Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number | | | | | |

Candidate Number | | | | |

Thursday 20 June 2019

Morning (Time: 2 hours) | Paper Reference **4PM1/02**

Further Pure Mathematics

Paper 2

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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P 5 8 3 7 3 A 0 1 3 2

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\quad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\quad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\quad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Referred to a fixed origin O, the point A has position vector $(4\mathbf{i} + 3\mathbf{j})$ and the point B has position vector $(\mathbf{i} + 7\mathbf{j})$

(a) Find $\overrightarrow{AB}$ as a simplified expression in terms of $\mathbf{i}$ and $\mathbf{j}$

(2)

(b) Find a unit vector that is parallel to $\overrightarrow{AB}$

(2)

(Total for Question 1 is 4 marks)

2 Oil is leaking from a pipe and forms a circular pool on a horizontal surface. The area of the surface of the pool is increasing at a constant rate of $8\,cm^2/s$. Find, in cm/s to 3 significant figures, the rate at which the radius of the pool is increasing when the area of the pool is $50\,cm^2$

(6)

Question 2 continued

(Total for Question 2 is 6 marks)

3 A particle P moves in a straight line. At time t seconds, the velocity, v m/s, of P is given by

$$v = t^2 - 4t + 7$$

(a) Find the acceleration of P, in m/s^2, when $t = 3$

(2)

(b) Find the distance, in m, that P travels in the interval $0 \leqslant t \leqslant 6$

(4)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 3 continued

(Total for Question 3 is 6 marks)

4 In triangle ABC, $AB = 5x$ cm, $BC = (3x - 1)$ cm, $AC = (2x + 5)$ cm and angle $ABC = 60°$

Find, to 3 significant figures, the value of x.

(5)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 5 marks)

5 Use algebra to solve the equations

$$xy = 36$$

$$xy + x + 2y = 53$$

(6)

Question 5 continued

(Total for Question 5 is 6 marks)

6 (a) Given that $y = (4x - 3)e^{2x}$

(i) find $\frac{dy}{dx}$

(3)

(ii) show that $(4x - 3)\frac{dy}{dx} = (8x - 2)y$

(2)

(b) Differentiate $\frac{\sin 5x}{(x - 3)^2}$ with respect to x

(3)

Question 6 continued

(Total for Question 6 is 8 marks)

7 The sum of the first n terms of an arithmetic series is A_n where

$$A_n = \sum_{r=1}^{n} (4r + 5)$$

(a) For this arithmetic series, find

(i) the first term,

(ii) the common difference.

(2)

The sum of the first n terms of a geometric series is G_n where

$$G_n = \sum_{r=1}^{n} 4(3)^{r-1}$$

(b) For this geometric series, find

(i) the first term,

(ii) the common ratio.

(2)

(c) Find the value of n for which $A_{14} - 6 = G_n$

(5)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 9 marks)

8 The point A has coordinates (2, 6), the point B has coordinates (6, 8) and the point C has coordinates (4, 2).

(a) Find the exact length of

(i) AB (ii) BC (iii) AC

(4)

(b) Find the size of each angle of triangle ABC in degrees.

(3)

The points A, B and C lie on a circle with centre P.

(c) Find the coordinates of P.

(2)

(d) Find the exact length of the radius of the circle in the form $\sqrt{a}$, where a is an integer.

(2)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 11 marks)

9 The curve C, with equation $y = f(x)$, passes through the point with coordinates $\left(-2, -\frac{28}{3}\right)$

Given that $f'(x) = x^3 - x^2 - 4x + 4$

(a) show that C passes through the origin.

(4)

(b) (i) Show that C has a minimum point at $x = 2$ and a maximum point at $x = 1$

(ii) Find the exact value of the y coordinate at each of these points.

(7)

The curve has another turning point at A.

(c) (i) Find the coordinates of A.

(ii) Determine the nature of this turning point.

(3)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 14 marks)

10 The roots of the equation $x^2 + 3x - 5 = 0$ are α and β.

(a) Without solving the equation, find

(i) the value of $\alpha^2 + \beta^2$

(ii) the value of $\alpha^4 + \beta^4$

(5)

Given that $\alpha > \beta$ and without solving the equation

(b) show that $\alpha - \beta = \sqrt{29}$

(2)

(c) Factorise $\alpha^4 - \beta^4$ completely.

(3)

(d) Hence find the exact value of $\alpha^4 - \beta^4$

(2)

Given that $\beta^4 = p + q\sqrt{29}$ where p and q are positive constants

(e) find the value of p and the value of q.

(3)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 15 marks)

11

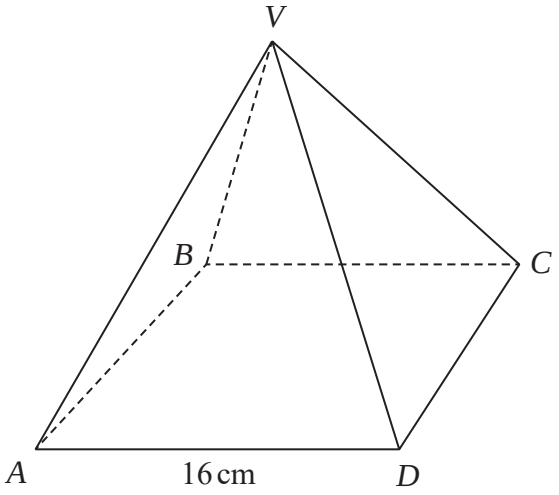

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a right pyramid with vertex V and square base, $ABCD$, of side 16 cm.

The size of angle AVC is 90°

(a) Show that the height of the pyramid is $8\sqrt{2}$ cm.

(4)

(b) Find, in cm, the length of VA.

(3)

(c) Find, in cm, the exact length of the perpendicular from D onto VA.

(3)

Find, in degrees to one decimal place, the size of

(d) the angle between the plane VAB and the base $ABCD$,

(3)

(e) the obtuse angle between the plane VAB and the plane VAD.

(3)

Question 11 continued

Question 11 continued

(Total for Question 11 is 16 marks)

TOTAL FOR PAPER IS 100 MARKS