and the 2<sup>nd</sup> as a disadvantage</p> <p>1<sup>st</sup> B1: for advantage</p> <p>2<sup>nd</sup> B1: for disadvantage</p> <p>“it requires a sampling frame” is 2<sup>nd</sup> B0 since the alphabetical list is given.</p> <p>Note: Do not score both B1 marks for opposing advantages and disadvantages.</p>	

Question Number	Scheme										Marks																																																																																																																																																																																																								
<p>2. (a)</p> <p>Judge 1</p> <p>Judge 2</p> <p>or</p> <p>Judge 1</p> <p>Judge 2</p> <p><math>\sum d^2 = 4 + 1 + 1 + 4 + 1 + 1 + 0 + 9 + 1</math></p> <p>or <math>0 + 1 + 1 + 1 + 9 + 4 + 1 + 1 + 4 = 22</math></p> <p><math>r_s = 1 - \frac{6(22)}{9(80)}; = 0.8166666...</math></p> <p>(b)</p> <p><math>H_0 : \rho = 0, H_1 : \rho &gt; 0</math></p> <p>Critical Value = 0.7833 <u>or</u> CR: <math>r_s \geq 0.7833</math></p> <p>Since <math>r_s = 0.8166...</math> it lies in the CR, <u>or</u> reject <math>H_0</math> (o.e.)</p> <p>The two <u>judges</u> (or “they”) are in <u>agreement</u> <u>or</u> there is a <u>positive correlation</u> between the ranks of the two <u>judges</u>.</p>	<table style="width:100%; border-collapse: collapse;"> <tr> <td style="text-align: center; width: 10%;">A</td> <td style="text-align: center; width: 10%;">B</td> <td style="text-align: center; width: 10%;">C</td> <td style="text-align: center; width: 10%;">L</td> <td style="text-align: center; width: 10%;">N</td> <td style="text-align: center; width: 10%;">R</td> <td style="text-align: center; width: 10%;">S</td> <td style="text-align: center; width: 10%;">T</td> <td style="text-align: center; width: 10%;">Y</td> <td colspan="2"></td> </tr> <tr> <td style="text-align: center;">6</td> <td style="text-align: center;">3</td> <td style="text-align: center;">4</td> <td style="text-align: center;">9</td> <td style="text-align: center;">2</td> <td style="text-align: center;">8</td> <td style="text-align: center;">1</td> <td style="text-align: center;">5</td> <td style="text-align: center;">7</td> <td colspan="2"></td> </tr> <tr> <td style="text-align: center;">8</td> <td style="text-align: center;">4</td> <td style="text-align: center;">5</td> <td style="text-align: center;">7</td> <td style="text-align: center;">3</td> <td style="text-align: center;">9</td> <td style="text-align: center;">1</td> <td style="text-align: center;">2</td> <td style="text-align: center;">6</td> <td colspan="2"></td> </tr> <tr> <td colspan="10" style="text-align: center;">or</td> <td></td> </tr> <tr> <td style="text-align: center;">S</td> <td style="text-align: center;">N</td> <td style="text-align: center;">B</td> <td style="text-align: center;">C</td> <td style="text-align: center;">T</td> <td style="text-align: center;">A</td> <td style="text-align: center;">Y</td> <td style="text-align: center;">R</td> <td style="text-align: center;">L</td> <td colspan="2"></td> </tr> <tr> <td style="text-align: center;">1</td> <td style="text-align: center;">2</td> <td style="text-align: center;">3</td> <td style="text-align: center;">4</td> <td style="text-align: center;">5</td> <td style="text-align: center;">6</td> <td style="text-align: center;">7</td> <td style="text-align: center;">8</td> <td style="text-align: center;">9</td> <td colspan="2"></td> </tr> <tr> <td style="text-align: center;">1</td> <td style="text-align: center;">3</td> <td style="text-align: center;">4</td> <td style="text-align: center;">5</td> <td style="text-align: center;">2</td> <td style="text-align: center;">8</td> <td style="text-align: center;">6</td> <td style="text-align: center;">9</td> <td style="text-align: center;">7</td> <td colspan="2"></td> </tr> <tr> <td colspan="10"></td> <td style="text-align: center;">M1</td> </tr> <tr> <td colspan="10"></td> <td style="text-align: center;">M1</td> </tr> <tr> <td colspan="10"></td> <td style="text-align: center;">A1</td> </tr> <tr> <td colspan="10"></td> <td style="text-align: center;">M1;</td> </tr> <tr> <td colspan="10"></td> <td style="text-align: center;">A1</td> </tr> <tr> <td colspan="10"></td> <td style="text-align: center;">[5]</td> </tr> <tr> <td colspan="10"></td> <td style="text-align: center;">B1</td> </tr> <tr> <td colspan="10"></td> <td style="text-align: center;">B1</td> </tr> <tr> <td colspan="10"></td> <td style="text-align: center;">M1</td> </tr> <tr> <td colspan="10"></td> <td style="text-align: center;">A1ft</td> </tr> <tr> <td colspan="10"></td> <td style="text-align: center;">[4]</td> </tr> <tr> <td colspan="10"></td> <td style="text-align: center;">(Total 9)</td> </tr> </table>	A	B	C	L	N	R	S	T	Y			6	3	4	9	2	8	1	5	7			8	4	5	7	3	9	1	2	6			or											S	N	B	C	T	A	Y	R	L			1	2	3	4	5	6	7	8	9			1	3	4	5	2	8	6	9	7													M1											M1											A1											M1;											A1											[5]											B1											B1											M1											A1ft											[4]											(Total 9)	
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<p>(a)</p> <p><b>False Ranking</b></p> <p>(b)</p> <p><b> cv &gt;1</b></p>	<p>1<sup>st</sup> M1 for an attempt to rank at least one row (at least 4 correct)</p> <p>2<sup>nd</sup> M1 for an attempt at <math>d^2</math> row (may be implied by sight of <math>\sum d^2 = 22</math> or 221 for reverse ranks)</p> <p>1<sup>st</sup> A1 for <math>\sum d^2 = 22</math> (or 221 if reverse ranking is used) Can be implied by correct answer.</p> <p>3<sup>rd</sup> M1 for use of the correct formula with their <math>\sum d^2</math> (if it is clearly stated)</p> <p>If the answer is not correct then a correct expression is required</p> <p>e.g Alphabetic ranking: Gives Judge 1: 7 5 2 3 8 1 9 6 4</p> <p style="padding-left: 150px;">Judge 2: 7 8 5 2 3 9 4 1 6 <math>\sum d^2 = 162</math> and <math>r_s = -0.35</math></p> <p><b>Scores:</b> M0(for ranking), M1(for attempt at <math>d^2</math> row), A0, M1 (for use of their <math>\sum d^2</math>), A0 i.e. 2 out of 5</p> <p>Can follow through their <math>r_s</math> in (b)</p> <p>1<sup>st</sup> B1 for both hypotheses stated correctly in terms of <math>\rho</math> (allow <math>\rho_s</math>) <math>H_1</math> must be compatible with ranking</p> <p>2<sup>nd</sup> B1 for cv = 0.7833 (independent of their <math>H_1</math> (no 2-tail value in tables) <u>but</u> compatible sign with their <math>r_s</math>)</p> <p>M1: for a correct statement (in words) relating their <math>r_s</math> with their critical value.</p> <p>e.g. “reject <math>H_0</math>”, “in critical region”, “significant”, “positive correlation”</p> <p>May be implied by a correct contextual comment.</p> <p>If their cv is <math> cv  &gt; 1</math> (often from using normal tables) award M0A0</p> <p>If <math> their r_s  &gt;  their cv </math> then “significant” (o.e.) for M1 and “<u>judges are in agreement</u>” (o.e.) for A1ft</p> <p>If <math> their r_s  &lt;  their cv </math> then “not significant” (o.e.) for M1 and “<u>judges don’t agree</u>” (o.e.) for A1ft</p> <p>A1ft: for a correct follow through conclusion in context.</p> <p>“positive correlation” alone scores M1 A0</p> <p>For reverse ranking should still say “<u>judges are in agreement</u>”</p>																																																																																																																																																																																																																		

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<p>3. (a)</p> <p>(b)</p> <p>(c)</p>	<p><math>\hat{\lambda} = \frac{0(47) + 1(57) + 2(46) + 3(35) + 4(9) + 5(6)}{200} = \frac{320}{200} = 1.6</math> * Full exp' or at least 2 products and 320/200 seen</p> <p><math>r = 200 \times \frac{e^{-1.6}(1.6)^2}{2!} \{= 51.68550861...\}</math> Using <math>r = 200 \times \frac{e^{-1.6}(1.6)^2}{2!}</math></p> <p><math>s = 200 - (40.38 + 64.61 + \text{their } r + 27.57 + 11.03) \{= 4.72449139...\}</math> <u>or</u> their <math>r + s = 56.41</math></p> <p><math>r = 51.68550861...</math> and <math>s = 4.72449139...</math> <math>r = \text{awrt } \mathbf{51.69}</math> and <math>s = \text{awrt } \mathbf{4.72}</math></p> <p><math>H_0</math> : Poisson (distribution) is a suitable/ sensible (model)  <math>H_1</math> : Poisson (distribution) is not a suitable/ sensible (model).</p> <table border="1" data-bbox="236 689 1361 1025"> <thead> <tr> <th>Number of accidents</th> <th>Observed</th> <th>Expected</th> <th>Combined Observed</th> <th>Combined Expected</th> <th><math>\frac{(O - E)^2}{E}</math></th> <th><math>\frac{O^2}{E}</math></th> </tr> </thead> <tbody> <tr> <td>0</td> <td>47</td> <td>40.38</td> <td>47</td> <td>40.38</td> <td>1.0853</td> <td>54.7053</td> </tr> <tr> <td>1</td> <td>57</td> <td>64.61</td> <td>57</td> <td>64.61</td> <td>0.8963</td> <td>50.2863</td> </tr> <tr> <td>2</td> <td>46</td> <td>51.69</td> <td>46</td> <td>51.69</td> <td>0.6264</td> <td>40.9364</td> </tr> <tr> <td>3</td> <td>35</td> <td>27.57</td> <td>35</td> <td>27.57</td> <td>2.0024</td> <td>44.4324</td> </tr> <tr> <td>4</td> <td>9</td> <td>11.03</td> <td rowspan="2">15</td> <td rowspan="2">15.75</td> <td rowspan="2">0.0357</td> <td rowspan="2">14.2857</td> </tr> <tr> <td><math>\geq 5</math></td> <td>6</td> <td>4.72</td> </tr> <tr> <td colspan="5" style="text-align: right;">Totals</td> <td>4.6461</td> <td>204.6461</td> </tr> </tbody> </table> <p><math>X^2 = \sum \frac{(O - E)^2}{E}</math> or <math>\sum \frac{O^2}{E} - 200 ; = 4.6461</math> awrt <b>4.65</b></p> <p><math>v = 5 - 1 - 1 = 3</math> <b>3</b></p> <p><math>\chi^2_3(0.10) = 6.251 \Rightarrow \text{CR: } X^2 \geq 6.251</math> <b>6.251</b></p> <p>[Since <math>X^2 = 4.6461</math> does not lie in the CR, then there is insufficient evidence to reject <math>H_0</math> ]</p> <p>The number of <b>accidents</b> per day can be modelled by a Poisson distribution <u>or</u> the <b>supervisor's</b> belief is correct.</p>	Number of accidents	Observed	Expected	Combined Observed	Combined Expected	$\frac{(O - E)^2}{E}$	$\frac{O^2}{E}$	0	47	40.38	47	40.38	1.0853	54.7053	1	57	64.61	57	64.61	0.8963	50.2863	2	46	51.69	46	51.69	0.6264	40.9364	3	35	27.57	35	27.57	2.0024	44.4324	4	9	11.03	15	15.75	0.0357	14.2857	$\geq 5$	6	4.72	Totals					4.6461	204.6461	<p>B1 * [1]</p> <p>M1</p> <p>M1</p> <p>A1 [3]</p> <p>B1</p> <p>M1</p> <p>M1; A1</p> <p>B1 ft</p> <p>B1 ft</p> <p>A1 ft</p> <p>[7] (Total 11)</p>
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<p>(b)</p> <p>(c)</p> <p><b>No pooling</b></p>	<p>Note: Allow A1 for <math>s = \text{awrt } 4.74</math> (found as a result of using expected values to full accuracy.)</p> <p>1<sup>st</sup> B1: for <u>both</u> hypotheses and mentioning Poisson at least once.                  Allow Poisson is a “good fit/model” but <u>not</u> “good method”                  Inclusion of 1.6 for mean in hypotheses is B0 but condone in conclusion.</p> <p>1<sup>st</sup> M1: For an attempt to pool 4 accidents and <math>\geq 5</math> accidents <u>or</u> pool when <math>E_i &lt; 5</math> No pooling is M0</p> <p>2<sup>nd</sup> M1: For an attempt at the test statistic, at least 2 correct expressions/values (to awrt 2 d.p.)</p> <p>1<sup>st</sup> A1: For awrt 4.65 (score M1M1A1 if awrt 4.65 seen)                  If no pooling can allow 2<sup>nd</sup> M1 if <math>X^2 = 5.33</math> is seen</p> <p>2<sup>nd</sup> B1ft: For <math>n - 1 - 1</math> i.e. subtracting 2 from their <math>n</math>. B1B1 may be implied by 6.251 (if pooling) or 7.779 for no pooling</p> <p>3<sup>rd</sup> B1ft: For a correct ft for their <math>\chi^2_k(0.10)</math>, where <math>k = n - 1 - 1</math> from their <math>n</math>.</p> <p>2<sup>nd</sup> A1ft: (<b>Dep. on the 2<sup>nd</sup> M1</b>) For correct comment in context based on their test statistic and their critical value that mentions <b>accidents</b> or <b>supervisor</b>. Condone mention of Po(1.6) in conclusion                  Score A0 for inconsistencies e.g. “significant” followed by “supervisor’s belief is justified”</p> <p><b>Note:</b> Full accuracy gives a combined expected frequency of 15.76, <math>\frac{(O - E)^2}{E} = 0.0366</math>, <math>\frac{O^2}{E} = 14.2766</math>,  <math>X^2 = 4.64855...</math> and p-value 0.199</p>																																																					

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<p>4. (a)</p> <p>(b)</p>	<p>Let <math>X =</math> weight of a sack of potatoes, <math>X \sim N(25.6, 0.24^2)</math></p> <p>So <math>D = X_1 - X_2 \sim N(0, 2(0.24)^2)</math> or <math>D \sim N(0, 0.1152)</math></p> <p><math>\{P( D  &gt; 0.5) = \}</math> <math>2P(D &gt; 0.5)</math></p> $= 2 \times P\left(Z > \frac{0.5}{\sqrt{0.1152}}\right)$ $= 2 \times P(Z > 1.4731\dots)$ <u>or</u> $= 2(1 - 0.9292)$ $= 0.1416$ <p>Let <math>Y =</math> weight of an empty pallet, <math>Y \sim N(20.0, 0.32^2)</math></p> <p>So <math>T = X_1 + X_2 + \dots + X_{30} + Y</math></p> <p><math>T \sim N(30(25.6) + 20, 30(0.24)^2 + 0.32^2)</math></p> <p><math>T \sim N(788, 1.8304)</math></p> <p><math>\{P(T &gt; 785) = \}</math> <math>P\left(Z &gt; \frac{785 - 788}{\sqrt{1.8304}}\right)</math></p> $= P(Z > -2.2174\dots)$ $= 0.9868$	<p>Attempt at <math>D</math> and <math>D \sim N(0, \dots)</math> M1</p> <p><math>(0.24)^2 + (0.24)^2</math>; <b>0.1152</b> A1; A1</p> <p><math>2 \times P(D &gt; 0.5)</math> can be implied dM1</p> <p><b>awrt 0.141</b> or <b>awrt 0.142</b> A1</p> <p><b>[6]</b></p> <p><u>or</u> <b>788</b> B1</p> <p><math>30(0.24)^2 + 0.32^2</math> M1</p> <p>N and 1.8304 or <b>awrt 1.83</b> A1</p> <p>M1</p> <p><b>awrt 0.987</b> A1</p> <p><b>[5]</b></p> <p><b>(Total 11)</b></p>
<b>Notes</b>		
<p>(a)</p> <p>(b)</p>	<p>1<sup>st</sup> M1: For clear definition of <math>D</math> and normal distribution with mean of 0 (Can be implied by 3<sup>rd</sup> M1)</p> <p>1<sup>st</sup> A1: for correct use of <math>\text{Var}(X_1 - X_2)</math> formula</p> <p>2<sup>nd</sup> A1: for 0.1152</p> <p>2<sup>nd</sup> dM1: For realising need <math>2 \times P(D &gt; 0.5)</math> (Dependent on 1<sup>st</sup> M1 i.e. must be using suitable <math>D</math>)</p> <p>3<sup>rd</sup> dM1: Dep on 1st M1 for standardising with 0.5, 0 and their s.d. (<math>\neq 0.24</math>) Must lead to <math>P(Z &gt; +ve)</math> (o.e.)</p> <p><math>P(Z &gt; 1.47)</math> implies 1<sup>st</sup> M1 1<sup>st</sup> A1 2<sup>nd</sup> A1 and 3<sup>rd</sup> M1</p> <p>Correct answer only will score 6 out of 6</p> <p>B1: For a mean of <math>30(25.6) + 20</math>. Can be implied by 788.</p> <p>1<sup>st</sup> M1: For <math>30(0.24)^2 + 0.32^2</math>. Can be implied by 1.8304 or awrt 1.83</p> <p>Allow M1 for swapping error i.e. <math>30 \times 0.32^2 + 0.24^2</math> if the expression is seen</p> <p>1<sup>st</sup> A1: For normal and correct variance of 1.8304 or awrt 1.83.</p> <p>Normality may be implied by standardisation</p> <p>2<sup>nd</sup> M1: For standardising with 785 with their mean and st. dev. (<math>\neq 0.24</math>) Must lead to <math>P(Z &gt; -ve)</math> oe.</p> <p>2<sup>nd</sup> A1: awrt 0.987</p> <p>Correct answer only will score 5 out of 5</p> <p><b>Note: Calculator answers are</b> (a) 0.14071..., (b) 0.98670...</p>	

Question Number	Scheme		Marks																																																																
<p><b>5.</b></p>	<p><math>H_0</math> : Grades and gender are independent (or not associated)  <math>H_1</math> : Grades and gender are dependent (or associated)</p> <table border="1" data-bbox="236 371 756 546"> <thead> <tr> <th>Observed</th> <th>Male</th> <th>Female</th> </tr> </thead> <tbody> <tr> <td>Distinction</td> <td>37</td> <td>44</td> </tr> <tr> <td>Merit</td> <td>127</td> <td>96</td> </tr> <tr> <td>Unsatisfactory</td> <td>36</td> <td>20</td> </tr> </tbody> </table> <table border="1" data-bbox="236 600 890 815"> <thead> <tr> <th>Expected</th> <th>Male</th> <th>Female</th> <th>Totals</th> </tr> </thead> <tbody> <tr> <td>Distinction</td> <td>45</td> <td>36</td> <td>81</td> </tr> <tr> <td>Merit</td> <td>123.889</td> <td>99.111</td> <td>223</td> </tr> <tr> <td>Unsatisfactory</td> <td>31.111</td> <td>24.889</td> <td>56</td> </tr> <tr> <td>Totals</td> <td>200</td> <td>160</td> <td>360</td> </tr> </tbody> </table> <table border="1" data-bbox="236 864 890 1245"> <thead> <tr> <th>Observed</th> <th>Expected</th> <th><math>\frac{(O - E)^2}{E}</math></th> <th><math>\frac{O^2}{E}</math></th> </tr> </thead> <tbody> <tr> <td>37</td> <td>45</td> <td>1.422</td> <td>30.422</td> </tr> <tr> <td>44</td> <td>36</td> <td>1.778</td> <td>53.778</td> </tr> <tr> <td>127</td> <td>123.889</td> <td>0.078</td> <td>130.189</td> </tr> <tr> <td>96</td> <td>99.111</td> <td>0.098</td> <td>92.987</td> </tr> <tr> <td>36</td> <td>31.111</td> <td>0.768</td> <td>41.657</td> </tr> <tr> <td>20</td> <td>24.889</td> <td>0.960</td> <td>16.071</td> </tr> <tr> <td colspan="2">Totals</td> <td>5.104</td> <td>365.104</td> </tr> </tbody> </table> <p><math>X^2 = \sum \frac{(O - E)^2}{E}</math> or <math>\sum \frac{O^2}{E} - 360</math> ;= awrt 5.1</p> <p><math>\nu = (3 - 1)(2 - 1) = 2</math></p> <p><math>\chi^2_2(0.05) = 5.991 \Rightarrow</math> CR: <math>X^2 \geq 5.991</math></p> <p>Since <math>X^2 = 5.1</math> does not lie in the CR, then there is insufficient evidence to reject <math>H_0</math></p> <p>Business Studies <u>grades</u> and <u>gender</u> are independent <u>or</u>  There is no association between Business Studies <u>grades</u> and <u>gender</u>. <u>Or</u>  <u>Head of department's</u> (belief) is correct</p>	Observed	Male	Female	Distinction	37	44	Merit	127	96	Unsatisfactory	36	20	Expected	Male	Female	Totals	Distinction	45	36	81	Merit	123.889	99.111	223	Unsatisfactory	31.111	24.889	56	Totals	200	160	360	Observed	Expected	$\frac{(O - E)^2}{E}$	$\frac{O^2}{E}$	37	45	1.422	30.422	44	36	1.778	53.778	127	123.889	0.078	130.189	96	99.111	0.098	92.987	36	31.111	0.768	41.657	20	24.889	0.960	16.071	Totals		5.104	365.104	<p>“grades” and “gender” mentioned at least once.</p> <p>An attempt to convert percentages to observed frequencies.</p> <p>All observed frequencies are correct.</p> <p>Some attempt at  (Row Total)(Column Total)  (Grand Total)</p> <p>Can be implied by a correct <math>E_i</math></p> <p>All expected frequencies are correct to nearest integer.</p> <p>At least 2 correct terms for <math>\frac{(O - E)^2}{E}</math> or <math>\frac{O^2}{E}</math> or correct expressions with their <math>E_i</math>.</p> <p>Accept 2 sf accuracy for the M1 mark.</p> <p>All correct <math>\frac{(O - E)^2}{E}</math> or <math>\frac{O^2}{E}</math> terms to either 2 dp or better.  Allow truncation.  (<math>\Rightarrow</math> by awrt 5.1 if 3<sup>rd</sup> M1 seen)</p> <p>awrt <b>5.1</b></p> <p>(<math>\nu =</math>) <b>2</b> (Can be implied by 5.991)</p> <p>For <b>5.991</b> only</p>	<p>B1 (1)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1ft</p> <p>(4)</p> <p>[12]  <b>(Total 12)</b></p>
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<p><b>5.10 only</b></p>	<p>Final M1: For a correct statement linking their test statistic and their critical value (<math>&gt; 3.8</math>)  Note: Contradictory statements score M0. E.g. “significant, do not reject <math>H_0</math>”.</p> <p>Final A1ft: For a correct ft statement in context –  must mention “grades” and “gender” or “sex” <u>or</u> “head of department”  Condone “relationship” or “connection” here but <b>not</b> “correlation”.  e.g. “There is no evidence of a relationship between grades and gender”</p> <p>Just seeing 5.10... only can imply 1<sup>st</sup> 3 Ms but loses 1<sup>st</sup> 3 As so can score 4 out of 7 (Qu says “show..”)</p> <p><b>Note: Full accuracy gives <math>X^2 = 5.104356...</math> and p-value 0.0779</b></p>																																																																		



Question Number	Scheme	Marks
<p>6. (a)</p>	$\left\{ \hat{\mu} = \frac{\sum x}{n} = \frac{1570}{50} = \right\} \bar{x} = 31.4$ $\left\{ \hat{\sigma}^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1} = \right\} s_x^2 = \frac{49467.58 - 50(31.4)^2}{50 - 1}$ $= 3.460816...$ <p style="text-align: right;"><math>\bar{x} = \mathbf{31.4}</math></p>	<p>B1 cao</p> <p>M1 A1ft</p> <p>awrt <b>3.46</b></p> <p>A1</p> <p style="text-align: right;"><b>[4]</b></p>
<p>(b)</p>	<p>[Let <math>Y =</math> time taken to complete obstacle course in the afternoon.]</p> <p><math>H_0 : \mu_x = \mu_y</math> , <math>H_1 : \mu_x &gt; \mu_y</math></p> $(z =) \frac{31.4 - 30.9}{\sqrt{\frac{3.46}{50} + \frac{3.03}{50}}}$ <p style="text-align: right;"><math>= 1.38781...</math> awrt <b>1.39</b></p> <p>CR: <math>Z \geq 1.6449</math> or probability = awrt 0.082 or awrt 0.083 <span style="float: right;"><b>1.6449</b> or better</span></p> <p>Since <math>z = 1.38781...</math> does not lie in the CR, then there is insufficient evidence to reject <math>H_0</math></p> <p>Conclude that the <u>mean time</u> to complete the obstacle course is the same for the early <u>morning</u> and late <u>afternoon</u>.</p>	<p>B1</p> <p>M1 A1ft</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;"><b>[7]</b></p>
<p>(c)</p>	<p><math>\bar{X}</math> and <math>\bar{Y}</math> are both approx. <u>normally</u> distributed or <math>\bar{X} - \bar{Y}</math> normal (Condone <math>\bar{x}</math> and <math>\bar{y}</math>)</p>	<p>B1</p> <p style="text-align: right;"><b>[1]</b></p>
<p>(d)</p>	<p>Have assumed <math>s^2 \approx \sigma^2</math> or variance of sample <math>\approx</math> variance of population</p>	<p>B1</p> <p style="text-align: right;"><b>[1]</b></p>
<b>Notes</b>		
<p>(a)</p>	<p>B1: 31.4 cao Allow 31 minutes, 24 seconds.</p> <p>1<sup>st</sup> M1: A correct expression for either <math>s</math> or <math>s^2</math> (ignore label)</p> <p>1<sup>st</sup> A1ft: A correct expression for <math>s^2</math> with their ft <math>\bar{x}</math>.</p> <p>3<sup>rd</sup> A1: awrt 3.46 (Correct answer scores 3 out of 3)</p>	
<p>(b)</p>	<p>1<sup>st</sup> B1: Both hypotheses stated correctly, with some indication of which <math>\mu</math> is which. Eg: <math>\mu_M, \mu_A</math></p> <p>1<sup>st</sup> M1: For an attempt at <math>\frac{a-b}{\sqrt{\frac{c}{50} + \frac{d}{50}}}</math> with at least 3 of <math>a, b, c</math> or <math>d</math> correct. Allow <math>\pm</math></p> <p>1<sup>st</sup> A1ft: for <math>\pm \frac{\text{their } 31.4 - 30.9}{\sqrt{\frac{\text{their } 3.46}{50} + \frac{3.03}{50}}}</math> Allow <math>D = \bar{x} - \bar{y}</math> <math>1.64 \sim 1.65 = \frac{D - 0}{\sqrt{\frac{3.46}{50} + \frac{3.03}{50}}}</math> [SE = 0.360277..]</p> <p>2<sup>nd</sup> A1: for awrt 1.39 (possibly <math>\pm</math>)(Allow for CV <math>D =</math> awrt 0.593) (NB <math>d = 0.5</math>)</p> <p>Correct answer scores M1A1ftA1 <u>but</u> <math>0 - (31.4 - 30.9) \rightarrow -1.39</math> loses this 2<sup>nd</sup> A mark</p> <p>2<sup>nd</sup> B1: Critical value of 1.6449 or better (seen). Allow for probability = awrt 0.082 or awrt 0.083</p> <p>Note: p-values are 0.0823 (tables) and 0.0826 (calculator).</p> <p>2<sup>nd</sup> M1: For a correct statement linking their test statistic and their critical value.</p> <p>Note: Contradictory statements score M0. E.g. “significant, do not reject <math>H_0</math>”.</p> <p>3<sup>rd</sup> A1: For a correct statement in context that accepts <math>H_0</math> (no ft) Condone “no difference in mean times”</p> <p>Must mention “<u>mean time</u>”, “<u>morning</u>” and “<u>afternoon</u>” or “<u>both times of day</u>”</p>	
<p>(c)</p>	<p>B1 E.g. <math>\bar{X} \sim N(...)</math> need both. Allow in words e.g “sample means are normally distributed”</p>	
<p>(d)</p>	<p>B1 condone only mentioning “<math>x</math>” or “<math>y</math>” <u>but</u> watch out for <math>s_x = s_y</math> or <math>\sigma_x = \sigma_y</math> which scores B0</p>	

Question Number	Scheme	Marks
<p><b>7.</b></p> <p>(a)</p> <p>(b)</p>	<p>Let <math>X =</math> score on a die</p> <p><math>E(S) = 3.5</math>, <math>Var(S) = \frac{35}{12}</math></p> <p>So, <math>\bar{S} \sim N\left(3.5, \frac{\left(\frac{35}{12}\right)}{40}\right)</math> or <math>\bar{S} \sim N\left(3.5, \frac{7}{96}\right)</math></p> <p><math>P(\bar{S} &lt; 3) = P\left(Z &lt; \frac{3 - 3.5}{\sqrt{\frac{7}{96}}}\right) \{= P(Z &lt; -1.85164...)\}</math></p> <p><math>\{= 1 - 0.9678\} = 0.0322</math></p>	<p><math>E(S) = 3.5</math> B1</p> <p><math>Var(S) = \frac{35}{12}</math> or awrt <b>2.92</b> B1</p> <p>[2]</p> <p>B1ft</p> <p>M1</p> <p><b>0.032 to 0.0322</b> A1</p> <p>[3]</p> <p><b>(Total 5)</b></p>
<b>Notes</b>		
<p>(a)</p> <p>(b)</p>	<p>2<sup>nd</sup> B1 allow awrt 2.92</p> <p>B1ft for <math>\bar{S} \sim N\left(3.5, \frac{\left(\frac{35}{12}\right)}{40}\right)</math> seen or implied. Follow through their <math>E(S)</math> and their <math>Var(S)</math></p> <p>NB <math>\frac{7}{96} = 0.07291\dot{6}</math> accept awrt 0.0729</p> <p>M1 for an attempt to standardise with 3, their mean (<math>&gt;3</math>) and <math>\sqrt{\frac{\text{their } Var(S)}{40}}</math>. Must lead to <math>P(Z &lt; -ve)</math></p> <p>A1 for 0.032 ~ 0.0322</p>	
<b>ALT ES</b>	<p>B1ft for <math>\sum S \sim N\left(140, \frac{350}{3}\right)</math> where 140 is <math>40 \times</math> their <math>E(S)</math> and variance is <math>40 \times</math> their <math>Var(S)</math></p> <p>M1 for <math>P\left(Z &lt; \frac{120 - 140}{\sqrt{\frac{350}{3}}}\right)</math> or <math>P\left(Z &lt; \frac{119.5 - 140}{\sqrt{\frac{350}{3}}}\right) \{= P(Z &lt; -1.8979...)\}</math></p> <p>A1 for 0.032~0.0322 or (with continuity correction) 0.0287 (tables) or 0.0289 (calculator).</p>	

Question Number	Scheme	Marks
<p><b>8. (a)</b></p> <p><b>(b)</b></p> <p><b>(c)</b></p>	$\left\{ \bar{x} = \frac{29.74 + 31.86}{2} \right\} \Rightarrow \bar{x} = 30.8$ <p style="text-align: right;"><math>\bar{x} = 30.8</math></p> <p style="text-align: center;">This can be implied. See note.</p> $"1.96" \left( \frac{\sigma}{\sqrt{n}} \right) = 31.86 - 30.8 \quad \text{or} \quad 2("1.96") \left( \frac{\sigma}{\sqrt{n}} \right) = 31.86 - 29.74$ $SE_{\bar{x}} = \frac{31.86 - 30.8}{1.96} = 0.540816\dots = 0.54 \text{ (2dp)}$ <p style="text-align: right;">awrt <b>0.54</b></p> <p>A 90% CI for <math>\mu</math> is <math>\bar{x} \pm 1.6449 \left( \frac{\sigma}{\sqrt{n}} \right)</math></p> $= 30.8 \pm 1.6449(0.54) \quad \text{(their } \bar{x} \text{)} \pm \text{(their } z \text{)} \text{(their } SE_{\bar{x}} \text{ from (a))}$ $= (29.91, 31.69) \quad \text{(awrt } \mathbf{29.9}, \text{ awrt } \mathbf{31.7})$ <p>Let <math>X</math> = number of confidence intervals containing <math>\mu</math>  or <math>Y</math> = number of confidence intervals not containing <math>\mu</math>  So <math>X \sim \text{Bin}(4, 0.9)</math> or <math>Y \sim \text{Bin}(4, 0.1)</math></p> $P(X \geq 3) \text{ or } P(Y \leq 1) = {}^4C_3(0.9)^3(0.1) + (0.9)^4 \quad {}^4C_3(0.9)^3(0.1) + (0.9)^4$ $= 0.2916 + 0.6561 = 0.9477 \quad \mathbf{0.9477} \text{ or } \mathbf{0.948}$	<p>B1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;"><b>[3]</b></p> <p>B1</p> <p>M1</p> <p>A1</p> <p style="text-align: right;"><b>[3]</b></p> <p>M1</p> <p>A1 oe</p> <p>A1</p> <p style="text-align: right;"><b>[3]</b></p> <p style="text-align: right;"><b>(Total 9)</b></p>
<b>Notes</b>		
<p><b>(a)</b></p> <p><b>(b)</b></p> <p><b>(c)</b></p>	<p><b>B1:</b> <math>\bar{x} = 30.8</math> may be implied by <math>1.96 \left( \frac{\sigma}{\sqrt{n}} \right) = [31.86 - 30.8] = 1.06</math> <u>or</u> <math>2(1.96) \left( \frac{\sigma}{\sqrt{n}} \right) = 31.86 - 29.74</math></p> <p><b>M1:</b> A correct equation for either a width or a half-width involving a <math>z</math>-value <math>1.5 \leq z \leq 2</math></p> <p>Eg: "their <math>z</math>" <math>\left( \frac{\sigma}{\sqrt{n}} \right) = 31.86 - "30.8"</math> ft their <math>\bar{x}</math> <u>or</u> <math>2("their z") \left( \frac{\sigma}{\sqrt{n}} \right) = 31.86 - 29.74</math></p> <p>or "their <math>z</math>" <math>(SE_{\bar{x}}) = 31.86 - "30.8"</math> <u>or</u> <math>2("their z")(SE_{\bar{x}}) = 31.86 - 29.74</math> are fine for M1.</p> <p><b>A1:</b> 0.54 or awrt 0.54 Must be seen as final answer to (a) NB <math>\frac{53}{98}</math> as final answer is A0</p> <p>Condone <math>\bar{x} \pm 1.96\sigma = \dots</math> for B1 and M1 but A0 even if they say "<math>\sigma</math> = standard error = 0.54"</p> <p>Otherwise answer only of 0.54 scores 3 out of 3</p> <p><b>B1</b> for use of 1.6449 or better in an attempt at a CI formula. Need at least 1.6449 (their SE)</p> <p><b>M1</b> for attempt at CI ft their values and provided <math>1 \leq z \leq 1.7</math></p> <p><b>M1:</b> States or applies either <math>X \sim \text{Bin}(4, 0.9)</math> <u>or</u> <math>Y \sim \text{Bin}(4, 0.1)</math></p> <p>Condone M1 for <math>0.9^4 + 0.9^3 \times 0.1</math> (o.e.)</p> <p><b>1<sup>st</sup> A1:</b> <math>{}^4C_3(0.9)^3(0.1) + (0.9)^4</math> or <math>(0.9)^4 + {}^4C_1(0.1)(0.9)^3</math> oe</p> <p><b>2<sup>nd</sup> A1:</b> 0.9477 or 0.948</p>	

