Mark Scheme (Results)

January 2022

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 2

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at www.edexcel.com or www.btec.co.uk. Alternatively, you can get in touch with us using the details on our contact us page at www.edexcel.com/contactus.

Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

January 2022
Question Paper Log Number P66308A
Publications Code 4PM1_02_2201_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme - not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed out work should be marked **unless** the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to $x=....$

$(ax^2+bx+c)=(mx+p)(nx+q)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=....$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*
leading to $x=....$

3. Completing the square:

$x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=....$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n\to x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:
When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)
Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question	Scheme	Mark
1(a)	$A = \frac{1.2}{2} \times 7^2 = 29\frac{2}{5}$	M1A1 [2]
(b)	$P = 7 + 7 + 7 \times 1.2 = 22\frac{2}{5}$ (cm)	M1A1 [2]
	Total	**4 marks**

Part	Mark	Notes
(a)	M1	For using the correct formula for the area of a sector with correct substitution of the given values. $A = \frac{1.2}{2} \times 7^2 = ...$
	A1 [2]	For $A = 29\frac{2}{5}$ o.e. (cm^2)
(b)	M1	For a complete method to find the perimeter of the sector with correct substitution of the given values $P = 7 + 7 + 7 \times 1.2 = ...$
	A1 [2]	$P = 22\frac{2}{5}$ o.e. (cm)

Question	Scheme	Mark
2	$\sin(2\theta-20)^{\circ}=\sqrt{3}\cos(2\theta-20)^{\circ}\Rightarrow\tan(2\theta-20)^{\circ}=\sqrt{3}$	M1
	$(2\theta-20)^{\circ}=60^{\circ},\ 240^{\circ},\ 420^{\circ},...$	M1A1
	$\theta=\dfrac{'60'+20}{2}=40^{\circ},\quad \theta=\dfrac{'240'+20}{2}=130^{\circ}$	M1A1 [5]
	Total	**5 marks**

Mark	Notes
M1	For using the identity $\dfrac{\sin A}{\cos A}=\tan A$ to reach $\tan(2\theta-20)^{\circ}=k$ where k is a numerical value $\sin(2\theta-20)^{\circ}=\sqrt{3}\cos(2\theta-20)^{\circ}\Rightarrow\tan(2\theta-20)^{\circ}=\sqrt{3}$
M1	For finding at least one **correct** angle for $(2\theta-20)^{\circ}$ $(2\theta-20)^{\circ}=60^{\circ},\ 240^{\circ},\ 420^{\circ},...$ Allow even for eg., -120° This mark can be implied by correct final answers.
A1	For both of the angles 60° and 240° Ignore any extra values, even if within range. $0\leqslant(2\theta-20)\leqslant360$ This mark can be implied by correct final answers.
M1	For correct processing of their values for $(2\theta-20)^{\circ}$ $\theta=\dfrac{'60'+20}{2}=...$ **or** $\theta=\dfrac{'240'+20}{2}=...$
A1 [5]	For both correct values of $\theta=$ 40 and 130 Ignore other angles out of range, penalise extra angles within the range by the loss of this A mark

Question	Scheme	Mark
3	$9-x^2=0 \Rightarrow x=\pm 3$	B1
	$A=\int_{'-3'}^{'3'} 9-x^2 \ \mathrm{d}x=\left[9x-\dfrac{x^3}{3}\right]_{'-3'}^{'3'}$	M1A1
	$A=\left(9\times 3-\dfrac{3^3}{3}\right)-\left(9\times -3-\dfrac{[-3]^3}{3}\right)=36$	M1A1 [5]
	Total	**5 marks**

Mark	Notes
You may see the working without $A=$ …. . Please accept for full marks.	
B1	Find the intersections of C with the x-axis. $9-x^2=0 \Rightarrow x=\pm 3$
M1	Attempt to integrate the given expression only i.e., $9-x^2$ A squared expression integrated is M0. See General Guidance for the definition of an attempt. $(A)=\int_{'-3'}^{'3'} 9-x^2 \ \mathrm{d}x=\left[9x-\dfrac{x^3}{3}\right]_{'-3'}^{'3'}$ Ignore limits for this mark, even if they are completely missing.
A1	For the correct integral $(A)=\left[9x-\dfrac{x^3}{3}\right]$ [ignore limits for this mark].
M1	For substituting **their** limits into **their** integrated expression provided it is changed from $9-x^2$ and is not a differentiated expression. $(A)=\left(9\times '3'-\dfrac{'3'^3}{3}\right)-\left(9\times '[-3]'-\dfrac{['-3']^3}{3}\right)=\ldots$ This mark can be implied by the correct final answer following correct integration. Allow also double 18 for this mark. If the final answer is incorrect and their integrated expression is incorrect, do not award this mark unless substitution is explicitly seen.
A1 [5]	For the correct answer only $(A)=36 \ \left(\mathrm{cm}^2\right)$

Question	Scheme	Mark
4(a)	$FC = \sqrt{10^2 + 10^2 + 10^2} = \sqrt{300}$	M1A1 [2]
(b)	$\cos FCA = \dfrac{\sqrt{200}}{\sqrt{300}} \Rightarrow \angle FCA = 35.3^{\circ}$	M1A1 [2]
(c)	$CX = \dfrac{\sqrt{200}}{2} = \sqrt{50}$, $FX = \sqrt{50+100} = \sqrt{150}$	M1,M1A1
	$\cos \angle FXC = \dfrac{'150' + '50' - '300'}{2 \times \sqrt{'150'} \times \sqrt{'50'}} \Rightarrow \angle FXC = 125.2643...^{\circ} \Rightarrow \text{awrt } 125^{\circ}$	M1A1 [5]
	Total	**9 marks**

Part	Mark	Notes
(a)	M1	For using Pythagoras theorem or any appropriate trigonometry to find *FC*. $FC = \sqrt{10^2 + 10^2 + 10^2} = ...$ OR $AC = \sqrt{10^2 + 10^2} = \sqrt{200}$ $FC = \sqrt{10^2 + 200} = \sqrt{300}$
	A1	For the correct exact length of $FC = \sqrt{300}\left(= 10\sqrt{3}\right)$
(b)	M1	For using any appropriate trigonometry to find the required angle. They must find a value for the award of this mark. $\cos \angle FCA = \dfrac{\sqrt{200}}{\sqrt{300}} \Rightarrow \angle FCA = \left(35.2643^{\circ}\right)$, $\sin \angle FCA = \dfrac{10}{\sqrt{300}} \Rightarrow \angle FCA = \left(35.2643^{\circ}\right)$, $\tan \angle FCA = \dfrac{10}{\sqrt{200}} \Rightarrow \angle FCA = \left(35.2643^{\circ}\right)$ Allow awrt 14.1 for $\sqrt{200}$ and awrt 17.3 for $\sqrt{300}$
	A1	$\angle FCA = 35.2643...^{\circ} \approx 35.3^{\circ}$ (awrt) Accept answers which round to 35.3°, but they must round to this value.

(c)	M1	Let X be the midpoint of BH. For any method (Pythagoras or trigonometry) to find the length CX $CX = \frac{\sqrt{200}}{2} = \left(\sqrt{50} \quad \text{or} \quad 5\sqrt{2}\right)$
	M1	For any method (Pythagoras or trigonometry) to find the length FX $FX = \sqrt{50+100} = \left(\sqrt{150} \quad \text{or} \quad 5\sqrt{6}\right)$ or $FX = \sqrt{200-50} = \left(\sqrt{150} \quad \text{or} \quad 5\sqrt{6}\right)$
	A1	For both correct lengths $\sqrt{50}$ and $\sqrt{150}$
	M1	For the correct cosine rule to find angle FXC using their lengths $\cos\angle FXC = \frac{'150'+'50'-'300'}{2\times\sqrt{'150'}\times\sqrt{'50'}} \Rightarrow \angle FXC = \left(125.2643...^\circ\right)$ NB: Check that the cosine rule they are using is to find the required angle.
	A1	For the correct angle with awrt. $\angle FXC = 125^\circ$ Accept answers which round to 125º, but they must round to this value.
	ALT – Uses the right-angled triangle *FGX*	
	M1	Let X be the midpoint of BH. For any method (Pythagoras or trigonometry) to find the length GX $GX = \frac{\sqrt{200}}{2} = \left(\sqrt{50} \quad \text{or} \quad 5\sqrt{2} \quad \text{or awrt } 7.07\right)$ If they use the sin ratio to find the angle, identify FG = 10 (cm)
	M1	For any method (Pythagoras or trigonometry) to find the length FX $FX = \sqrt{50+100} = \left(\sqrt{150} \quad \text{or} \quad 5\sqrt{6} \quad \text{awrt } 12.2\right)$ or $FX = \sqrt{200-50} = \left(\sqrt{150} \quad \text{or} \quad 5\sqrt{6} \quad \text{awrt } 12.2\right)$ **OR** If they use the tan ratio to find the angle, identify FG = 10 (cm)
	A1	For both correct lengths $\sqrt{50}$ and 10 for tan **or** $\sqrt{50}$ and $\sqrt{150}$ for cos **or** $\sqrt{150}$ and 10 for sin Allow decimal equivalents, i.e., 7.07 and 12.2
	M1	$\cos\angle FXG = \frac{\sqrt{50}}{\sqrt{150}} \Rightarrow \angle FXG = \left(54.7356...^\circ\right)$, $\sin\angle FXG = \frac{10}{\sqrt{150}} \Rightarrow \angle FXG = \left(54.7356...^\circ\right)$, $\tan\angle FXG = \frac{10}{\sqrt{50}} \Rightarrow \angle FXG = \left(54.7356...^\circ\right)$ So the required angle is $\angle FXC = 180 - '54.7356...'^\circ = \left(125.264...^\circ\right)$ We must see this calculation for the award of this mark. Do **not** award just for 54.7356…º
	A1	For the correct angle with awrt. $\angle FXC = 125^\circ$ Accept answers which round to 125º, but they must round to this value.

Question	Scheme	Mark
5(a)	$3t^2 - 23t + 30 = 0 \Rightarrow (3t-5)(t-6) = 0 \Rightarrow t = 6, \quad \frac{5}{3}$	M1A1A1 [3]
(b)	$a = \frac{\mathrm{d}v}{\mathrm{d}t} = 6t - 23$ $(6t - 23 > 0) \Rightarrow t > \frac{23}{6}$	M1 A1 [2]
(c)	$s = \int 3t^2 - 23t + 30 \ \mathrm{d}t = \frac{3t^3}{3} - \frac{23t^2}{2} + 30t + c$ $26 = \frac{3(8)^3}{3} - \frac{23(8)^2}{2} + 30(8) + c \Rightarrow c = '10'$ $d = 0 + 0 + 0 + '10' \Rightarrow d = 10$	M1A1 M1 A1 [4]
	Total	**9 marks**

Part	Mark	Notes
5 (a)	M1	For setting the given expression for $v = 0$ and attempting to solve the 3TQ See general guidance for the definition of an attempt $3t^2 - 23t + 30 = 0 \Rightarrow (3t-5)(t-6) = 0 \Rightarrow t = ..., ...$
	A1	For either $t = 6$ or $t = \frac{5}{3}$
	A1	For both $t = 6$ and $t = \frac{5}{3}$
(b)	M1	For differentiating the given expression for v which must be correct for this mark. $a = \frac{\mathrm{d}v}{\mathrm{d}t} = 6t - 23$
	A1	$(6t - 23 > 0) \Rightarrow t > \frac{23}{6}$ Accept equivalent exact values including $t > 3.8\dot{3}$
(c)	M1	For attempting to integrate the given expression for v See General Guidance for the definition of an attempt with no terms differentiated. $+c$ is not required for the award of this mark. $s = \int 3t^2 - 23t + 30 \ \mathrm{d}t = \frac{3t^3}{3} - \frac{23t^2}{2} + 30t + c$
	A1	For the correct integrated expression, which must include a constant term e.g.+ c $s = \frac{3t^3}{3} - \frac{23t^2}{2} + 30t + c$ Simplification is not required for this mark
	M1	For substituting the given values of $t = 8$ when $s = 26$ into their **integrated** expression to find c $26 = \frac{3(8)^3}{3} - \frac{23(8)^2}{2} + 30(8) + c \Rightarrow c = '10'$ $[d = 0 + 0 + 0 + '10' \Rightarrow d = ...]$
	A1	For the correct value of $d = 10$

Question	Scheme	Mark
6(a)	$S_{20}=20(3+2\times20)=860$	M1A1 [2]
(b)	$S_1=3\times1+2\times1^2=5$ $S_2=3\times2+2\times2^2=14$ $14=5+U_2\Rightarrow U_2=9$ $d=9-5=4$ $U_n='5'+(n-1)'4'$ $S_n=\sum_{r=1}^{n}(4r+1)\Rightarrow A=4,\quad B=1$	B1 M1 M1 A1 M1 A1 [6]
(c)	$T_n=\frac{n}{2}(2\times7+(n-1)4)=\left[\frac{n}{2}(10+4n)\right]\Rightarrow T_n=\frac{n}{2}(10+4n)$ or $T_n=n(5+2n)$ $n(5+2n)=3n+2n^2+252,\quad 5n=2n+252$ $5n=3n+252\Rightarrow n=126$	M1 M1,dM1 ddM1A1 [5]
	Total	**13 marks**

Part	Mark	Notes
6(a)	M1	For substituting 20 into the given $S_{20} = 20(3+2\times 20) = 860$
	A1	For $S_{20} = 860$ Sight of 860 with no working scores M1A1
(b)	B1	For finding the first term $S_1 = 3\times 1 + 2\times 1^2 = 5$ or $a = 5$ Award this mark even if it is not clear that they understand that S_1 is the first term.
	M1	For a complete method to find the second term $S_2 = 3\times 2 + 2\times 2^2 = 14$ $14 = 5 + U_2 \Rightarrow U_2 = 9$
	M1	For finding the common difference, they must reach a value for this mark. $d = '9' - '5' = (4)$
	A1	For $d = 4$
	M1	For either $A = 4$ or $B = 1$ Accept embedded values.
	A1	For both $A = 4$ and $B = 1$ Accept embedded values.
	For the final correct answer seen without any or minimal working, award full marks in part (b)	
(c)	M1	For an expression for T_n using the given values $T_n = \frac{n}{2}(2\times 7 + (n-1)4) = \left[\frac{n}{2}(10+4n)\right]$
	M1	For equating their expression for T_n in $T_n = n(3+2n)+252$ The correct expression for S_n must be used here. $\Rightarrow \frac{n}{2}(10+4n) = n(3+2n)+252 \Rightarrow (5n = 3n+252)$ **This is an A mark in Epen**
	dM1	For forming a linear equation. e.g. $5n = 3n + 252$ o.e. This mark is dependent on the previous M mark.
	ddM1	For solving **their** linear equation. $5n = 3n + 252 \Rightarrow n = (126)$ This mark is dependent on both previous M marks.
	A1	For $n = 126$

Question	Scheme	Mark
7 (a)	$\alpha+\beta=-\dfrac{p}{2}$ and $\alpha\beta=\dfrac{q}{2}$	B1
	$\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=-\dfrac{37}{14}$	B1
	$\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=\dfrac{\alpha^2+\beta^2}{\alpha\beta}=\dfrac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$	M1
	$\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=\dfrac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\dfrac{\left(-\dfrac{p}{2}\right)^2-2\left(\dfrac{q}{2}\right)}{\dfrac{q}{2}}=-\dfrac{37}{14}$	A1
	$\dfrac{\left(-\dfrac{p}{2}\right)^2-2\left(\dfrac{q}{2}\right)}{\dfrac{q}{2}}=\dfrac{p^2-4q}{2q}=\dfrac{p^2+4p+12}{2(-p-4)}=-\dfrac{37}{14}\Rightarrow 7p^2-9p-36=0$	dM1A1
	$[\,7p^2-9p-36=0$ OR $7q^2+65q+112=0\,]$	
	$p=\dfrac{-(-9)\pm\sqrt{(-9)^2-4\times7\times(-36)}}{2\times7}\Rightarrow p=3,\ q=-7$	M1A1A1ft [9]
(b)	$\alpha^2-\beta^2=(\alpha-\beta)(\alpha+\beta)$	M1
	$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$	M1
	$(\alpha-\beta)(\alpha+\beta)=\sqrt{\left('-\dfrac{3}{2}'\right)^2-4\times\left('-\dfrac{7}{2}'\right)}\times\left('-\dfrac{3}{2}'\right)=-\dfrac{3\sqrt{65}}{4}$	M1A1 [4]
	Total	**13 marks**

Part	Mark	Notes
(a)	B1	For the sum and product of roots of the equation f(x) = 0 in terms of p and q $\alpha+\beta=-\dfrac{p}{2}$ and $\alpha\beta=\dfrac{q}{2}$
	B1	For the sum of roots of the equation g(x) = 0 $\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=-\dfrac{37}{14}$
	M1	For the correct algebra to find the sum of roots of g(x) = 0 ready for substitution of values. $\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=\dfrac{\alpha^2+\beta^2}{\alpha\beta}=\dfrac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

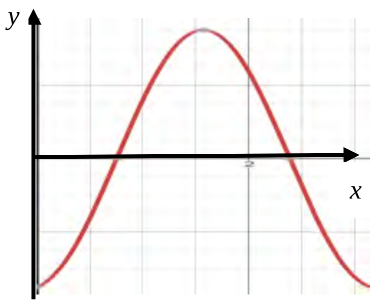
		This can be implied by correct substitution of their values for the sum and product.
	A1	For the correct value of $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ in terms of p and q [Simplification not required] $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{\left(-\frac{p}{2}\right)^2-2\left(\frac{q}{2}\right)}{\frac{q}{2}}=-\frac{37}{14}$
	dM1	For substituting in for either p or for q into the above equation **correctly** to form a 3TQ in just p or q [ft their $-\frac{37}{14}$ and their $-\frac{p}{2}$ and $\frac{q}{2}$]. The algebra must be correct here. $\frac{\left(-\frac{p}{2}\right)^2-2\left(\frac{q}{2}\right)}{\frac{q}{2}}=\frac{p^2-4q}{2q}=\frac{p^2+4p+12}{2(-p-4)}=-\frac{37}{14}\Rightarrow\left[7p^2-9p-36=0\right]$ This mark is dependent on the previous M mark.
	A1	For the correct 3TQ $7p^2-9p-36=0$ OR $7q^2+65q+112=0$
	M1	For attempting to solve their 3TQ by any valid method $(7p+12)(p-3)=0\Rightarrow p=\ldots\ (p=3)$ OR $p=\frac{-(-9)\pm\sqrt{(-9)^2-4\times7\times(-36)}}{2\times7}\Rightarrow p=\ldots,\ldots$ Negative value of p not required
	A1	For the correct value of $p=3$ **or** value of $q=-7$
	A1ft	For the correct value of $p=3$ **and** value of $q=-7$ Ft on their value of p or q Ignore mislabelling of (i) and (ii) or even no labelling of parts at all.
(b)	M1	For factorising $\alpha^2-\beta^2=(\alpha-\beta)(\alpha+\beta)$ correctly
	M1	For the correct algebra on $(\alpha-\beta)^2$ i.e. $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$ Look for this expansion which may be embedded in their working.
	M1	For the correct substitution of their sum and product into $(\alpha-\beta)(\alpha+\beta)$ $(\alpha-\beta)(\alpha+\beta)=\sqrt{\left('-\frac{3}{2}'\right)^2-4\times\left('-\frac{7}{2}'\right)}\times\left('-\frac{3}{2}'\right)=\left[\sqrt{\frac{65}{4}}\times\left(-\frac{3}{2}\right)\right]$
	A1	For the correct answer only $\alpha^2-\beta^2=-\frac{3\sqrt{65}}{4}$

<table>
<tr><th>Question</th><th>Scheme</th><th>Mark</th></tr>
<tr><td>8(a)</td><td>

x	0	0.5	0.8	1	1.6	2	2.5	3
y	**–3.5**	**–1.6**	0.3	1.6	**3.5**	**2.4**	–0.8	–3.2

</td><td>B2</td></tr>
<tr><td>(b)</td><td>Their points plotted correctly on the graph
Points are joined to form a smooth curve</td><td>B1ft
B1ft
[2]</td></tr>
<tr><td>(c)</td><td>$\cos(A+B)=\cos A\cos B-\sin A\sin B \Rightarrow \cos 2A=\cos^2 A-\sin^2 A=1-\sin^2 A-\sin^2 A$
$\cos(2A)=1-2\sin^2 A$</td><td>M1

A1
cso
[2]</td></tr>
<tr><td>(d)</td><td>$2\sin x+6(1-\cos 2x)-x-5=0$
$\sin x-3\cos 2x-\frac{1}{2}=\frac{x}{2}\pm k$ where k is a constant $\Rightarrow y=\frac{x}{2}-1$
Draws line $y=\frac{x}{2}-1$
$x=0.6$ or $x=0.7$ AND $x=2.3$ or $x=2.4$</td><td>M1
dM1A1
M1
A1
[5]</td></tr>
<tr><td></td><td align="right">Total</td><td>11 marks</td></tr>
</table>

<table>
<tr><th>Part</th><th>Mark</th><th>Scheme</th></tr>
<tr><td>(a)</td><td>B2</td><td>
<table>
<tr><td>x</td><td>0</td><td>0.5</td><td>0.8</td><td>1</td><td>1.6</td><td>2</td><td>2.5</td><td>3</td></tr>
<tr><td>y</td><td>–3.5</td><td>–1.6</td><td>0.3</td><td>1.6</td><td>3.5</td><td>2.4</td><td>–0.8</td><td>–3.2</td></tr>
</table>
For all 4 values correct OR If B2 not scored then award B1 for any 2 values correct
Note: These values must be rounded to one decimal place only. Accept only the above values.</td></tr>
<tr><td>(b)</td><td>B1ft
B1ft</td><td>


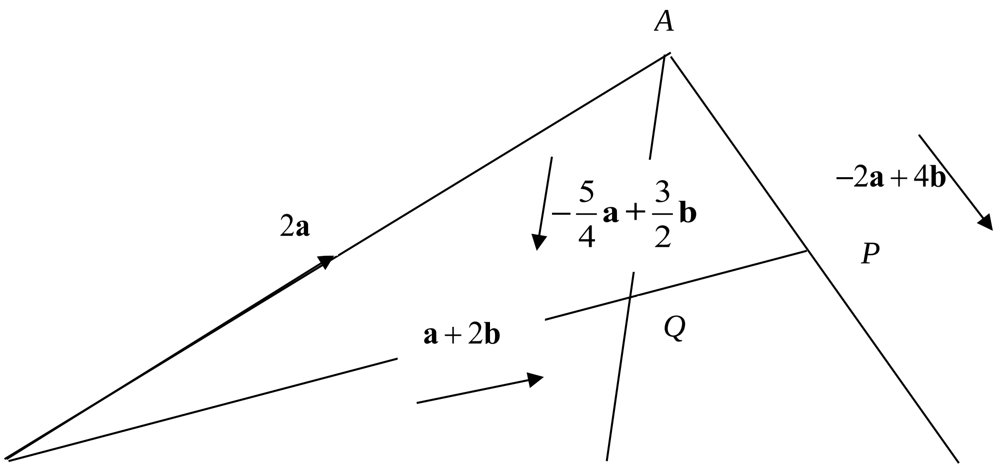
All points plotted correctly (ft their values) on the graph within half of one square.
All of their points are joined to form a smooth curve within half of a square of a point. There must be at least six points plotted and joined.</td></tr>
<tr><td>(c)</td><td>M1</td><td>For using the identity $\cos(A+B)=\cos A\cos B-\sin A\sin B=\cos^2 A-\sin^2 A$ correctly and the identity $\sin^2 A+\cos^2 A=1$ also correctly.</td></tr>
<tr><td></td><td>A1
cso</td><td>For simplifying to the required identity $\cos 2A=1-2\sin^2 A$*
This is a given result so there must be no errors in their solution.</td></tr>
<tr><td>(d)</td><td>M1</td><td>$2\sin^2 x=1-\cos 2x$
For substituting the above into the given f(x) with no errors.
e.g., $2\sin x+6(1-\cos 2x)-x-5=0$</td></tr>
<tr><td></td><td>dM1</td><td>For reaching the equation of the curve on one side and the equation of a straight line on the other.
$\sin x-3\cos 2x-\frac{1}{2}=\frac{x}{2}\pm k$ where k is a constant o.e
Note: This mark is dependent on the previous M mark.</td></tr>
<tr><td></td><td>A1</td><td>For the correct equation of the equation of the straight line required $y=\frac{x}{2}-1$</td></tr>
<tr><td></td><td>M1</td><td>For drawing their line which must be in the form $y=\frac{x}{2}\pm k$
Correct coordinates for $y=\frac{x}{2}-1$ are; $(0,\ -1)$, $\left(1,\ -\frac{1}{2}\right)$, $(2,\ 0)$, $\left(3,\ \frac{1}{2}\right)$</td></tr>
<tr><td></td><td>A1</td><td>For both correct roots of $x=0.6$ or $x=0.7$ AND $x=2.3$ or $x=2.4$</td></tr>
</table>

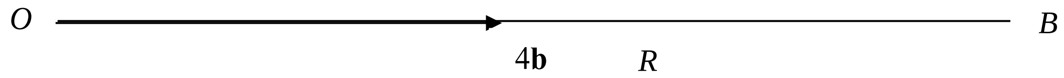
Question	Scheme	Mark
9(a)	$\overrightarrow{AB}=-\overrightarrow{OA}+\overrightarrow{OB}\Rightarrow\overrightarrow{AB}=-2\mathbf{a}+4\mathbf{b}$	M1A1 [2]
(b)	$\overrightarrow{OP}=\overrightarrow{OA}+\overrightarrow{AP}\Rightarrow\overrightarrow{OP}=2\mathbf{a}+\frac{1}{2}(4\mathbf{b}-2\mathbf{a})=\mathbf{a}+2\mathbf{b}$	M1A1 [2]
(c)	$\overrightarrow{AQ}=\overrightarrow{AO}+\overrightarrow{OQ}$ $\overrightarrow{OQ}=\frac{3}{4}(\overrightarrow{OP})$ $\overrightarrow{AQ}=-2\mathbf{a}+\frac{3}{4}(\mathbf{a}+2\mathbf{b})=-\frac{5}{4}\mathbf{a}+\frac{3}{2}\mathbf{b}$	M1 B1 A1 [3]
(d)	$\overrightarrow{OR}=\lambda 4\mathbf{b}$ or $\overrightarrow{OR}=\phi\mathbf{b}$ $\overrightarrow{OR}=\overrightarrow{OA}+\mu\overrightarrow{AQ}=2\mathbf{a}+\mu\left(-\frac{5}{4}\mathbf{a}+\frac{3}{2}\mathbf{b}\right)$ $2\mathbf{a}+\mu\left(-\frac{5}{4}\mathbf{a}+\frac{3}{2}\mathbf{b}\right)=\lambda 4\mathbf{b}$ $\mathbf{a}\left(2-\frac{5\mu}{4}\right)+\frac{3\mu}{2}\mathbf{b}=\lambda 4\mathbf{b}\Rightarrow\frac{3\mu}{2}=4\lambda$ and $2-\frac{5\mu}{4}=0$ $2-\frac{5\mu}{4}=0\Rightarrow\mu=\left(\frac{8}{5}\right)$ $\frac{3}{2}\times\frac{8}{5}=4\lambda\Rightarrow\lambda=\frac{12}{20},\left(\frac{3}{5}\right)$ $OR:RB=3:2$	M1 M1 M1 M1A1 A1 [6]
	Total	13 marks

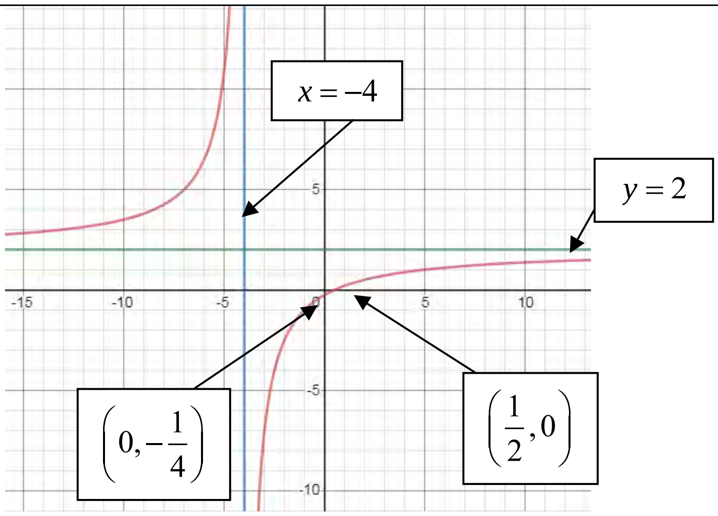
Part	Mark	Scheme
(a)	M1	For the correct vector statement $\overrightarrow{AB}=-\overrightarrow{OA}+\overrightarrow{OB}$
	A1	$\overrightarrow{AB}=-2\mathbf{a}+4\mathbf{b}$
(b)	M1	For a correct vector statement e.g $\overrightarrow{OP}=\overrightarrow{OA}+\overrightarrow{AP}$
	A1	For the correct simplified vector $\overrightarrow{OP}=2\mathbf{a}+(2\mathbf{b}-\mathbf{a})=\mathbf{a}+2\mathbf{b}$
(c)	M1	For a correct vector statement e.g., $\overrightarrow{AQ}=\overrightarrow{AO}+\overrightarrow{OQ}$ or $\overrightarrow{AQ}=\overrightarrow{AP}+\overrightarrow{PQ}$
	B1	For the correct vector statement for $\overrightarrow{OQ}$ $\overrightarrow{OQ}=\frac{3}{4}\left(\overrightarrow{OP}\right)$ or $\overrightarrow{PQ}=-\frac{1}{4}\left(\overrightarrow{OP}\right)$
	A1	For the correct simplified vector

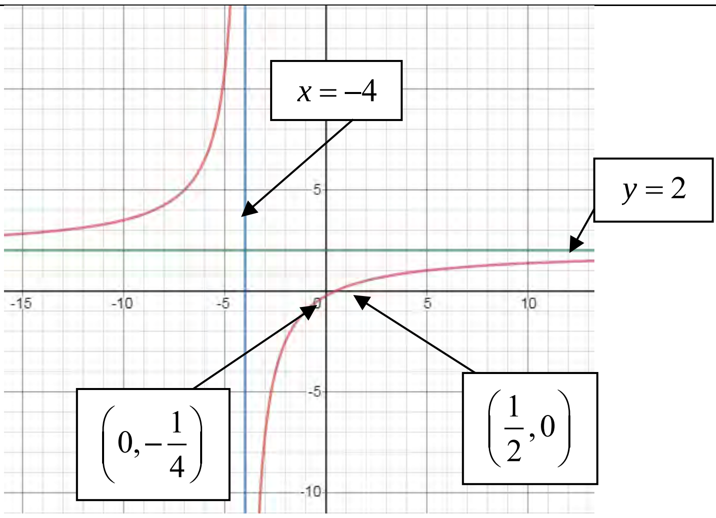
		$\overrightarrow{AQ} = -2\mathbf{a} + \frac{3}{4}(\mathbf{a}+2\mathbf{b}) = -\frac{5}{4}\mathbf{a} + \frac{3}{2}\mathbf{b}$ or $\overrightarrow{AQ} = \frac{1}{2}(-2\mathbf{a}+4\mathbf{b}) - \frac{1}{4}(\mathbf{a}+2\mathbf{b}) = -\frac{5}{4}\mathbf{a} + \frac{3}{2}\mathbf{b}$
(d)	In this part there is more than one route to finding the required ratio, but any route must involve $\overrightarrow{OR}$ or $\overrightarrow{RB}$ As the general principle: The first M mark is for one vector using a parameter. The second M mark is for a second vector using a different parameter. The third M mark is for equating coefficients and forming two equations in both parameters. The fourth M mark is for solving their equations. Check their working and do not allow an erroneous method. The following is using one path. Please trace their path on the sketch to check that it is valid. The path they use must involve the vector $\overrightarrow{OR}$ or $\overrightarrow{RB}$ eg., $\overrightarrow{AR} = \overrightarrow{AO} + \overrightarrow{OR}$ or $\overrightarrow{AR} = \overrightarrow{AB} + \overrightarrow{BR}$	
	M1	For the correct statement $\overrightarrow{OR} = \lambda 4\mathbf{b}$ or $\overrightarrow{OR} = \phi\mathbf{b}$
	M1	For the correct statement for $\overrightarrow{OR}$ $\overrightarrow{OR} = \overrightarrow{OA} + \mu\overrightarrow{AQ} = 2\mathbf{a} + \mu\left(-\frac{5}{4}\mathbf{a} + \frac{3}{2}\mathbf{b}\right)$
	M1	For equating the two statements for $\overrightarrow{OR}$ and equating coefficients $2\mathbf{a} + \mu\left(-\frac{5}{4}\mathbf{a} + \frac{3}{2}\mathbf{b}\right) = \lambda 4\mathbf{b}$ $\mathbf{a}\left(2 - \frac{5\mu}{4}\right) + \frac{3\mu}{2}\mathbf{b} = \lambda 4\mathbf{b} \Rightarrow \frac{3\mu}{2} = 4\lambda$ and $2 - \frac{5\mu}{4} = 0$
	M1	For solving their two simultaneous equations **correctly** to find the value of λ (the parameter for *OR*) $2 - \frac{5\mu}{4} = 0 \Rightarrow \mu = \frac{8}{5}$ $\frac{3}{2} \times \frac{8}{5} = 4\lambda \Rightarrow \lambda = ...$
	A1	For the correct value of λ $\lambda = \frac{12}{20} = \left(\frac{3}{5}\right)$
	A1	For the correct ratio $OR : RB = 3 : 2$

Useful Sketch





Question	Scheme	Mark
10(a)	(i) $y=2$ (ii) $x=-4$	B1B1 [2]
(b)	$\left(\frac{1}{2},\ 0\right)$, $\left(0,-\frac{1}{4}\right)$	B1B1 [2]
(c)		B3 [3]
(d)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2(x+4)-(2x-1)}{(x+4)^2}$ $m=1$ $\frac{\mathrm{d}y}{\mathrm{d}x}=1\Rightarrow\frac{9}{(x+4)^2}=1\Rightarrow(x+4)^2=9\Rightarrow x=-7,-1$ $P\ (-1,-1)$, $Q\ (-7,5)$	M1A1 B1 dM1ddM1A1 A1A1 [8]
(e)	$y=x+k$ $'-1'='-1'+k_1\Rightarrow k_1=0\quad '5'='-7'+k_2\Rightarrow k_2=12$	M1A1A1 [3]
	Total	**18 marks**

Part	Mark	Notes
(a)	B1 B1	For correct **equations** only (i) $y = 2$ These must be clearly labelled. (ii) $x = -4$
(b)	B1	For either $\left(\frac{1}{2},\ 0\right)$ OR $\left(0, -\frac{1}{4}\right)$ accept $y = -\frac{1}{4}$ OR $x = \frac{1}{2}$
	B1	For both $\left(\frac{1}{2},\ 0\right)$ AND $\left(0, -\frac{1}{4}\right)$ accept $y = -\frac{1}{4}$ AND $x = \frac{1}{2}$
(c)		
	B1	For the correct shape in the correct 'quadrants'. Do not allow the curves to turn back on themselves, but be reasonable in your judgement. If they all turn back on themselves clearly – withold the mark. If there is doubt over just one end, allow the mark. If you are really not sure, then please send to Review.
	B1ft	Their asymptotes drawn and labelled and there must be at least one part of the curve present that is asymptotic in nature. Any curve that crosses the asynptotes does not score this mark. Accept -4 written on the x-axis and 2 written on the y-axis.
	B1ft	Their intersections labelled. Accept $y = -\frac{1}{4}$ AND $x = \frac{1}{2}$ labelled correctly and their curve must pass through these points.
(d)	M1	For attempting to use Quotient rule: An attempt is defined as both $(2x-1)$ and $(x+4)$ differentiated CORRECTLY and the correct formula used (subtracted either way around in the numerator) with the denominator squared. $\frac{dy}{dx} = \frac{2(x+4)-(2x-1)}{(x+4)^2}$ [Correct] or $\frac{dy}{dx} = \frac{(2x-1)-2(x+4)}{(x+4)^2}$ [Incorrect] Or for an attempt to use product rule: Both terms differentiated correctly with the correct formula used. Allow a maximum of one sign error.

		$\frac{dy}{dx}=(2x-1)(1)(-1)(x+4)^{-2}+(2)(x+4)^{-1}\Rightarrow\left[\frac{dy}{dx}=\frac{-(2x-1)+2(x+4)}{(x+4)^2}\right]$
	A1	$\frac{dy}{dx}=\frac{2(x+4)-(2x-1)}{(x+4)^2}$ Fully correct
	B1	For $m=1$
		ALT 1 for next **two marks** only
	dM1	For setting their $\frac{dy}{dx}=1$ and rearranging to reach a 3TQ. If there is no squared term in their $\frac{dy}{dx}$ then this is M0. $x^2+8x+7=0$
	ddM1	For attempting to solve their 3TQ $x^2+8x+7=0\Rightarrow(x+1)(x+7)=0\Rightarrow x=...,...$
		ALT 2 for next **two marks** only
	dM1	For setting their $\frac{dy}{dx}=1\Rightarrow\frac{9}{(x+4)^2}=1\Rightarrow(x+4)^2=9$
	ddM1	$(x+4)^2=9\Rightarrow x+4=\pm3\Rightarrow x=...,\ ...$
	A1	For **BOTH** correct values of x, -7 and -1
	A1	For using their values of x to find either the **coordinates** of P or Q, they must be given as coordinates but allow missing or incorrect labels i.e, if they label P as Q or vice versa, allow the marks. At P $y=\frac{2\times('-1')-1}{-1+4}=-1\Rightarrow$ Coordinates are $(-1,-1)$ **OR** At Q $y=\frac{2\times('-7')-1}{-7+4}=5\Rightarrow$ Coordinates are $(-7,5)$
	A1	At P $y=\frac{2\times('-1')-1}{-1+4}=-1\Rightarrow$ Coordinates are $(-1,-1)$ **BOTH** At Q $y=\frac{2\times('-7')-1}{-7+4}=5\Rightarrow$ Coordinates are $(-7,5)$
(e)	M1	For substituting either of **their** coordinates into $y=x+k$ $'-1'='-1'+k_1\Rightarrow k_1=...$ $'5'='-7'+k_2\Rightarrow k_2=...$
	A1	$k_1=0$
	A1	$k_2=12$

Pearson Education Limited. Registered company number 872828
with its registered office at 80 Strand, London, WC2R 0RL, United Kingdom