Pearson Edexcel

Mark Scheme (Results)

January 2019

Pearson Edexcel International GCSE in Further Pure Mathematics (4PM0) Paper 02

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.

  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks (dependent on the preceding M mark)
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - eeoo – each error or omission
- **No working**

  If no working is shown then correct answers normally score full marks.

  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

  If there is a wrong answer indicated always check the working in the body of the script and award any marks appropriate from the mark scheme.

  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

  Any case of suspected misread loses two A (or B) marks on that part, but can gain the M marks. Mark all work on follow through but enter A0 (or B0) for the first two A or B marks gained.

  If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

  If there are multiple attempts shown, then all attempts should be marked and the highest score on a single attempt should be awarded.

- **Follow through marks**

  Follow through marks which involve a single stage calculation can be awarded without working since you can check the answer yourself, but if ambiguous do not award.

  Follow through marks which involve more than one stage of calculation can only be awarded on sight of the relevant working, even if it appears obvious that there is only one way you could get the answer given.

- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially shows that the candidate did not understand the demand of the question.

- **Linear equations**

  Full marks can be gained if the solution alone is given, or otherwise unambiguously indicated in working (without contradiction elsewhere). Where the correct solution only is shown substituted, but not identified as the solution, the accuracy mark is lost but any method marks can be awarded.

- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$ leading to $x = ...$

$(ax^2 + bx + c) = (mx + p)(nx + q)$ where $|pq| = |c|$ and $|mn| = |a|$ leading to $x = ...$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x = ...$

3. Completing the square:

$x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$ leading to $x = ...$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

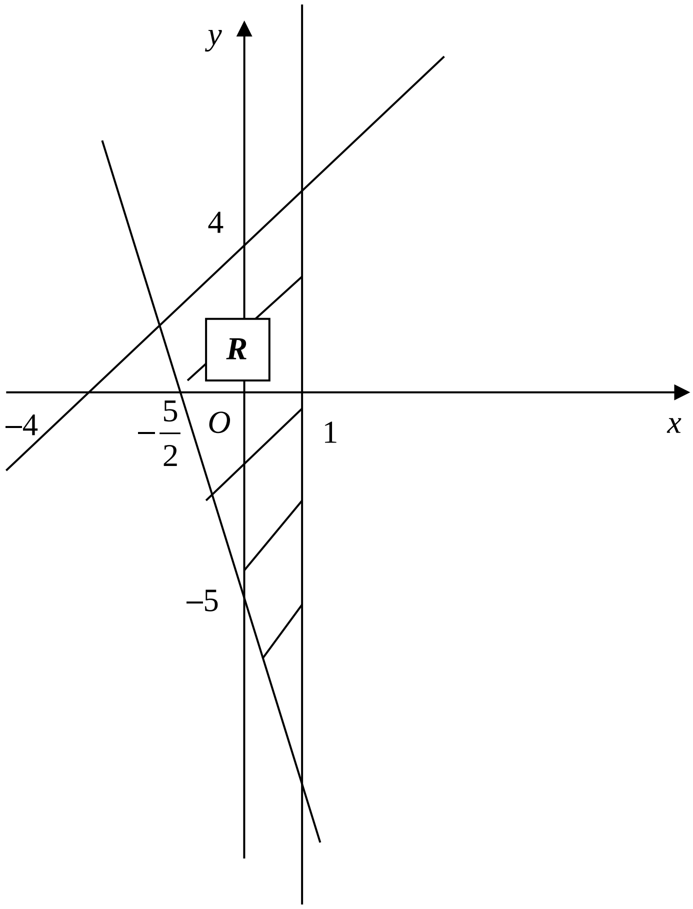
Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question number	Scheme	Marks
1	$3\log_3 x - 8\log_x 3 = 10 \Rightarrow 3\log_3 x - 8\dfrac{\log_3 3}{\log_3 x} - 10 = 0$	M1
	$\Rightarrow 3(\log_3 x)^2 - 10(\log_3 x) - 8 = 0$	M1
	OR : $3\dfrac{\log_x x}{\log_x 3} - 8\log_x 3 = 10 \Rightarrow 3 - 8(\log_x 3)^2 = 10\log_x 3$	
	$(3\log_3 x + 2)(\log_3 x - 4) = 0 \Rightarrow \log_3 x = -\dfrac{2}{3}, 4$	
	OR: $(4\log_x 3 - 1)(2\log_x 3 + 3) = 0 \Rightarrow \log_x 3 = \dfrac{1}{4}, -\dfrac{3}{2}$	M1A1
	$x = 3^4 = 81 \quad x = 3^{-\frac{2}{3}} \left[= \dfrac{1}{\sqrt[3]{9}} = \dfrac{\sqrt[3]{9}}{9} \approx 0.4807... \right]$	M1A1 [6]
M1	Use correct change of base formula so that all logs have the same base. May have 1 instead of $\log_3 3$ or $\log_x x$	
M1	Obtain a corresponding 3TQ, brackets here can be implied by subsequent working	
M1	Solve their 3TQ to $\log_3 x = ...$ or $\log_x 3 = ...$ If a substitution has been used it must be reversed before this mark can be awarded.	
A1	Either correct answer obtained	
M1	"Undo" at least one log correctly and obtain at least one value for x	
A1	2 correct values for x. These can be in any form inc decimals (min 3 sf)	
NB	This question can be solved using any base. For the first M mark all logs must have the same base and at least one change of base must be correct. If in doubt about the marking, send to review.	

Question number	Scheme	Marks
2 (a)		B1 B1 (2)
(b)		B1ft (1) [3]
(a) B1 B1	One mark per correct line, including intersections. Award B1B1, B1B0 or B0B0 The intersections must be numerical but can be shown below or at side of the sketch.	
(b)		
B1ft	Correct region shaded in or out. Must be clear that $x = 1$ is being used. Follow through their lines. Accept solid or dotted lines for boundaries (and in (a))	

Question number	Scheme	Marks
3 (a)	$\overrightarrow{PQ} = -(5\mathbf{i}+6\mathbf{j})+(3\mathbf{i}-4\mathbf{j}) = -2\mathbf{i}-10\mathbf{j}$	M1A1 (2)
(b)	$\left\lvert\overrightarrow{PQ}\right\rvert = \sqrt{(-2)^2+(-10)^2}\ \left(=\sqrt{104}\right)$ oe Unit vector parallel to $\overrightarrow{PQ}$: $\overrightarrow{XY} = \frac{1}{\sqrt{104}}(-2\mathbf{i}-10\mathbf{j})$ or $\overrightarrow{XY} = -\frac{1}{\sqrt{104}}(-2\mathbf{i}-10\mathbf{j})$ or $\frac{1}{\sqrt{104}}(2\mathbf{i}+10\mathbf{j})$ oe eg $\pm\frac{(\mathbf{i}+5\mathbf{j})}{\sqrt{26}}$	M1 A1 (2)
(c)	$\overrightarrow{QR} = \overrightarrow{QP}+\overrightarrow{PR}$ $5(2\mathbf{i}+10\mathbf{j}) = (2\mathbf{i}+10\mathbf{j})+(8\mathbf{i}+\mathbf{j}(a-6))$ $\Rightarrow 50 = 10+a-6 \Rightarrow a = 46$ $\overrightarrow{QR} = \overrightarrow{OR}-\overrightarrow{OQ}$ $5(2\mathbf{i}+10\mathbf{j}) = (13\mathbf{i}+a\mathbf{j})-(3\mathbf{i}-4\mathbf{j})$ $\Rightarrow 50 = a+4 \Rightarrow a = 46$ **ALT** $\sqrt{(13-3)^2+(a--4)^2} = 5\times 2\sqrt{26}$ $\sqrt{10^2+(a+4)^2} = 10\sqrt{26}$ $100+(a+4)^2 = 2600 \Rightarrow (a+4) = \pm 50$ $a>0$, so $a = 46$	M1 A1 (2) [6] M1 A1cao {2}
	Allow column vectors throughout. Deduct max 2A marks if final vectors are column vectors inc **i**, **j**	
(a)		
M1	Attempt $\overrightarrow{PO}+\overrightarrow{OQ}$ (oe)	
A1	Correct answer	
(b)		
M1	Attempt the modulus of their $\overrightarrow{PQ}$ using +/- their components squared and added	
A1	Correct unit vector in any equivalent form. (parallel or anti-parallel)	
(c)		
M1	Any complete correct method that leads to a value of *a* (value to be shown)	
A1cao	*a* = 46	

Question number	Scheme	Marks
4 (a)	$2=4\sin 2t \Rightarrow \sin 2t=0.5 \Rightarrow 2t=\frac{\pi}{6} \Rightarrow t=\frac{\pi}{12}$ $[\approx 0.26179...]$	M1A1 (2)
(b)	$a=\frac{dv}{dt}=8\cos 2t \Rightarrow a=8\cos\left(2\times\frac{\pi}{12}\right)=4\sqrt{3}$ $(6.928...)\left(\text{m/s}^2\right)$	M1dM1 A1cao (3)
(c)	$s=\int 4\sin 2t\ dt=-2\cos 2t\ (+c)$	M1A1
	$t=\frac{\pi}{4} \Rightarrow 3=-2\cos\frac{\pi}{2}+c \Rightarrow c=3$ $s=3-2\cos 0 \Rightarrow s=3-2=1$ (m)	dM1 A1cao (4) [9]

(a)	
M1	Equate v to 2 and solve the equation by any valid method to obtain at least one value of t (not nec the least, but must be radians) Allow degrees only if then changed to radians.
A1	Correct, least value. Can be exact or decimal – 3 sf minimum
(b)	
M1	Differentiate v. $4\sin 2t \to k\cos 2t$, $k=\pm 8$ or ± 4
dM1	Substitute their answer from (a) and obtain a **positive** value for a. Depends on the previous M mark OR: use $\cos 2x=\sqrt{1-\sin^2 2x}$ with their value for $\sin 2x$ from (a)
A1cao	$4\sqrt{3}$ or 6.928... 6.93 (3 sf minimum) Allow all marks here if their answer from (a) is in degrees
(c)	
M1	Integrate v. $4\sin 2t \to k\cos 2t$, $k=\pm 2, \pm 4$. If definite integration ignore limits here.
A1	Correct integration, constant (or limits) not needed
dM1	Substitute $t=\frac{\pi}{4}$ and $s=3$ to obtain the value of c Definite integration: Substitute correct limits $t=0, \frac{\pi}{4}$ and $s=3$ Depends on the previous M mark.
A1cao	s = 1 (m)

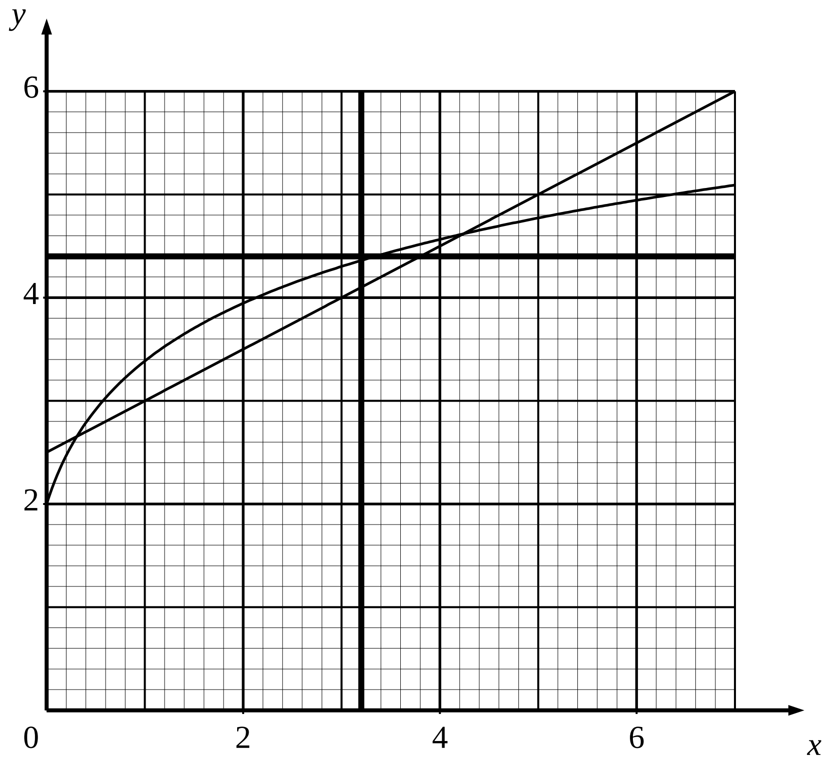
Qu numb	Scheme	Marks
5 (a)	Applies Pythagoras Theorem $5^2+15^2=250 \Rightarrow 5\sqrt{10}\,(=AC)$ $\Rightarrow \angle ABC = 90^\circ$ *	 M1 A1cso (2)
	ALT $\cos\angle ABC = \dfrac{15^2+5^2-250}{2\times 15\times 5}=0$ $\Rightarrow \angle ABC = 90^\circ$ *	{M1 A1}(2)
(b)	$DC=\sqrt{5^2+10^2}\,(=5\sqrt{5})$ and $DA=\sqrt{15^2+10^2}\,(=5\sqrt{13})$	B1B1
	$\angle DAC=\cos^{-1}\left(\dfrac{(5\sqrt{13})^2+(5\sqrt{10})^2-(5\sqrt{5})^2}{2\times 5\sqrt{13}\times 5\sqrt{10}}\right)=37.874...\approx 37.9^\circ$	M1A1cao (4)
(c)	$\angle BCA=\tan^{-1}\left(\dfrac{15}{5}\right)=71.5650...^\circ$ or find $\angle BAC=\tan^{-1}\left(\dfrac{5}{15}\right)=18.43^\circ$ $XB=5\sin 71.565=4.74341...$ $XB=5\cos 18.43=4.743...$ Required angle $\angle DXB=\tan^{-1}\left(\dfrac{10}{4.74341}\right)=64.6230...^\circ\approx 64.6^\circ$	M1 dM1 M1A1cao (4) [10]
	Alternatives:	
ALT 1	Δs ABC and BCX are similar $\Rightarrow \dfrac{BX}{5}=\dfrac{15}{5\sqrt{10}}\Rightarrow BX=\dfrac{15}{\sqrt{10}}$ M2 (may not be stated, just used) Find angle as main scheme M1A1	
ALT 2	Use area formula twice for triangle ABC $\dfrac{1}{2}AC\times BX=\dfrac{1}{2}AB\times BC\Rightarrow BX=\dfrac{15}{\sqrt{10}}$ M2 Find angle as main scheme M1A1	
ALT 3	DX is perpendicular to AX (Stated or used. No explanation/proof needed) $DX=AD\sin\angle DAC\;\left(=5\sqrt{13}\sin 37.9...^\circ\right)$ M2 $\sin\angle DXB=\dfrac{10}{DX}\Rightarrow \angle DXB=64.6^\circ$ M1A1	

(a)		
M1	Use Pythagoras with correct signs in ΔABC or use cosine rule (formula correct) or any other complete method	
A1cso	Correct conclusion stated and no errors in their method	
(b)		
B1	Correct length of *DC* or *DA*	
B1	Second length correct	
M1	Use cosine rule in either form, formula must be correct, and reach a value for the size of $\angle DAC$	
A1cao	37.9^{o} (Must be 1 dp)	
(c)		
M1	Use any trig ratio to obtain a value for the size of $\angle BCA$ (not nec correct)	
dM1	Use their value for $\angle BCA$ to obtain the length of *XB* Depends on the first M mark	
M1	Use $\tan DXB$ (or any other complete method) to obtain a value for the size of $\angle DXB$ (not nec correct)	
A1cao	64.6^{o} Must be 1 dp unless rounding already penalised in (b)	
	For the alternatives:	
	Getting directly to *XB* or *DX* scores M2	
	Completion to the angle M1A1	

Question number	Scheme	Marks
6	$\frac{\mathrm{d}V}{\mathrm{d}t}=0.03$	B1
	$\tan 30=\frac{r}{h}\Rightarrow r=h\tan 30^{\circ}=\left(\frac{h}{\sqrt{3}}\right)$	B1
	$V=\frac{1}{3}\pi r^2 h\Rightarrow V=\frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h\quad\left(=\frac{1}{9}\pi h^3\right)$	M1
	$\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{1}{3}\pi h^2$	dM1
	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\times\frac{\mathrm{d}h}{\mathrm{d}V},\Rightarrow 0.03\times\frac{1}{\frac{1}{3}\pi 1.5^2}=0.012732...\approx 0.0127\text{ (m/s)}$	M1,A1 (6) [6]
B1	For $\frac{\mathrm{d}V}{\mathrm{d}t}=0.03$ seen anywhere	
B1	Correct expression for r in terms of h. Can be in any form	
M1	Obtain an expression for V in terms of h only. Must have used trig for their expression for r in terms of h. Can include a trig ratio instead of the corresponding number.	
dM1	Attempt to differentiate their expression wrt h. Depends on the M mark above.	
M1	Correct (useful) chain rule, terms in any order. (Quoted, need not be used)	
A1	Substitute for all variables and obtain the correct value for $\frac{\mathrm{d}h}{\mathrm{d}t}$ **Must** be 3 sf	
NB	The question does not define the volume to be V, so allow any other letter (inc A) provided this is used consistently throughout the question or changed to V part way through.	

Question number	Scheme	Marks
7 (a)	| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | | y | 2 | **3.39** | 3.95 | 4.30 | **4.56** | **4.77** | 4.94 |	B1B1 (2)
(b)	All points plotted correctly All points joined together in a smooth curve	B1ft B1ft (2)
(c)	$3x+1=10.6 \Rightarrow x=3.2$ Draws line $x=3.2 \Rightarrow y=4.4$ on graph deduces $\ln 10.6 = 4.4-2 = 2.4$ or 2.3	B1 M1 A1cao (3)
(d)	$(3x+1)^2 = e^{(x+1)} \Rightarrow 2\ln(3x+1) = x+1 \Rightarrow \ln(3x+1)+2 = \frac{x}{2}+\frac{5}{2}$ Draws line $y=\frac{x}{2}+\frac{5}{2} \quad \Rightarrow x = 4.2$ or 4.3, 0.3 or 0.4	M1A1 dM1A1A1 (5) [12]

(a)	
B1	Any 2 values correct to at least 2 dp
B1	All 3 values correct and all to 2 dp
(b)	
B1ft	Their values plotted correctly or a smooth graph correct for their table of values drawn
B1ft	Smooth curve drawn through their points. Do not award this mark if it is clear that a ruler has been used on lhs (can be used at rhs).
NB	These 2 marks can be awarded for a correct graph if the table values are incorrect or missing.
(c)	
B1	For $x=3.2$ (Award if correct line is drawn)
M1	Draws the line $x=3.2$ on their graph and obtains the corresponding y value (horizontal line may be omitted). Without evidence that the graph has been used, give M0
A1cao	ln10.6 = 2.4, or 2.3 **Must** be 1 dp unless rounding already penalised in (a)
(d)	
M1	Attempt to rearrange the equation to $\ln(3x+1)+2=...$ with a linear function on RHS.
A1	Correct rearrangement. Need not be simplified eg $\ln e^{\frac{1}{2}(x+1)}+2$ is a linear function and a correct rearrangement
dM1	Draw their line on their graph. Depends on the first M mark
A1	Either value correct
A1	Second value correct Award A1A0 if both correct but one or both given to more than 1dp (unless rounding already penalised)


y
6
4
2
0
2
4
6
x

Question number	Scheme	Marks
8 (a)	$\alpha+\beta=\frac{2}{3}, \quad \alpha\beta=-\frac{1}{3}$	B1
	$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta \Rightarrow \left(\frac{2}{3}\right)^2-2\left(-\frac{1}{3}\right)=\frac{10}{9}$	M1A1 (3)
(b)	$\alpha-\beta=\sqrt{(\alpha-\beta)^2}=\sqrt{\left(\alpha^2+\beta^2-2\alpha\beta\right)}=\sqrt{\left(\frac{10}{9}-2\left(-\frac{1}{3}\right)\right)}=\frac{4}{3}$	M1A1cso (2)
	OR: $\sqrt{\left((\alpha+\beta)^2-4\alpha\beta\right)}=\sqrt{\left(\frac{4}{9}+\frac{4}{3}\right)}=\frac{4}{3}$ *	
(c)	Sum $=\frac{\alpha+\beta}{\alpha}+\frac{\alpha-\beta}{\beta}=\frac{\alpha\beta+\beta^2+\alpha^2-\alpha\beta}{\alpha\beta}=\frac{\frac{10}{9}}{-\frac{1}{3}}=-\frac{10}{3}$	M1A1
	Product $\left(\frac{\alpha+\beta}{\alpha}\right)\times\left(\frac{\alpha-\beta}{\beta}\right)=\frac{\frac{2}{3}\times\frac{4}{3}}{-\frac{1}{3}}=-\frac{8}{3}$	M1A1
	Equation $x^2-\left(-\frac{10}{3}\right)x+\left(-\frac{8}{3}\right)=0 \Rightarrow 3x^2+10x-8=0$	M1A1 (6) [11]
	"Without solving the equation" applies throughout this question. All work must be based on the sum and product of the roots.	
(a)B1	Correct sum and product of roots. May be shown explicitly or just used but must be clear that $\alpha+\beta=\frac{2}{3}$. Award if seen anywhere.	
M1	Using the sum and product to obtain a value for $\alpha^2+\beta^2$ Algebra used must be correct.	
A1	Correct value. Allow if $\alpha+\beta=-\frac{2}{3}$ used **NB** B1 lost in this case.	
(b)M1	For correct algebra leading to a value for $\alpha-\beta$ or $(\alpha-\beta)^2$ May use their value for $\alpha^2+\beta^2$ or use the sum and product values	
A1cso	Correct **given** value for $\alpha-\beta$ obtained from a correct solution	
(c)		
M1	Correct algebra used to reach a value for the sum	
A1	Correct sum	
M1	Form the product and use previous results to obtain a value for the product. Algebra must be correct.	
A1	Correct product	
M1	Use $"x^2-(\text{sum of roots})x+\text{product of roots}"$ with or without = 0 with their sum and product	
A1	Correct equation, including = 0. Can be as shown or any integer multiple of this.	

Question number	Scheme	Marks
9 (a)	$(2x+3)^{\frac{1}{2}}=\frac{x}{2}+\frac{3}{2}\Rightarrow 4(2x+3)=(x+3)^2, \Rightarrow 0=x^2-2x-3$ oe	M1,A1
	$x^2-2x-3=(x-3)(x+1)=0\Rightarrow x=3,-1$	M1A1
	$y=1,3$ so coordinates are $(-1,1)$ and $(3,3)$	A1 (5)
(b)	Vol = $\pi\int_{-1}^{3}(2x+3)\ \mathrm{d}x-\pi\int_{-1}^{3}\left(\frac{x}{2}+\frac{3}{2}\right)^2\ \mathrm{d}x$	M1
	$\pi\int_{-1}^{3}(2x+3)\ \mathrm{d}x-\pi\int_{-1}^{3}\left(\frac{x}{2}+\frac{3}{2}\right)^2\ \mathrm{d}x=\frac{\pi}{4}\int_{-1}^{3}3+2x-x^2\ \mathrm{d}x=\frac{\pi}{4}\left[3x+x^2-\frac{x^3}{3}\right]_{-1}^{3}$	M1A1
	$\Rightarrow\frac{\pi}{4}\left[(9+9-9)-\left(-3+1+\frac{1}{3}\right)\right]=\frac{8}{3}\pi$	dM1A1 cao (5) [10]
	For separate integrals:	
	$\pi\int_{-1}^{3}(2x+3)\ \mathrm{d}x-\pi\int_{-1}^{3}\left(\frac{x}{2}+\frac{3}{2}\right)^2\ \mathrm{d}x=\pi\left[x^2+3x\right]_{-1}^{3}-\pi\left[\frac{x^3}{12}+\frac{3x^2}{4}+\frac{9x}{4}\right]_{-1}^{3}$	
	$=\ldots$	
	ALT:	
	Vol $=\pi\int_{-1}^{3}(2x+3)\ \mathrm{d}x-$ vol of truncated cone	M1
	Vol $=\pi\left[x^2+3x\right]_{-1}^{3}-\left(\frac{1}{3}\pi 3^2\times 6-\frac{1}{3}\pi 1^2\times 2\right)$	M1A1
	$=\pi(9+9-(1-3))-\frac{52\pi}{3}=\frac{8}{3}\pi$	dM1A1

(a)	
M1	Eliminate y and obtain a quadratic in x. Need not be simplified. Allow if $(x+3)^2\to x^2+9$
A1	Correct 3TQ, as shown or equivalent.
M1	Solve their 3TQ by factorising, formula or completing the square (see general guidance)
A1	Two correct values for x
A1	Corresponding y coordinates. No need to write in coordinate brackets but pairing must be clear.
	If one x and its corresponding y are correct, award A1A0, provided M mark has been gained
ALT:	Elimination of x gives $y^2=2(2y-3)+3\Rightarrow y^2-4y+3=0\Rightarrow(y-1)(y-3)=0$ etc

(b)	
M1	Correct expression for volume. If the integrals are evaluated separately or π omitted here award only when the correct difference has been obtained (and π included). Limits not needed.
M1	Attempt all the required integration (ie volume for the curve and volume for the line or a combination of these as on the mark scheme), π and limits not needed – ignore any shown
A1	Correct integration (can be one or 2 integrals); ignore limits, π may be missing
dM1	Substitute their x coordinates in their integrated expression(s). Depends on the second M mark. Substitution must be shown for both limits.
A1cao	Correct final answer. All 3 M marks needed
ALT	
M1	Correct expression for the volume including some attempt at the truncated cone. π needed for the cone but may appear later for the integral/
M1	Attempt the integration - π and limits not needed – ignore any shown – and attempt the vol of the truncated cone.
A1	Correct integration and correct difference of 2 cones
dM1	Substitute their x coordinates in their integrated expression. Depends on the second M mark. Substitution must be shown for both limits.
A1cao	Correct final answer. All 3 M marks needed

Question number	Scheme	Marks
10 (a)(i)	$a+3ar=8$	B1
	$ar\times ar^2=4ar^4\Rightarrow(a=4r)$	B1
	Solves simultaneous equations $4r(1+3r)=8\Rightarrow 12r^2+4r-8=0\Rightarrow(3r-2)(r+1)=0$	M1
	$\Rightarrow r=\frac{2}{3}\ \ (r=-1)$	A1
(ii)	$a=4\times\frac{2}{3}=\frac{8}{3}$	A1 (5)
(b)	$U_n=\frac{8}{3}\times\left(\frac{2}{3}\right)^{n-1}\Rightarrow U_n=\frac{2^3\times 2^{n-1}}{3\times 3^{n-1}}=\frac{2^{n+2}}{3^n}$ *	M1A1cso (2)
(c)	$U_n<0.05\Rightarrow\frac{2^{n+2}}{3^n}<0.05\quad\left(\Rightarrow\left(\frac{2}{3}\right)^n\times 4<0.05\right)$	M1
	$\Rightarrow n>\log_{\left(\frac{2}{3}\right)}\frac{0.05}{4}\Rightarrow n>10.807...\Rightarrow n=11$	dM1A1cao (3) [10]
	ALT $\frac{8}{3}\left(\frac{2}{3}\right)^{n-1}=\frac{2^{n+2}}{3^n}=\left(\frac{2}{3}\right)^n\times 4<\frac{1}{20}$	{ M1
	So $\left(\frac{2}{3}\right)^n<\frac{1}{80}$ or $\left(\frac{3}{2}\right)^n=(1.5)^n>80$	dM1
	Leading to $n>\frac{\log 80}{\log 1.5}=10.8...\ \ n=11$	A1cao}(3)

(a)	
(i) B1	For $a+3ar=8$
B1	For $ar\times ar^2=4ar^4$
M1	Solving the simultaneous equations by any valid method. Must get to $r=...$ or $a=...$ Must solve a 3TQ by the usual rules
A1	Correct value for *r*. $r=-1$ need not be seen, but if shown it must be eliminated or made clear that $r=\frac{2}{3}$ is the only correct answer by eg underlining
(ii) A1	$a=\frac{8}{3}$
(b)	
M1	Use the correct formula for the *n*th term with their *r* and *a*
A1cso	Simplify to the correct **given** result, no errors in the work. Must see 8 changed to 2^3

(c)	
M1	Use the result in (b) to form an inequality or equation ALT: use the formula for the *n*th term
dM1	Attempt to solve their inequality, using logs (any base) or trial and error. Log work must be correct for their inequality or equation. If an equation is used the values of *n* either side of their answer must be tested before this mark can be awarded. Depends on first M mark of (c)
A1cao	Correct answer ($n = 11$) from correct working. Trial and error can be done on a calculator, so correct answer may get M1dM1A1

Question number	Scheme	Marks
11 (a) (i)	$\cos 2x = \cos^2 x - \sin^2 x$ $\cos 2x = 1 - \sin^2 x - \sin^2 x = 1 - 2\sin^2 x$ *	M1 M1A1cso
(ii)	$\dfrac{13\sin x - 2\cos 2x - 10}{(4\sin x - 3)} = \dfrac{13\sin x - 2(1 - 2\sin^2 x) - 12}{(4\sin x - 3)}$	M1
	$\Rightarrow \dfrac{4\sin^2 x + 13\sin x - 12}{(4\sin x - 3)} = \dfrac{(4\sin x - 3)(\sin x + 4)}{(4\sin x - 3)} = \sin x + 4$	M1M1A1 cso (7)
(b)	Let $A = \left(\theta + \dfrac{\pi}{6}\right)$ in either method:	
ALT 1	Uses (a) (i): $10 + 2\cos 2A - 13\sin A = 2\sin A + 8$ $2(1 - 2\sin^2 A) - 15\sin A + 2 = 0$	
	$4\sin^2 A + 15\sin A - 4 = 0$	M1
	$(4\sin A - 1)(\sin A + 4) = 0$	
	$\sin A = \dfrac{1}{4}$ $(\sin A = -4 \text{ not poss})$	dM1
	$\left(\theta + \dfrac{\pi}{6}\right) = 0.252680..., 2.888912..., 6.535865...$	ddM1A1
	$\theta = 6.01$	A1 (5)
ALT 2	Uses (a) (ii): $10 + 2\cos 2A - 13\sin A = 2(\sin A + 4) \Rightarrow \dfrac{13\sin A - 2\cos 2A - 10}{\sin A + 4} = -2$ M1	
	$\Rightarrow 4\sin A - 3 = -2 \Rightarrow \sin A = \dfrac{1}{4}$ dM1	
	$\sin\left(\theta + \dfrac{\pi}{6}\right) = \dfrac{1}{4} \Rightarrow \left(\theta + \dfrac{\pi}{6}\right) = 0.252680..., 2.888912..., 6.535865...$ ddM1A1	
	$\theta = 6.01$ A1cao (5)	

(c)	$\int_0^{\frac{\pi}{2}} \frac{13\sin x - 2\cos 2x - 10 + 4x\sin x - 3x}{4\sin x - 3}\,\mathrm{d}x = \int_0^{\frac{\pi}{2}} \frac{13\sin x - 2\cos 2x - 10}{4\sin x - 3} + \frac{x(4\sin x - 3)}{4\sin x - 3}\,\mathrm{d}x$ $\Rightarrow \int_0^{\frac{\pi}{2}} \frac{13\sin x - 2\cos 2x - 10}{4\sin x - 3} + x\,\mathrm{d}x = \int_0^{\frac{\pi}{2}} \sin x + 4 + x\,\mathrm{d}x$ $\int_0^{\frac{\pi}{2}} \sin x + 4 + x\,\mathrm{d}x = \left[-\cos x + 4x + \frac{x^2}{2}\right]_0^{\frac{\pi}{2}}$ $\int_0^{\frac{\pi}{2}} \sin x + 4 + x\,\mathrm{d}x = \left(0 + 2\pi + \frac{\pi^2}{8}\right) - (-1) = 2\pi + \frac{\pi^2}{8} + 1$ oe	M1A1 dM1 ddM1A1cao (5) [17]

(a)	
(i)M1	Set $A = B = x$ in the given identity Allow with *x* or any other single variable. Can have $x + x$ or $2x$
M1	Use $\cos^2 x + \sin^2 x = 1$ to eliminate $\cos^2 x$ Allow with *x* or any other single variable and $x + x$ or $2x$
A1cso	Obtain the **given** result with no errors in the working. The variable must be *x* now and $x + x$ must have become $2x$
(ii)M1	Use the result given in (a) to eliminate $\cos 2x$
M1	Simplify the numerator to a 3TQ
M1	Factorise the numerator. Correct factorisation implies the previous M mark.
	These M marks can be awarded for work on the numerator alone – award if denominator not seen yet.
A1cso	Obtain the **given** result with no errors in the working. **Must** have seen the denominator for evidence of the cancellation.
(b)	
ALT 1	
M1	Use (a) (i) to obtain a quadratic in sin *A*. Terms in any order. (Can be done w/o the substitution.)
dM1	Solve their 3TQ and reach sin *A* = ... sin *A* = - 4 need not be seen. Depends on the first M mark.
ddM1	Obtain at least one value for $\left(\theta + \frac{\pi}{6}\right)$ or $(\theta + 30°)$ (Need not be the one to give a final answer in the required range). Depends on both M marks above.
A1	For $\theta + \frac{\pi}{6} = 6.535\ldots$
A1	For $\theta = 6.01$ Ignore answers outside the range, extras inside score A0. If final answer is in degrees, both A marks are lost. If degrees are changed to radians both A marks are available even if penultimate answer is in degrees.

ALT 2	
M1	Set $\theta+\frac{\pi}{6}=A$ in the given equation and rearrange to the expression shown. (Can be done w/o the substitution.)
dM1	Use the identity from (a) (ii) to obtain a value for $\sin A$ or $\sin\left(\theta+\frac{\pi}{6}\right)$ or $(\theta+30°)$ Depends on the first M mark.
ddM1	Obtain at least one value for $\left(\theta+\frac{\pi}{6}\right)$ (Need not be the one to give a final answer in the required range)
A1	For $\theta+\frac{\pi}{6}=6.535...$
A1cao	For $\theta=6.01$ Ignore answers outside the range, extras inside score A0. If final answer is in degrees, both A marks are lost. If degrees are changed to radians both A marks are available even if penultimate answer is in degrees.
NB	If compound angle formulae used – send to review.
(c)	
M1	Use the identity from (a) (ii) to simplify the integrand from the given function. Must **not** ignore $4x\sin x-3x$ so $\int(4+\sin x)\mathrm{d}x$ scores M0
A1	Correct changed integrand.
dM1	Attempt the integration. $x\to\frac{x^2}{k}$, $k=1$ or 2 **and** $\sin x\to\pm\cos x$ Depends on first M mark of (c)
ddM1	Substitute the given limits. Depends on both M marks of (c)
A1cao	For $2\pi+\frac{\pi^2}{8}+1$ Must be exact but any equivalent accepted provided the trig functions have been replaced with their numerical values.
	Decimal answer, 8.516...may score 4/5 but w/o working implies from a calculator and scores 0/5

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