S3 June 2017 (MA)

Q1a) more representative of the entire population.

b) No. from each group = $\dfrac{\text{group size}}{\text{population size}} \times \text{sample size}$.

managers = $\dfrac{72}{720} \times 40 = \boxed{4}$

drivers = $\dfrac{108}{720} \times 40 = \boxed{6}$

admins = $\dfrac{180}{720} \times 40 = \boxed{10}$

warehouse = $\dfrac{360}{720} \times 40 = \boxed{20}$

c) Label all managers 1-72 then use random numbers to select 4 managers. in range.

Q2a) Angle(degrees) range = 360.

Expected = $\dfrac{\text{Group range}}{\text{Total range}} \times 100$ $(n=100)$.

$a = \dfrac{45}{360} \times 100 = 12.5$

$b = \dfrac{90}{360} \times 100 = 25$

$c = \dfrac{135}{360} \times 100 = 37.5$

b) H_0: Continuous Uniform Distribution is a suitable model.

H_1: Continuous Uniform Distribution is not a suitable model.

Angle	0-45	45-90	90-180	180-315	315-360
Frequency	18	16	18	29	19
Expected	12.5	12.5	25	37.5	12.5
$\frac{(O-E)^2}{E}$	2.42	0.98	1.96	1.93	3.38

$$x^2 = \sum \frac{(O-E)^2}{E} = 10.67$$

$$Y = 5-1 = 4$$
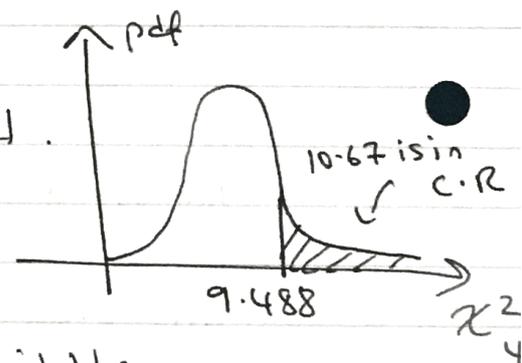
$\therefore$ critical value $= x^2_4 (5\%) = 9.488$

10.67 > 9.488
$\therefore$ Result is significant.
Reject H_0.
Evidence suggests
a continuous uniform
distribution is not suitable.

10.67 is in
C.R

Q3a)

Skater	Senior	Junior	d	d^2
A	1	3	2	4
B	2	1	1	1
C	4	5	1	1
D	3	2	1	1
E	6	6	0	0
F	5	4	1	1
				8

$$\sum d^2 = 8 \quad \therefore \quad r_s = 1 - \frac{6(8)}{6(35)}$$

$$= \boxed{0.771}$$

b) $H_0 : p = 0$

$H_1 : p > 0$

⌃

agreement indicated
by +ve correlation

Critical value : ± 0.8286

(5%, 1-tail)

$0.771 < 0.8286$

$\therefore$ Result is insignificant

Accept H_0.

Evidence suggests that
no positive correlation
exists — no agreement.

c) Ineffective as result of test was insignificant

Q4a) expected no- = $\dfrac{\text{Row Total} \times \text{Column Total}}{\text{Grand Total}}$

	Under	About Right	Over
Male	17·28	24	30·72
Female	18·72	26	33·28

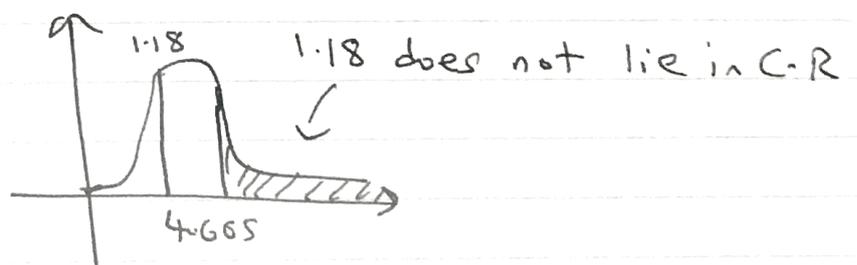b) H_0 : Perceived weight is independent of gender.

H_1 : Perceived weight is not independent of gender.

O_i	E_i	$\dfrac{(O-E)^2}{E}$
20	17·28	0·428
22	24	0·167
30	30·72	0·0169
16	18·72	0·395
28	26	0·154
34	33·28	0·0156
		1·18

$\gamma = (3-1)(2-1) = 2$
$= (\text{columns} - 1)(\text{rows} - 1)$

critical value $= \chi^2_2 \, (10\%) = 4·605$

1·18 does not lie in C·R

$$1.18 < 4.605$$

∴ Result is insignificant
Accept H_0
Evidence suggests perceived body weight
is independent of gender.

c)

O_i	E_i	$\frac{(O-E)^2}{E}$
36	50	3.92
50	50	0
64	50	3.92

$$\frac{}{7.84}$$

[expected values will all be 50 if weight types are chosen equally]

$$x^2 = \sum \frac{(O-E)^2}{E} = 7.84$$

$$\gamma = 3-1 = 2$$

$$x^2_2 (2.5\%) = 7.378$$
$$x^2_2 (1\%) = 9.210$$

hence smallest significance level where
result is significant is $\boxed{2.5\%}$

Q5a) mean $= \frac{\sum x}{n} = \frac{60}{15} = \boxed{4}$

$\hat{\sigma}^2 = s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{(\sum x)^2}{n}\right)$

$= \frac{1}{14}\left(1946 - \frac{(60)^2}{15}\right)$

$= 121.86\ldots = \boxed{122}$

bi) 95% C.I : $\left[\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}\right]$

$\left[121.86 \pm 1.96\left(\frac{10}{\sqrt{15}}\right)\right]$

$[-1.06 , 9.06]$

as a time: $[08:30 , 08:40]$ to nearest min.

ii) times of buses are sampled independently
     of each other.

c) 08:31 is within the confidence interval so
    Paul's belief is not supported.

Q6a) H_0: $\mu_{new} - \mu_{old} = 8$ critical value: ± 1.6449
                                           (5%, 1-tail)

H_1: $\mu_{new} - \mu_{old} > 8$

Test Statistic $= \dfrac{\bar{X} - \bar{Y} - (\mu_X - \mu_Y)}{\sqrt{\dfrac{\sigma_x^2}{n_x} + \dfrac{\sigma_y^2}{n_y}}}$

$= \dfrac{83 - 74 - (8)}{\sqrt{\dfrac{7}{50} + \dfrac{6}{40}}}$

$= 1.86$

$1.86 > 1.6449$
∴ Result is significant
Reject H_0.
Evidence suggests that the engineer's
claim is correct.

b) Allows us to assume the sample means for
new/old batteries are normally distributed
as n is large.

Q7a) let M = medium bags $\underset{\text{(weight)}}{\text{bags}}$, $M \sim N(520, 10^2)$

and L = large $\underset{\text{(weight)}}{\text{bags}}$, $L \sim N(1510, 20^2)$

$P(\text{required}) = P(L > M_1 + M_2 + M_3 + 15)$

$= P(L - (M_1 + M_2 + M_3 + 15) > 0)$

let $A = L - (M_1 + M_2 + M_3) - 15$,

$E(A) = 1510 - 3(520) - 15 = -65$ //

$\text{Var}(A) = \text{Var}(L) + 3\text{Var}(M)$ $\boxed{(M_1 + M_2 + M_3 \neq 3M)}$

$= 20^2 + 3(10^2) = 700$

$\therefore A \sim N(-65, 700)$

$P(\text{required}) = P(A > 0) = P\left(Z > \dfrac{0 - -65}{\sqrt{700}}\right)$

$= P(Z > 2.46) = 1 - P(Z < 2.46)$

$= \boxed{0.0069}$

b) $P(\text{required}) = P(L < 3M) = P(L - 3M < 0)$

let $B = L - 3M$,

$$E(B) = E(L) - 3E(M)$$

$$= 1510 - 3(520) = -50$$

$$Var(B) = Var(L) + 3^2 Var(M)$$

$$= 20^2 + 3^2(10^2) = 1300$$

$$\therefore B \sim N(-50, 1300)$$

$$P(\text{required}) = P(B < 0) = P\left(z < \frac{0 - -50}{\sqrt{1300}}\right)$$

$$= P(z < 1.39) = \boxed{0.9177}$$

c) $$\left[P(M > 520)\right]^5 = P(\bar{M} > d)$$

$$\bar{M} \sim N\left(520, \frac{10^2}{5}\right) \text{ by C.L.T}$$

$$P(M > 520) = 0.5 \quad (\text{since } 520 \text{ is the mean})$$

$$\therefore \quad 0.5^5 = P(\bar{M} > d)$$

$$P(\bar{M} > d) = P\left(z > \frac{d - 520}{\frac{10}{\sqrt{5}}}\right) = 0.5^5$$

$$P\left(z > \frac{\sqrt{5}(d - 520)}{10}\right) = 0.03125$$

$$\therefore \quad P\left(z < \frac{\sqrt{5}(d - 520)}{10}\right) = 0.96875$$

$P(z < z) = 0.96875$ is not in the normal distribution tables so find the closest probability.

from tables : $P(z < 1.86) = 0.9686$

$\therefore 1.86 = \dfrac{\sqrt{5}(d - 520)}{10}$

$\dfrac{1.86 \times 10}{\sqrt{5}} = d - 520$

$d = \dfrac{1.86(10)}{\sqrt{5}} + 520$

$\boxed{d \approx 528.3}$