Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

**Pearson Edexcel International GCSE**

Centre Number | | | | | |

Candidate Number | | | | |

# Friday 10 January 2020

Morning (Time: 2 hours) | Paper Reference **4PM1/01R**

# Further Pure Mathematics

## Paper 1R

**Calculators may be used.**

Total Marks

## Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

## Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

*Turn over* ▶

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P 6 0 4 7 5 A 0 1 3 6

Pearson

# International GCSE in Further Pure Mathematics Formulae sheet

## Mensuration

**Surface area of sphere** = $4\pi r^2$

**Curved surface area of cone** = $\pi r \times$ slant height

**Volume of sphere** = $\frac{4}{3}\pi r^3$

## Series

### Arithmetic series

Sum to $n$ terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

### Geometric series

Sum to $n$ terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

### Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

## Calculus

### Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

## Trigonometry

### Cosine rule

In triangle $ABC$: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\quad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\quad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\quad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

## Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

**Answer all ELEVEN questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

**1** Given that $\frac{a+\sqrt{3}}{2-\sqrt{3}} = 11 + b\sqrt{3}$ where $a$ and $b$ are integers,

find the value of $a$ and the value of $b$.

**(4)**

**(Total for Question 1 is 4 marks)**

**2** $f(x) = 7 + 4x - x^2$

(a) Write $f(x)$ in the form $a - b(x + c)^2$ where $a$, $b$ and $c$ are integers to be found.

**(3)**

(b) Hence, or otherwise, find

(i) the maximum value of $f(x)$

(ii) the value of $x$ for which this maximum occurs.

**(2)**

**Question 2 continued**

**(Total for Question 2 is 5 marks)**

**3** Given that $y = e^{2x}(x^2 + 1)$

(a) find $\frac{dy}{dx}$

**(3)**

The straight line $l$ is the tangent to the curve with equation $y = e^{2x}(x^2 + 1)$ at the point on the curve where $x = 0$

(b) Find an equation for $l$ in the form $y = mx + c$

**(3)**

**Question 3 continued**

**(Total for Question 3 is 6 marks)**

**4** $f(x) = 2x^3 + ax^2 + bx + 18$ where $a$ and $b$ are constants

When $f'(x)$ is divided by $(x - 2)$ the remainder is 5

Given that $(x - 2)$ is a factor of $f(x)$

(a) find the value of $a$ and the value of $b$.

**(6)**

(b) Express $f(x)$ as a product of linear factors.

**(3)**

(c) Hence use algebra to solve the equation $f(x) = 0$

**(2)**

**Question 4 continued**

**(Total for Question 4 is 11 marks)**

**5** (a) Show that $\log_4 32 = \frac{5}{2}$

**(2)**

(b) Hence, or otherwise, find the exact solutions of the equation

$$\log_2 x - \log_4 32 + \frac{1}{4}\log_x 16 = 0$$

**(7)**

**Question 5 continued**

**(Total for Question 5 is 9 marks)**

**6** (a) Complete the table of values for

$$y = x - \frac{3}{x^2}$$

giving your answers to one decimal place where appropriate.

| $x$ | 0.5 | 1 | 1.5 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|---|
| $y$ | –11.5 | | | 1.3 | 2.7 | | | 5.9 |

**(2)**

(b) On the grid opposite, draw the graph of $y = x - \frac{3}{x^2}$ for $0.5 \leqslant x \leqslant 6$

**(2)**

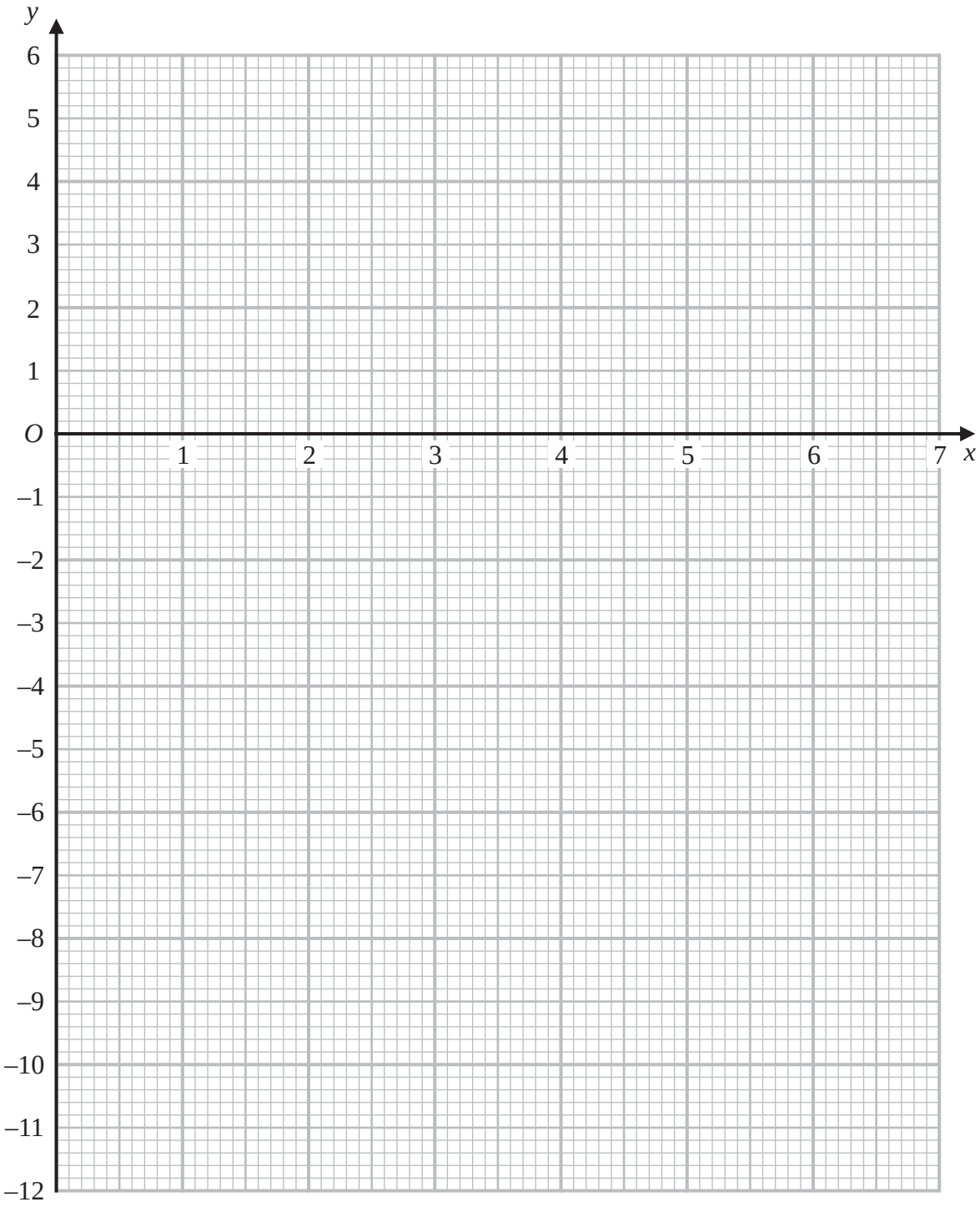
(c) By drawing a suitable straight line on the grid, obtain estimates, to one decimal place, of each of the two roots of the equation

$$2x^3 - 6x^2 + 3 = 0$$

in the interval $0.5 \leqslant x \leqslant 6$

**(5)**

**Question 6 continued**

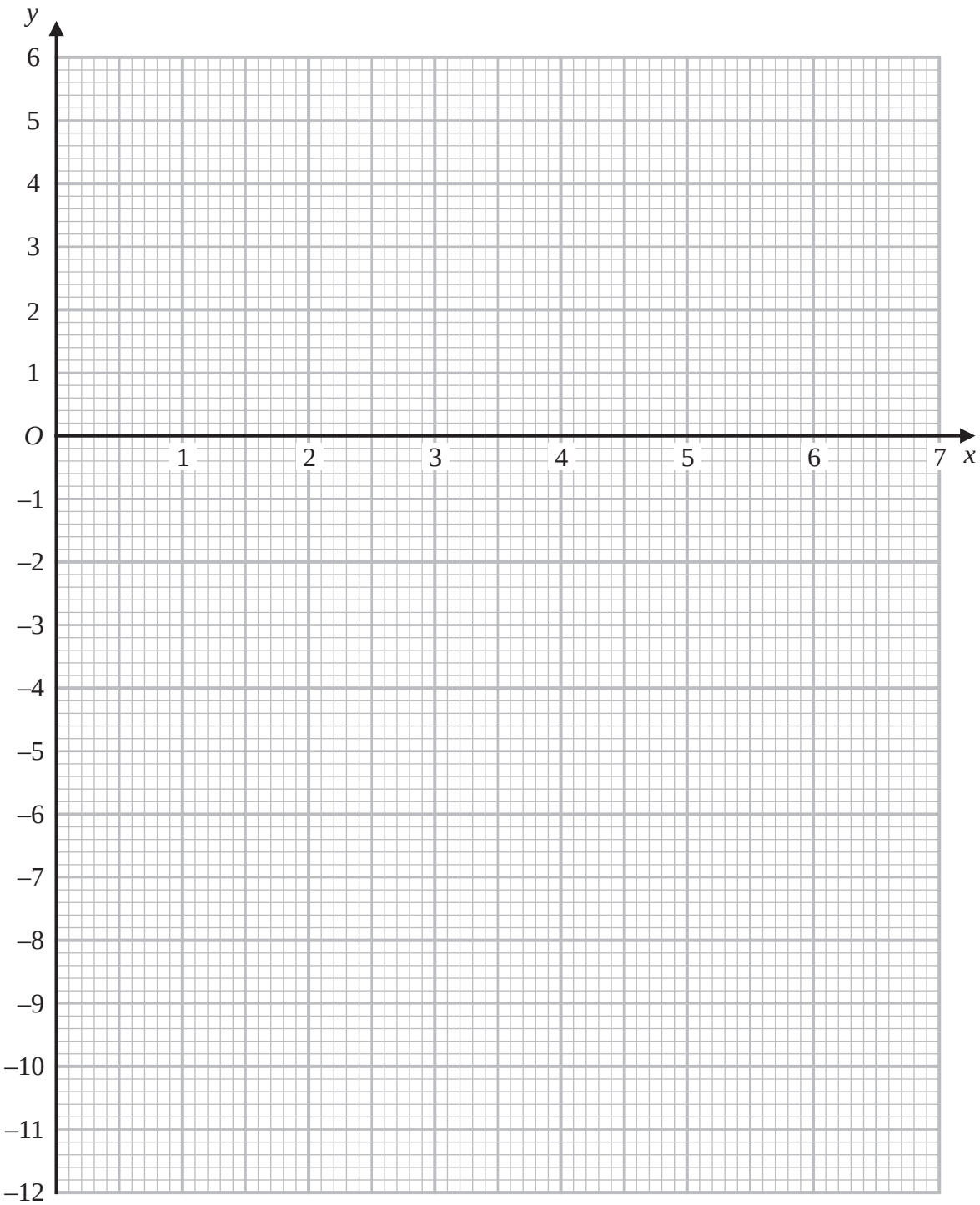



**Turn over for a spare grid if you need to redraw your graph.**

**Question 6 continued**

**Question 6 continued**

**Only use this grid if you need to redraw your graph.**




**(Total for Question 6 is 9 marks)**

**7** An arithmetic series $P$ has first term $a$, common difference $d$ and $n$th term $u_n$

Given that $u_5 = 4x + 6$ and that $u_8 = 7x + 3$

(a) (i) show that $d = x - 1$

(ii) find the value of $a$

**(4)**

Given further that $u_9 = 42$

(b) find the value of $x$

**(2)**

The sum of the first $n$ terms of $P$ is $S_n$

(c) Find the value of $n$ for which $S_{(n+1)} = 12u_n + 18$

**(5)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**(Total for Question 7 is 11 marks)**

**8** A particle $P$ moves along the positive $x$-axis. At time $t$ seconds ($t \geqslant 0$) the velocity, $v$ m/s, of $P$ is given by $v = 3 + 5t - 2t^2$

At time $t$ seconds, $P$ is at the point with coordinates $(x, 0)$.

Given that at time $t = 0$, $P$ is at the point with coordinates $(5, 0)$, find the maximum value of $x$, justifying that this is a maximum value.

**(8)**

**Question 8 continued**

**Question 8 continued**

**Question 8 continued**

**(Total for Question 8 is 8 marks)**

**9** The line $l_1$ with equation $y + 2x - 4 = 0$ passes through the point $P$ with coordinates $(a, 6)$ and the point $Q$ with coordinates $(3, b)$.

(a) Find the value of $a$ and the value of $b$.

**(2)**

The line $l_2$ passes through point $P$ and is perpendicular to $l_1$

The point $R$, with coordinates $(e, f)$ lies on $l_2$ such that $PR = 6\sqrt{5}$

(b) Find the two possible pairs of values of $e$ and $f$.

**(8)**

Given that $e < 0$,

(c) find the area of triangle $PQR$.

**(3)**

The points $P$, $Q$ and $R$ lie on a circle $C$.

(d) Find the coordinates of the centre of $C$.

**(2)**

**Question 9 continued**

**Question 9 continued**

**Question 9 continued**

**(Total for Question 9 is 15 marks)**

**10**

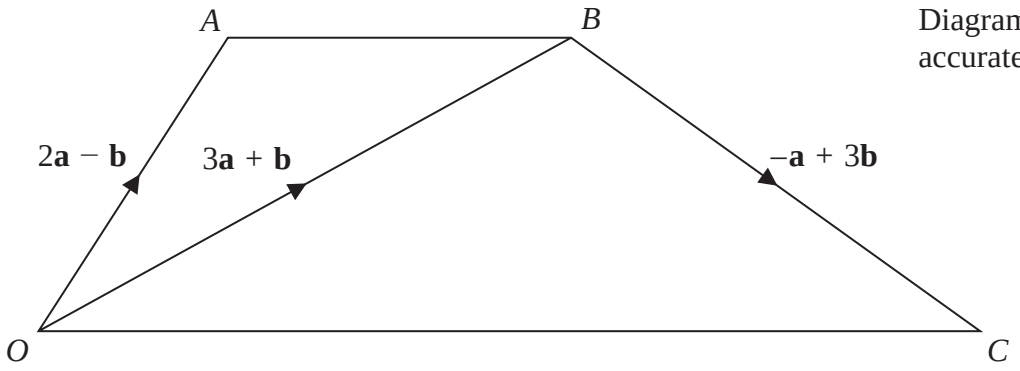



Diagram **NOT** accurately drawn

**Figure 1**

Figure 1 shows quadrilateral $OABC$ with

$$\overrightarrow{OA} = 2\mathbf{a} - \mathbf{b} \qquad \overrightarrow{OB} = 3\mathbf{a} + \mathbf{b} \qquad \overrightarrow{BC} = -\mathbf{a} + 3\mathbf{b}$$

(a) Find $\overrightarrow{AB}$ as a simplified expression in terms of **a** and **b**.

**(2)**

(b) Prove that $\overrightarrow{OC}$ is parallel to $\overrightarrow{AB}$

**(2)**

The diagonals, $OB$ and $AC$, intersect at the point $X$.

(c) Using a vector method find the ratio $AX : XC$

**(7)**

**Question 10 continued**

**Question 10 continued**

**Question 10 continued**

**(Total for Question 10 is 11 marks)**

**11**

Diagram **NOT** accurately drawn

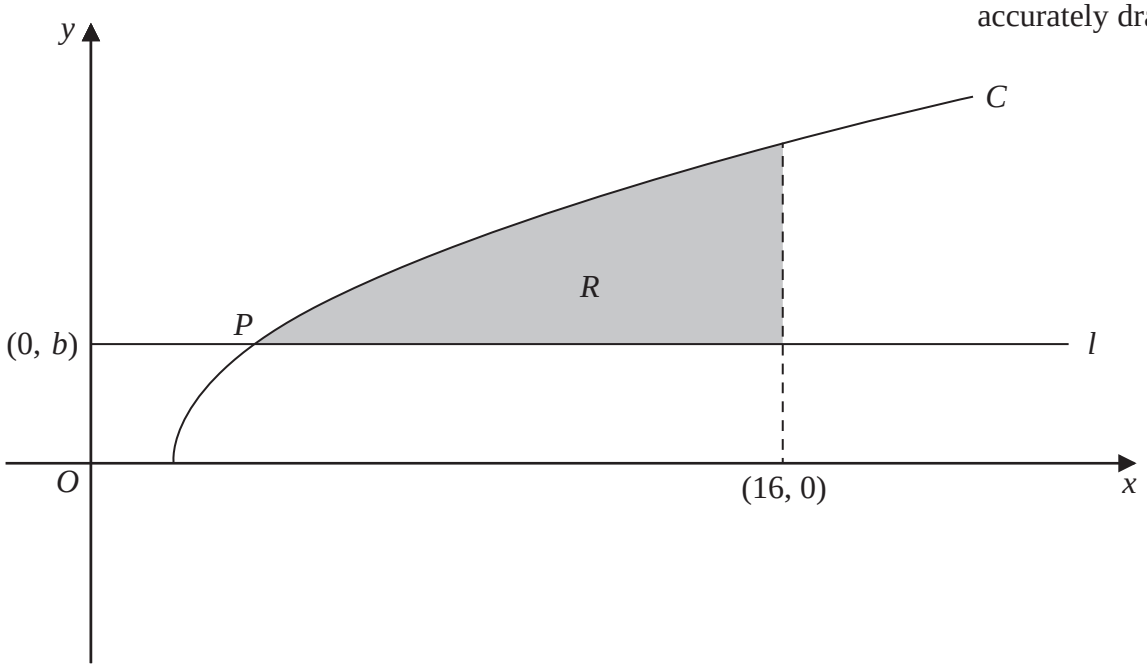



**Figure 2**

Figure 2 shows part of the curve $C$ with equation $y = \sqrt{x - 2}$
Figure 2 also shows the straight line $l$ with equation $y = b$ for $x > 0$ where $b > 0$

Given that $C$ and $l$ intersect at the point $P$ with coordinates $(a, b)$, where $2 < a < 16$

(a) show that $b^2 = a - 2$

**(2)**

The finite region $R$ bounded by $C$, the straight line with equation $x = 16$ and $l$, shown shaded in Figure 2, is rotated through 360° about the $x$-axis to form a solid $S$.

Given that the volume of the solid formed is $50\pi$

(b) use algebraic integration to find the value of $a$ and the value of $b$.

**(9)**

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**Question 11 continued**

**Question 11 continued**

**Question 11 continued**

**Question 11 continued**

**(Total for Question 11 is 11 marks)**

**TOTAL FOR PAPER IS 100 MARKS**