

Mark Scheme (Results)

January 2013

International GCSE Further Pure Mathematics (4PM0/02)

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications come from Pearson, the world's leading learning company. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk for our BTEC qualifications.

Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

If you have any subject specific questions about this specification that require the help of a subject specialist, you can speak directly to the subject team at Pearson.

Their contact details can be found on this link: www.edexcel.com/teachingservices.

You can also use our online Ask the Expert service at www.edexcel.com/ask. You will need an Edexcel username and password to access this service.

Our website subject pages hold useful resources, support material and live feeds from our subject advisors giving you access to a portal of information. If you have any subject specific questions about this specification that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

www.edexcel.com/contactus

Pearson: helping people progress, everywhere

Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

January 2013

Publications Code UG034507

All the material in this publication is copyright

© Pearson Education Ltd 2013

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.

Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**

- M marks: method marks
- A marks: accuracy marks. Can only be awarded if the relevant method mark(s) has (have) been gained.
- B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**

- cao – correct answer only
- ft – follow through
- isw – ignore subsequent working
- SC - special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- eeoo – each error or omission

- **No working**

If no working is shown then correct answers may score full marks

If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

If there is a wrong answer indicated always check the working and award any marks appropriate from the mark scheme.

If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.

Any case of suspected misread which does not significantly simplify the question loses two A (or B) marks on that question, but can gain all the M marks. Mark all work on follow through but enter A0 (or B0) for the first two A or B marks gained.

If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

If there are multiple attempts shown, then all attempts should be marked and the highest score on a single attempt should be awarded.

- **Follow through marks**

Follow through marks which involve a single stage calculation can be awarded without working since you can check the answer yourself, but if ambiguous do not award.

Follow through marks which involve more than one stage of calculation can only be awarded on sight of the relevant working, even if it appears obvious that there is only one way you could get the answer given.

- **Ignoring subsequent work**

It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

It is not appropriate to ignore subsequent work when the additional work essentially shows that the candidate did not understand the demand of the question.

- **Linear equations**

Full marks can be gained if the solution alone is given, or otherwise unambiguously indicated in working (without contradiction elsewhere). Where the correct solution only is shown substituted, but not identified as the solution, the accuracy mark is lost but any method marks can be awarded.

- **Parts of questions**

Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another

June 2013
4PM0 Further Pure Mathematics
Mark Scheme

Question Number	Scheme	Marks
1.		
(a)	$\sin \theta = \frac{2}{6}$ $\sin \theta = \frac{1}{3}$ $\theta = 0.3398$	M1A1 A1 (3)
(b)	<p>Area of sector = $\frac{1}{2}r^2 \times 2\theta = \frac{1}{2} \times 64 \times 2\theta$ (= 21.649...)</p> <p>Shaded area = sector $-\pi \times 2^2$, = 9.18</p>	M1 M1,A1 (3) [6]
2		
(a)	$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$ <p style="text-align: center;">÷ by $\cos A \cos B$</p> $= \frac{\tan A + \tan B}{1 - \tan A \tan B}$	M1 M1 A1 (3)
(b)	<p>(i) $\tan 105 = \tan(60+45) = \frac{\tan 60 + \tan 45}{1 - \tan 60 \tan 45} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}}$ or see explicitly $\tan 60 = \sqrt{3}$ and $\tan 45 = 1$</p> <p>(ii) $\tan 15 = \tan(60-45) = \frac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}}$</p>	M1A1 M1A1 (4) [7]

Question Number	Scheme	Marks
<p>3(a)</p>	$(1+3x^2)^{\frac{1}{4}} = 1 + \left(-\frac{1}{4}\right)(3x^2) + \frac{\left(-\frac{1}{4}\right)\left(-\frac{5}{4}\right)}{2!}(3x^2)^2 + \frac{\left(-\frac{1}{4}\right)\left(-\frac{5}{4}\right)\left(-\frac{9}{4}\right)}{3!}(3x^2)^3$ $= 1 - \frac{3}{4}x^2 + \frac{45}{32}x^4 - \frac{405}{128}x^6$	<p>M1</p> <p>A2,1,0 (3)</p>
<p>(b)</p>	$x^2 < \frac{1}{3} \Rightarrow x < \frac{1}{\sqrt{3}}$	<p>B1 (1)</p>
<p>(c)</p>	$f(x) = (3+kx^2)\left(1 - \frac{3}{4}x^2 + \frac{45}{32}x^4 - \frac{405}{128}x^6\right)$ $= 3 + \left(k - \frac{9}{4}\right)x^2 + \left(\frac{135}{32} - \frac{3k}{4}\right)x^4 + \left(\frac{45k}{32} - \frac{1215}{128}\right)x^6$	<p>M1</p> <p>M1A1 (3)</p>
<p>(d)</p>	$\frac{135}{32} = \frac{3k}{4}, \quad k = \frac{135}{32} \times \frac{4}{3} = \frac{45}{8}$	<p>M1,A1 (2)</p>
[9]		
<p>4.</p> <p>(a)</p>	$3\sin 5x + 15x \cos 5x$	<p>M1A1A1 (3)</p>
<p>(b)</p>	$\frac{2e^{2x}(4-3x^2) - e^{2x}(-6x)}{(4-3x^2)^2}$	<p>M1A1A1 (3)</p>
[6]		

Question Number	Scheme	Marks
5(a)	$\cos 2A = \cos^2 A - \sin^2 A, = (1 - \sin^2 A) - \sin^2 A$ $\cos 2A = 1 - 2\sin^2 A \quad 2\sin^2 A = 1 - \cos 2A$	M1,M1 A1 (3)
(b)	$\cos 4A = 1 - 2\sin^2 2A \quad \sin^2 2A = \frac{1}{2}(1 - \cos 4A) \quad k = \frac{1}{2}$	B1 (1)
(c)	$\text{Volume} = \pi \int_0^{\frac{\pi}{6}} (3\sin 2x)^2 dx$ $= \pi \int_0^{\frac{\pi}{6}} \frac{9}{2} (1 - \cos 4x) dx$ $= \frac{9\pi}{2} \left[x - \frac{1}{4} \sin 4x \right]_0^{\frac{\pi}{6}}$ $= \frac{9\pi}{2} \left[\frac{\pi}{6} - \frac{1}{4} \sin \frac{2\pi}{3} \right]$ $= 4.3414... = 4.34$	M1 M1 M1A1ft (on k) M1 A1 (6) [10]
6	<p data-bbox="172 1144 213 1178">(a)</p> $V = 5x^2h$ $A = 2(5x^2 + 5xh + xh)$ $h = \frac{15}{5x^2} \quad A = 10x^2 + 12x \times \frac{3}{x^2} = 10x^2 + \frac{36}{x} \quad *$ <p data-bbox="172 1420 213 1453">(b)</p> $\frac{dA}{dx} = 20x - 36x^{-2}$ $\frac{dA}{dx} = 0 \quad 20x = \frac{36}{x^2}$ $x = \sqrt[3]{\frac{36}{20}} = 1.216... = 1.22$ $\frac{d^2A}{dx^2} = 20 + 72x^{-3}$ $x = 1.216 \Rightarrow \frac{d^2A}{dx^2} > 0 \quad \therefore \text{min}$ <p data-bbox="172 1890 213 1924">(c)</p> $x = 1.216 \quad A = 10 \times 1.216^2 + \frac{36}{1.216} = 44.4$ <p>(using $x = 1.22$ also gives 44.4)</p>	B1 M1A1 (3) M1 M1 A1 M1 M1A1 (6) M1A1 (2) [11]

Question Number	Scheme	Marks
<p>7</p> <p>(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	$\frac{y-2}{6-2} = \frac{x-3}{1-3}$ $-2(y-2) = 4(x-3)$ $y + 2x = 8$	<p>M1A1</p> <p>A1 (3)</p>
	$(8-2x)x = 8$ $x^2 - 4x + 4 = 0$ $(x-2)(x-2) = 0$ <p>equal roots \therefore tangent</p>	<p>M1</p> <p>M1</p> <p>A1 (3)</p>
	$x = 2, y = 4$	<p>B1B1 (2)</p>
	$\text{grad tgt} = -2$ $\text{grad normal} = \frac{1}{2}$ $\text{Equation normal : } y - 4 = \frac{1}{2}(x - 2)$ $2y = x + 6 \text{ (oe integer coeffs only)}$	<p>B1</p> <p>M1</p> <p>A1 (3)</p> <p>[11]</p>

Question Number	Scheme	Marks
8 (a)	(i) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \mathbf{b} - \mathbf{a}$	M1A1
	(ii) $\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$	M1A1
	(iii) $\overrightarrow{PM} = \overrightarrow{PA} + \overrightarrow{AM} = \frac{2}{5}\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}\mathbf{b} - \frac{1}{10}\mathbf{a}$	M1A1 (6)
	(b) $\overrightarrow{OP} + \lambda\overrightarrow{PM} = \overrightarrow{OX}$	M1
	$\frac{3}{5}\mathbf{a} + \lambda\left(\frac{1}{2}\mathbf{b} - \frac{1}{10}\mathbf{a}\right) = \mu\mathbf{b}$	A1
	$\frac{3}{5} = \frac{\lambda}{10} \quad \lambda = 6$	
	$\mu = \frac{1}{2}\lambda = 3$	M1
	$\therefore \overrightarrow{OX} = 3\mathbf{b}$	A1 (4)
	(c) area $\triangle OAM = \frac{1}{2}$ area $\triangle OAB$	
	area $\triangle OAB = \frac{1}{3}$ area $\triangle OAX$	M1
\therefore area $\triangle OAM = \frac{1}{2} \times \frac{1}{3}$ area $\triangle OAX = \frac{1}{6}$ area $\triangle OAX$	M1	
ratio = 1 : 6	A1 (3) [13]	

Question Number	Scheme	Marks
9		
(a)	$\frac{ar^4}{ar^2} = \frac{768}{48}$	M1A1
	$r^2 = 16 \quad r = \pm 4$	A1 (3)
(b)	$ar^2 = 48 \quad a = 3$	B1 (1)
(c)	$r = -4$	B1
	$S_9 = \frac{3((-4)^9 - 1)}{-4 - 1}, = 157287$	M1A1,A1 (4)
(d)	$r = \frac{1}{4}$	B1 (1)
(e)	$T_9 = \frac{3\left(1 - \left(\frac{1}{4}\right)^9\right)}{1 - \frac{1}{4}} = 3.999984741$	M1A1 (2)
(f)	$T_\infty = \frac{3}{1 - \frac{1}{4}} = 4$	M1A1
	$4 - 0.002 > \frac{3\left(1 - \left(\frac{1}{4}\right)^n\right)}{\frac{3}{4}}$	M1
	$\frac{3.998}{4} > 1 - \left(\frac{1}{4}\right)^n$	
	$\left(\frac{1}{4}\right)^n > 1 - \frac{3.998}{4} = 0.0005$	
	Solve by logs or trial and error	M1
	greatest n is 5	A1 (5) [16]

Question Number	Scheme	Marks
10 (a)	$\log_x 1024 = 5 \quad x^5 = 1024 \quad x = 4$	M1A1 (2)
(b)	$\log_5(6y+11) = 3 \quad 6y+11 = 5^3 = 125, \quad y = \frac{114}{6} = 19$	M1, M1A1 (3)
(c)	$2\log_3 t + \log_t 9 = 5$ $\frac{2}{\log_t 3} + 2\log_t 3 = 5$ $2(\log_t 3)^2 - 5\log_t 3 + 2 = 0$ $(2\log_t 3 - 1)(\log_t 3 - 2) = 0$ $\log_t 3 = \frac{1}{2} \quad 3 = \sqrt{t} \quad t = 9$ $\log_t 3 = 2 \quad 3 = t^2 \quad t = \sqrt{3}$	 M1 (change base) A1 M1 M1A1 A1 (must be positive) (6) [11]

Further copies of this publication are available from
Edexcel Publications, Adamsway, Mansfield, Notts, NG18 4FN

Telephone 01623 467467

Fax 01623 450481

Email publication.orders@edexcel.com

Order Code UG034507 January 2013

For more information on Edexcel qualifications, please visit our website
www.edexcel.com

Pearson Education Limited. Registered company number 872828
with its registered office at Edinburgh Gate, Harlow, Essex CM20 2JE

Ofqual




Llywodraeth Cynulliad Cymru
Welsh Assembly Government

