



Mark Scheme (Results)

January 2016

International GCSE Further Pure Mathematics 4PM0/02

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
 - M marks: method marks
 - A marks: accuracy marks
 - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
 - cao – correct answer only
 - ft – follow through
 - isw – ignore subsequent working
 - SC - special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - eeo – each error or omission

- **No working**
If no working is shown then correct answers may score full marks
If no working is shown then incorrect (even though nearly correct) answers score no marks.
- **With working**
Always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
Any case of suspected misread loses 2A (or B) marks on that part, but can gain the M marks.
If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.
- **Ignoring subsequent work**
It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question
- **Parts of questions**
Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another.

General Principles for Further Pure Mathematics Marking (but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$(x^2 + bx + c) = (x + p)(x + q) \text{ where } |pq| = |c|$$

$$(ax^2 + bx + c) = (mx + p)(nx + q) \text{ where } |pq| = |c| \text{ and } |mn| = |a|$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for a , b and c , leading to

3. Completing the square:

$$\text{Solving } x^2 + bx + c = \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c \text{ where } q \neq 0$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1.

2. Integration:

Power of at least one term increased by 1.

Use of a formula:

Generally, the method mark is gained by:

either quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show....")

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Jan 2016

4PM0 Further Pure Mathematics Paper 2

Mark Scheme

Question Number	Scheme	Marks
1.	$2^{2(x-2)} = 2^{3(3x-1)}$ $\Rightarrow 2(x-2) = 3(3x-1)$ $x = -\frac{1}{7}$	M1 dM1A1 A1cao (4)
M1 dM1 A1 A1cao ALT 1 ALT 2	<p>Attempt to change to powers of 2, 4 or 8 (both sides of equation) Equate powers Correct linear equation - unsimplified $x = -\frac{1}{7}$ (or equivalent fraction with integer numerator and denominator) NB: $\log_4 8 = 1.5$ is exact and so allowed</p> <p>Alternatives for no 1 Take logs base 4 each side Change $\log_4 8$ to 1.5 Correct linear equation 1.5 and any other non-rounded decimals allowed $x = -\frac{1}{7}$ decimals may have been used in working, provided none have been rounded</p> <p>$\log 4^{(x-2)} = \log 8^{(3x-1)}$ can be any base $(x-2)\log 4 = (3x-1)\log 8$ $(x-2) \times 2\log 2 = (3x-1) \times 3\log 2$ $2(x-2) = 3(3x-1)$ $x = -\frac{1}{7}$</p>	M1 dM1 A1 A1cao M1 dM1A1 A1cao

Question Number	Scheme	Marks
(1) Alt 3	$\frac{4^x}{4^2} = \frac{8^{3x}}{8} \Rightarrow \frac{4^x}{2} = 8^{3x}$ $4^x \times \frac{1}{2} = (8^3)^x \quad \frac{1}{2} = \left(\frac{8^3}{4}\right)^x$ $\frac{1}{2} = 128^x$ $x = \frac{\log \frac{1}{2}}{\log 128} = \frac{-\log 2}{7 \log 2} \quad (\text{any base})$ $x = -\frac{1}{7}$	M1 dM1A1 A1cao
2.	(i) $48 = \frac{1}{2}\theta r^2, \quad 8 = \theta r$ or equivalent equations $\frac{\theta r^2}{2} = \frac{48}{8} \Rightarrow r = 12$ (ii) $\theta = \frac{8}{12}, (= \frac{2}{3})$	B1B1 M1A1 A1 (5)
B1 B1 M1 A1 A1	B1B1 Two correct equations; B1B0 One correct equation Eliminate either variable and solve to obtain the other $r = 12$ $\theta = \frac{8}{12}$ oe Accept 0.667 or better (NB: decimal may be ignored under isw rule.)	

Question Number	Scheme	Marks
<p>3</p>	$3y = 12 - 4x \Rightarrow y = 4 - \frac{4}{3}x \quad \text{OR} \quad 4x = 12 - 3y \Rightarrow x = 3 - \frac{3}{4}y$ $(x+1)^2 + \left(4 - \frac{4}{3}x - 2\right)^2 = 4 \quad \left \quad \left(3 - \frac{3}{4}y + 1\right)^2 + (y-2)^2 = 4$ $\Rightarrow 25x^2 - 30x + 9 = 0 \text{ 3TQ} \quad \Rightarrow 25y^2 - 160y + 256 = 0 \text{ 3TQ}$ $(5x-3)(5x-3) = 0 \Rightarrow x = \frac{3}{5} \quad (5y-16)(5y-16) = 0 \Rightarrow y = \frac{16}{5}$ $y = 4 - \frac{4}{3} \times \frac{3}{5} = \frac{16}{5} \quad x = 3 - \frac{3}{4} \times \frac{16}{5} = \frac{3}{5}$	<p>B1</p> <p>M1</p> <p>M1A1</p> <p>M1A1</p> <p>A1</p> <p>(7)</p>
<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>Write the linear equation to read $x = \dots$ or $y = \dots$. May be seen explicitly or implied by subsequent working. (Equivalent forms accepted)</p> <p>Substitute to obtain a quad equation in one variable</p> <p>Simplify to a 3 term quadratic - terms in any order - coeffs need not be integers</p> <p>Correct 3 term quadratic - terms in any order - coeffs need not be integers</p> <p>Their 3 term quadratic solved by any valid method. (Can still be earned if the discriminant is negative.)</p> <p>Correct values for one variable</p> <p>(B1 on e-pen) Correct values for the second variable</p> <p>Equivalents accepted for both variables</p> <p>NB: Calculator solutions for the quadratic accepted provided both roots correct.</p>	
<p>4</p>	$f'(x) = 2e^{2x}(x+1)^{0.5} + e^{2x} \frac{(x+1)^{-0.5}}{2}$ $f'(x) = e^{2x} \left(2(x+1)^{0.5} + \frac{1}{2(x+1)^{0.5}} \right)$ $\Rightarrow e^{2x} \left(\frac{4(x+1)+1}{2(x+1)^{0.5}} \right) \Rightarrow \frac{e^{2x}(4x+5)}{2\sqrt{x+1}} \quad ***$	<p>M1A1A1</p> <p>dM1</p> <p>dM1A1cso</p> <p>(6)</p>
<p>M1</p> <p>A1A1</p> <p>dM1</p> <p>dM1</p> <p>A1cso</p>	<p>Attempt to differentiate using the product rule. Must be the sum of two terms both with $(x+1)^{+/-0.5}$ and e^{2x}. Constants may be incorrect</p> <p>If quotient rule is used the numerator must be the difference of two terms both with $(x+1)^{+/-0.5}$ and e^{2x} and the denominator must be $(x+1)^{-1}$.</p> <p>A1A1 Both terms fully correct; A1A0 one term fully correct</p> <p>Extract a common factor of form ke^{2x} where k is an integer</p> <p>Simplify the bracket by combining to a single term</p> <p>The above steps may be carried out in either order but marks must be entered in this order. These 2 M marks are dependent on the first M mark but not on each other.</p> <p>Obtain the GIVEN answer with no errors seen $(x+1)^{\frac{1}{2}}$ scores A0</p>	

Question Number	Scheme	Marks
<p>5 (a)</p> <p>(b)</p> <p>(c)</p>	$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta \Rightarrow \alpha\beta = \frac{5^2 - 19}{2} = 3 \quad \text{cso} \quad ***$ $\Rightarrow \frac{c}{a} = 3 \text{ and } -\frac{b}{a} = 5 \text{ let } a = 1 \Rightarrow x^2 - 5x + 3 = 0 \text{ oe}$ $\frac{\beta}{\alpha} + \frac{\alpha}{\beta} = \frac{\alpha^2 + \beta^2}{\alpha\beta}, = \frac{19}{3}$ $\frac{\beta}{\alpha} \times \frac{\alpha}{\beta} = 1$ $x^2 - \frac{19}{3}x + 1 (= 0), \quad 3x^2 - 19x + 3 = 0 \text{ oe}$	<p>M1A1cso (2)</p> <p>M1A1 (2)</p> <p>M1,A1</p> <p>B1</p> <p>M1,A1 (5) (9)</p>
<p>(a)M1 A1cso ALT: (b)M1 A1 (c)M1 A1 B1 M1 A1ft</p>	<p>Obtain an expression for $\alpha\beta$ in terms of $\alpha + \beta$ and $\alpha^2 + \beta^2$</p> <p>Correct value for $\alpha\beta$</p> <p>Solve the given equations for α and β M1 Fully correct to given answer A1</p> <p>Use $x^2 - (\text{sum of roots})x + \text{product of roots} (= 0)$</p> <p>A correct equation - any integer multiple of the one shown</p> <p>Write the sum of the roots as a single fraction. Algebra to be correct for this mark.</p> <p>Correct value for the sum of the roots</p> <p>Product = 1 Seen explicitly or used</p> <p>Use $x^2 - (\text{sum of roots})x + \text{product of roots} (= 0)$</p> <p>Correct equation. Follow through their sum and product. Any integer multiple accepted.</p>	
<p>6 (a)</p> <p>(b)</p> <p>(c)</p>	$\sin(2x) = \sin x \cos x + \cos x \sin x = 2 \sin x \cos x \quad *$ $\cos(2x) = \cos x \cos x - \sin x \sin x = \cos^2 x - \sin^2 x \quad *$ $\frac{\sin 2x}{1 + \cos 2x} = \frac{2 \sin x \cos x}{1 + (\cos^2 x - \sin^2 x)}$ $= \frac{2 \sin x \cos x}{\cos^2 x + \sin^2 x + \cos^2 x - \sin^2 x}$ $= \frac{2 \sin x \cos x}{2 \cos^2 x} = \tan x \quad ***$	<p>B1 B1 (2)</p> <p>M1</p> <p>dM1A1</p> <p>A1cso (4) (6)</p>
<p>(a)B1 (b)B1 (c)M1 dM1 A1 A1cso</p>	<p>For the correct result. Award only if evidence of use of the given formula is seen</p> <p>As for (a)</p> <p>Use the above identities to change "2x"s to "x"s</p> <p>Use $\cos^2 x + \sin^2 x = 1$ to eliminate $\sin^2 x$</p> <p>Min evidence is $(1 - \sin^2 x)$ changed to $\cos^2 x$ or $(1 - \sin^2 x) + \cos^2 x = 2 \cos^2 x$</p> <p>Denominator $1 + c^2 - s^2$ changed to either $c^2 + c^2$ or $2c^2$ is NOT sufficient</p> <p>But $1 - s^2 + c^2$ changed to $c^2 + c^2$ or $2c^2$ is sufficient</p> <p>Correct (unsimplified) fraction, as shown or equivalent (no trig functions of 2x)</p> <p>Both M marks must be gained for this A mark to be awarded</p> <p>Obtain the GIVEN result with no errors seen</p>	

Question Number	Scheme	Marks
<p>7 (a)</p> <p>(b)</p> <p>(c)</p>	$x = \frac{3}{2} \quad \left(\text{or eg } 2x = 3, x - \frac{3}{2} = 0 \right)$ $\frac{dy}{dx} = \frac{(2x-3)(2x) - (x^2-2)(2)}{(2x-3)^2} = \left(\frac{2x^2 - 6x + 4}{(2x-3)^2} \right)$ $\frac{dy}{dx} = 0 \Rightarrow \frac{(2x-3)(2x) - (x^2-2)(2)}{(2x-3)^2} = 0$ $\Rightarrow 2x^2 - 6x + 4 = 0 \Rightarrow (x-1)(x-2) = 0 \Rightarrow x = 1, x = 2$ $x = 1, y = 1 \quad (1,1) \quad x = 2, y = 2 \quad (2,2)$	<p>B1 (1)</p> <p>M1A1A1 (3)</p> <p>M1</p> <p>M1A1A1</p> <p>A1 (5)</p> <p>(9)</p>
<p>(a) B1</p> <p>(b) M1 A1 A1 ALT:</p> <p>(c) M1 M1 A1 A1 A1</p>	<p>For a correct equation for the asymptote. NB $x \neq \frac{3}{2}$ scores B0</p> <p>Attempt to differentiate by quotient rule. Denominator must be correct. Numerator must be the difference of two terms of the appropriate form.</p> <p>NB M1 on e-PEN First term correct</p> <p>Second term correct</p> <p>Use the product rule. M1 for the attempt, using $(x^2 - 2)(2x - 3)^{-1}$</p> <p>A1,A1 one for each correct term</p> <p>Equate their derivative to 0</p> <p>Solve their quadratic (numerator) by any valid method.</p> <p>A1A1 two correct values for x from a correct equation; A1A0 for one correct value Ignore extra values.</p> <p>NB B1 on e-PEN Find the corresponding y values. Coordinate brackets need not be shown. Give A0 if more than 2 stationary points shown.</p> <p>NB: Quadratic solved on a calculator: correct values for x, M1A1A1 One or both values incorrect, or only one value shown: M0A0A0</p> <p>Special Case for (c): Both correct answers only shown, Award B1B1 - in first two marks on e-PEN.</p>	

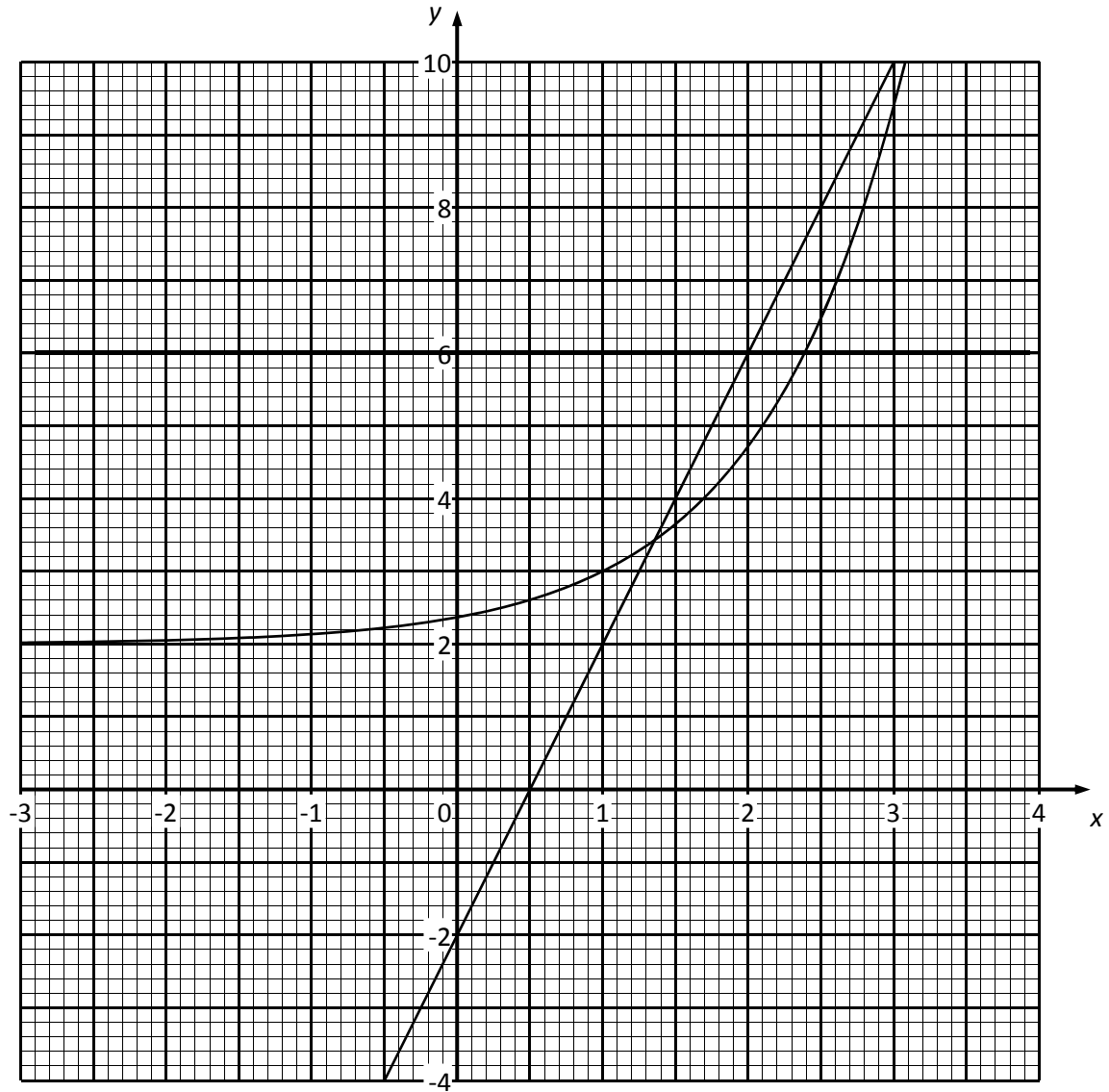
Question Number	Scheme	Marks
<p>8 (a)</p> <p>(b)</p>	$a = 2 - 3 = -1 \quad d = 2 \quad (l = 2n - 3)$ <p>Uses $S_n = \frac{n}{2}(a+l)$, $S_n = \frac{n}{2}(-1 + (2n - 3))$ $S_n = \frac{n}{2}(n - 2)$ ***</p> <p>OR</p> $S_n = \frac{n}{2}(2 \times -1 + (n - 1)2) \Rightarrow S_n = \frac{n}{2}(2n - 4) \Rightarrow S_n = n(n - 2)$ *** $5(2n + 4 - 3) = 3(n - 3)((n - 3) - 2)$ $3n^2 - 34n + 40 = 0 \quad 3\text{TQ} \Rightarrow (3n - 4)(n - 10) = 0 \Rightarrow n = 10$	<p>B1B1</p> <p>M1A1cso (4)</p> <p>M1A1</p> <p>M1 dM1A1 (5) (9)</p>
<p>(a)</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1cso</p> <p>(b)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>dM1</p> <p>A1</p>	<p>$a = -1$ No working needed - need not be shown explicitly</p> <p>$d = 2$ No working needed or if $S_n = \frac{n}{2}(a+l)$ used, give B1 for correct substitution if no value shown anywhere for d</p> <p>Using either formula for S_n with their a and d</p> <p>Obtaining the GIVEN result with no errors seen</p> <p>Using the GIVEN t_n and S_n in the equation or start from correct basic formulae</p> <p>Correct unsimplified equation</p> <p>Obtaining a three term quadratic, terms in any order</p> <p>NB A1 on e-pen Factorising their quadratic or correct use of formula/completing the square.</p> <p>Cao $n = 10$ Award A0 if single correct answer not identified.</p> <p>If final answers shown without working (implying calculator solution) give M1 only if both correct answers to the quadratic are shown. A1 then for identifying the single correct solution for this problem.</p>	

Question Number	Scheme	Marks
<p>9 (a)</p> <p>(b)</p>	$\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$ $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} \Rightarrow \overrightarrow{OC} = 2\mathbf{b} - 2\mathbf{a} = 2(\mathbf{b} - \mathbf{a}) (= 2\overrightarrow{AB}) \quad (\text{oe})$ <p>(i) Hence, \overrightarrow{OC} and \overrightarrow{AB} are in same direction</p> <p>(ii) And, \overrightarrow{OC} is twice the length of \overrightarrow{AB}</p> <p>Conclusions required *</p> $\frac{\text{area of triangle } ODC}{\text{area of triangle } OBC} = \frac{0.5 \times \text{height} \times 2}{0.5 \times \text{height} \times 5} = \frac{2}{5}$ $\frac{\text{area of triangle } OAB}{\text{area of triangle } OBC} = \frac{0.5 \times \text{height} \times 1}{0.5 \times \text{height} \times 2} = \frac{1}{2}$ <p>area of triangle $OBC = \frac{5}{2} \times \text{area of triangle } ODC$, and,</p> <p>area of triangle $OBC = 2 \times \text{area of triangle } OAB$</p> <p>Therefore, $\frac{\text{area of triangle } ODC}{\text{area of triangle } OAB} = \frac{4}{5}$</p> <p>{Or given as ratio, area of triangle ODC ; area of triangle $OAB = 4 : 5$}</p>	<p>B1 M1,</p> <p>A1 A1</p> <p>(4)</p> <p>M1A1</p> <p>M1A1</p> <p>dM1A1cso</p> <p>(6) (10)</p>
<p>(a) B1 M1 (i)A1 (ii)A1</p> <p>(b) M1 A1 M1 A1 dM1 A1cso</p>	<p>Correct expression for \overrightarrow{AB}</p> <p>Obtaining \overrightarrow{OC} in terms of \mathbf{a} and \mathbf{b}</p> <p>Using correct expressions for \overrightarrow{OC} and \overrightarrow{AB} to deduce that they are parallel</p> <p>NB B1 on e-PEN Deducing the GIVEN ratio $AB : OC$ or $OC : AB$ provided clear which is intended. No vector arrows here.</p> <p>Accept shown or # or similar as a conclusion provided clear which part it refers to.</p> <p>Finding the ratio of the areas of triangles ODC and OBC, either order</p> <p>Correct ratio (or fraction), triangles in either order</p> <p>Finding the ratio of the areas of triangles OAB and OBC, either order</p> <p>Correct ratio (or fraction), triangles in either order</p> <p>Eliminating area of triangle OBC to obtain a value for the required ratio (or fraction)</p> <p>Depends on both the preceding M marks.</p> <p>Correct ratio or fraction (any equivalent). Triangles to be in the correct order.</p> <p>Ratio can be in one of forms $1:1.25$, $1:5/4$, $0.8:1$, $4/5:1$</p> <p>NB: $\mathbf{b} - \mathbf{a}$ (whether bold, underlined or neither) is a vector, not the length of a line. M marks only can be awarded.</p>	

Alternatives for 9(b)	
ALT 1	<p>Area $\triangle OAB = \frac{1}{2} AB \times OB \sin OBA$ M1 (area either triangle)</p> <p>Area $\triangle ODC = \frac{1}{2} OD \times OC \sin DOC$ A1 (both areas correct)</p> <p>$2 \overline{AB} = \overline{OC}$ or $2AB = OC$, $\frac{2}{5} \overline{OB} = \overline{OD}$ M1 (either)</p> <p>$\angle OBA = \angle DOC$ A1 (all 3 statements correct or used correctly)</p> <p>$\therefore \triangle ODC : \triangle OAB = \left(\frac{1}{2}\right) AB \times OB : \left(\frac{1}{2}\right) \times 2AB \times \frac{2}{5} OB$ dM1 (their ratio of lengths)</p> <p>$= 4 : 5$ A1</p>
ALT 2	<p>If $\frac{1}{2} \times \text{base} \times \text{height}$ used:</p> <p>Area $\triangle OAB = \frac{1}{2} AB \times h$ M1</p> <p>Area $\triangle ODC = \frac{1}{2} OC \times h'$ A1</p> <p>$h' = \frac{2}{5} h$ $OC = 2AB$ M1A1</p> <p>$\triangle OCD : \triangle OAB = AB \times \frac{2}{5} h : \frac{1}{2} AB \times h$ dM1</p> <p>$= 4 : 5$ oe A1</p> <p>M1A1 areas of triangles (M1 either correct, A1 both correct)</p> <p>M1A1 ratio of bases and ratio of heights (M1 either correct, A1 both correct)</p> <p>dM1A1 correct completion</p>

Question Number	Scheme	Marks
<p>10 (a)</p> <p>(b)</p>	$f(2) = 2 \times 2^3 - p \times 2^2 - 13 \times 2 - q = -20 \quad (\Rightarrow 10 = 4p + q)$ $f(3) = 2 \times 3^3 - p \times 3^2 - 13 \times 3 - q = 0 \quad (\Rightarrow 15 = 9p + q)$ <p>Solves simultaneous equations by elimination or substitution; $\Rightarrow 5 = 5p \Rightarrow p = 1,$ so $q = 6$</p> $(2x^3 - x^2 - 13x - 6) \div (x - 3) = 2x^2 + 5x + 2$ $(2x^3 - x^2 - 13x - 6) = (x - 3)(2x + 1)(x + 2) \text{ (Factorises } 2x^2 + 5x + 2 \text{)}$ $x = 3, -\frac{1}{2}, -2 \text{ (all three roots)}$	<p>M1A1</p> <p>M1A1</p> <p>M1 A1 A1 (7)</p> <p>M1A1</p> <p>M1</p> <p>A1A1 (5) (12)</p>
<p>(a)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>(b)</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1A1</p>	<p>Substitute ± 2 in $f(x)$</p> <p>Correct equation using remainder -20 Need not be simplified</p> <p>Substitute ± 3 in $f(x)$</p> <p>Correct equation using remainder 0 Need not be simplified</p> <p>First 4 marks can be given for long division:</p> <p>Divide by $(x \pm 2)$ M1 Equate correct remainder to -20 A1</p> <p>Divide by $(x \pm 3)$ M1 Equate correct remainder to 0 A1</p> <p>Solve the simultaneous equations, any valid method</p> <p>p or q correct</p> <p>Second unknown correct</p> <p>Obtain the quadratic factor by division or inspection. Factor need not be fully correct but must be of form $2x^2 + kx \pm \frac{\text{their } q}{3}$ If by division, remainder need not be 0.</p> <p>Correct quadratic factor</p> <p>Attempt to factorise their quadratic factor</p> <p>A1A1 all three roots correct; A1A0 two roots correct</p>	

Question Number	Scheme	Marks														
<p>11(a)</p> <table border="1" data-bbox="355 304 1185 448"> <tr> <td>x</td> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> </tr> <tr> <td>$f(x)$</td> <td>2.05</td> <td>2.14</td> <td>2.37</td> <td>3</td> <td>4.72</td> <td>9.39</td> </tr> </table> <p>(b) Correct points plotted and graph drawn</p> <p>(c) $4 = e^{(x-1)} \Rightarrow 6 = e^{(x-1)} + 2 \quad y = 6$ Line $y = 6$ drawn $\Rightarrow x = 2.4$</p> <p>(d)</p> <p>$\ln(4x - 4) = x - 1 \Rightarrow (4x - 4) = e^{(x-1)}$, $\Rightarrow 4x - 2 = e^{(x-1)} + 2$ $y = 4x - 2$ drawn on graph</p> <p>accept $x = 1.3/1.4$</p>	x	-2	-1	0	1	2	3	$f(x)$	2.05	2.14	2.37	3	4.72	9.39		<p>B1B1 (2)</p> <p>B1ftB1ft (2)</p> <p>M1 A1 (2)</p> <p>M1,A1 A1ft dM1</p> <p>A1cso(5) (11)</p>
x	-2	-1	0	1	2	3										
$f(x)$	2.05	2.14	2.37	3	4.72	9.39										
<p>(a) B1B1</p> <p>(b) B1ft B1ft</p> <p>(c) M1 A1</p> <p>(d) M1 A1 A1ft dM1 A1cso</p>	<p>NB Read rounding rules at start of this document B1B1 three correct values; B1B0 two correct values</p> <p>Plot their points correctly Draw a smooth curve through their points. $-2 \leq x \leq 3$ only needed - ignore any points/graph outside this range.</p> <p>Attempt to deduce the value of y corresponding to the given equation, $y = 4 \pm 2$ should be seen Using $y = 6$ to obtain $x = 2.4$ Must be 1 dp unless already penalised (2.3862...) If the M mark is gained and $y = 6$ or $e^{(x-1)} + 2 = 6$ is seen this mark can be given without the line being drawn. If the line $y = 6$ is seen on the graph and correct answer given, award M1A1</p> <p>Change equation from log to exponential form Correct exponential equation Add 2 to each side of their equation Draw their line on their graph Obtain $x = 1.3$ or 1.4 Must be 1 dp unless already penalised (1.355...) Correct answers from incorrect lines score A0. Ignore extra answers outside the given range.</p>															



Question Number	Scheme	Marks
<p>12(a)</p> <p>(b)</p> <p>(c)</p> <p>(d)</p>	$BM = \sqrt{8^2 - 4^2}, = 4\sqrt{3} \text{ (oe eg } \sqrt{24} \times \sqrt{2})$ $p = 4 \quad q = 3$ $\cos BAM = \frac{4}{8} \Rightarrow BAM = 60^\circ$ $EM = \sqrt{12^2 + 20^2} \text{ (= } \sqrt{544} = 4\sqrt{34})$ $MEB = \tan^{-1} \left(\frac{4\sqrt{3}}{4\sqrt{34}} \right) = 16.5437\dots \Rightarrow MEB = 16.5^\circ$ <p>Angle between plane <i>BCEH</i> and <i>ADEH</i> =</p> $\tan^{-1} \left[\frac{4\sqrt{3}}{20} \right] = 19.1066\dots = 19.1^\circ$	<p>M1,A1A1 (3)</p> <p>M1A1 (2)</p> <p>M1A1</p> <p>dM1A1(4)</p> <p>M1 dM1A1 (3) (12)</p>
<p>(a)</p> <p>M1 A1 A1</p> <p>(b)</p> <p>M1</p> <p>A1</p> <p>(c)</p> <p>M1</p> <p>A1 dM1 A1</p> <p>(d)</p> <p>M1</p> <p>dM1 A1</p>	<p>Use Pythagoras Must have minus sign A1A1 for correct <i>p</i> and <i>q</i> equivalent values allowed as long as one is prime. A1A0 for one correct. Values need not be shown explicitly.</p> <p>Use any trig function correctly (eg $\sin = \frac{\text{opp}}{\text{hyp}}$) to find $\angle BAM$ If cos or tan used then <i>AM</i> must = 4 or working for length <i>AM</i> must be seen. Their <i>BM</i> if used</p> <p>Correct answer. 60° without working scores M1A1</p> <p>Use Pythagoras to find length <i>EM</i>. Must have + sign. If <i>BE</i> found without first finding <i>EM</i> this mark requires a complete method. Award M1 for $EM^2 = 16^2 + 20^2$ provided this is stated to be <i>EM</i> or implied by subsequent working.</p> <p>Correct length <i>EM</i> (need not be simplified) (or <i>BE</i> =24.33....)</p> <p>Use any trig function correctly with their values to find $\angle MEB$</p> <p>Correct answer. Must be to nearest 0.1°</p> <p>Identify the required angle. Can be stated explicitly or implied by subsequent working.</p> <p>Use any trig function correctly to obtain the size of a correct angle</p> <p>Correct answer. Must be to nearest 0.1° unless already penalised.</p>	

