

Please check the examination details below before entering your candidate information


Candidates surname					Other names				
Centre Number					Candidate Number				
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**Pearson Edexcel International GCSE**

Time 2 hours

Paper reference **4PM1/01**

**Further Pure Mathematics**  
**PAPER 1**



**Calculators may be used.**

Total Marks

### Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.  
Anything you write on the formulae page will gain NO credit.

### Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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## International GCSE in Further Pure Mathematics Formulae sheet

**Mensuration**

$$\text{Surface area of sphere} = 4\pi r^2$$

$$\text{Curved surface area of cone} = \pi r \times \text{slant height}$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

**Series****Arithmetic series**

$$\text{Sum to } n \text{ terms, } S_n = \frac{n}{2}[2a + (n-1)d]$$

**Geometric series**

$$\text{Sum to } n \text{ terms, } S_n = \frac{a(1-r^n)}{(1-r)}$$

$$\text{Sum to infinity, } S_\infty = \frac{a}{1-r} \quad |r| < 1$$

**Binomial series**

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$$

**Calculus****Quotient rule (differentiation)**

$$\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

**Trigonometry****Cosine rule**

$$\text{In triangle } ABC: a^2 = b^2 + c^2 - 2bc \cos A$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

**Logarithms**

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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**Answer all TEN questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

- 1 Given that  $\frac{2\sqrt{3} - 4}{3\sqrt{3} + 5}$  can be written in the form  $a + b\sqrt{3}$  where  $a$  and  $b$  are integers, find, without using a calculator, the value of  $a$  and the value of  $b$
- Show your working clearly.

(3)

(Total for Question 1 is 3 marks)



2

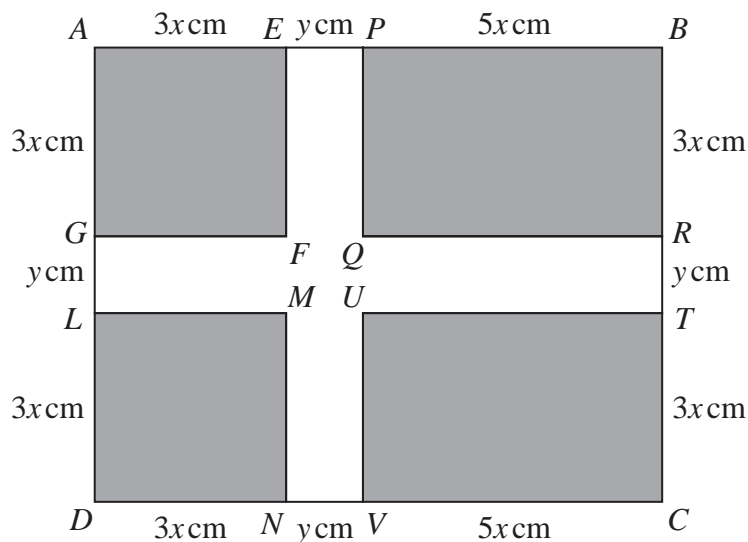


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the design for a flag consisting of a white cross on a grey background.

$AEFG$  and  $DLMN$  are squares with sides of length  $3x$  cm.

$BPQR$  and  $CTUV$  are rectangles with sides of length  $5x$  cm and  $3x$  cm.

The width of the cross is  $y$  cm.

The total area of the flag is  $H$  cm<sup>2</sup>

(a) Write down an expression, in terms of  $x$  and  $y$ , for  $H$ .

(1)

Given that the area of the cross is  $K$  cm<sup>2</sup>

(b) show that  $K = 14xy + y^2$

(3)

The total area of the flag is to be 3432 cm<sup>2</sup> and the area of the cross is to be 1080 cm<sup>2</sup>

(c) Find the value of  $x$  and the value of  $y$

(5)

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**Question 2 continued**

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**Question 2 continued**

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**(Total for Question 2 is 9 marks)**





**Question 3 continued**

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**(Total for Question 3 is 7 marks)**



4 The common ratio of a geometric series  $G$  is positive.

The sum of the first 4 terms of  $G$  is 80

The sum to infinity of  $G$  is 81

Show that the sum of the first 7 terms of  $G$  differs from the sum to infinity of  $G$  by  $\frac{1}{27}$  (7)

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**Question 4 continued**

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**(Total for Question 4 is 7 marks)**



5 Given that  $(2 + 3x)^{-1}$  can be expressed in the form  $p(1 + qx)^{-1}$  where  $p$  and  $q$  are constants,

(a) find the value of  $p$  and the value of  $q$

(2)

(b) Hence expand  $(2 + 3x)^{-1}$  in ascending powers of  $x$  up to and including the term in  $x^3$ , expressing each coefficient as an exact fraction in its lowest terms.

(3)

$$f(x) = \frac{1 + x}{2 + 3x}$$

(c) Obtain a series expansion for  $f(x)$ , in ascending powers of  $x$  up to and including the term in  $x^3$ , expressing each coefficient as an exact fraction in its lowest terms.

(2)

(d) Hence use algebraic integration to obtain an estimate, to 4 decimal places, of

$$\int_0^{0.5} f(x) \, dx$$

(4)

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**Question 5 continued**

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**Question 5 continued**

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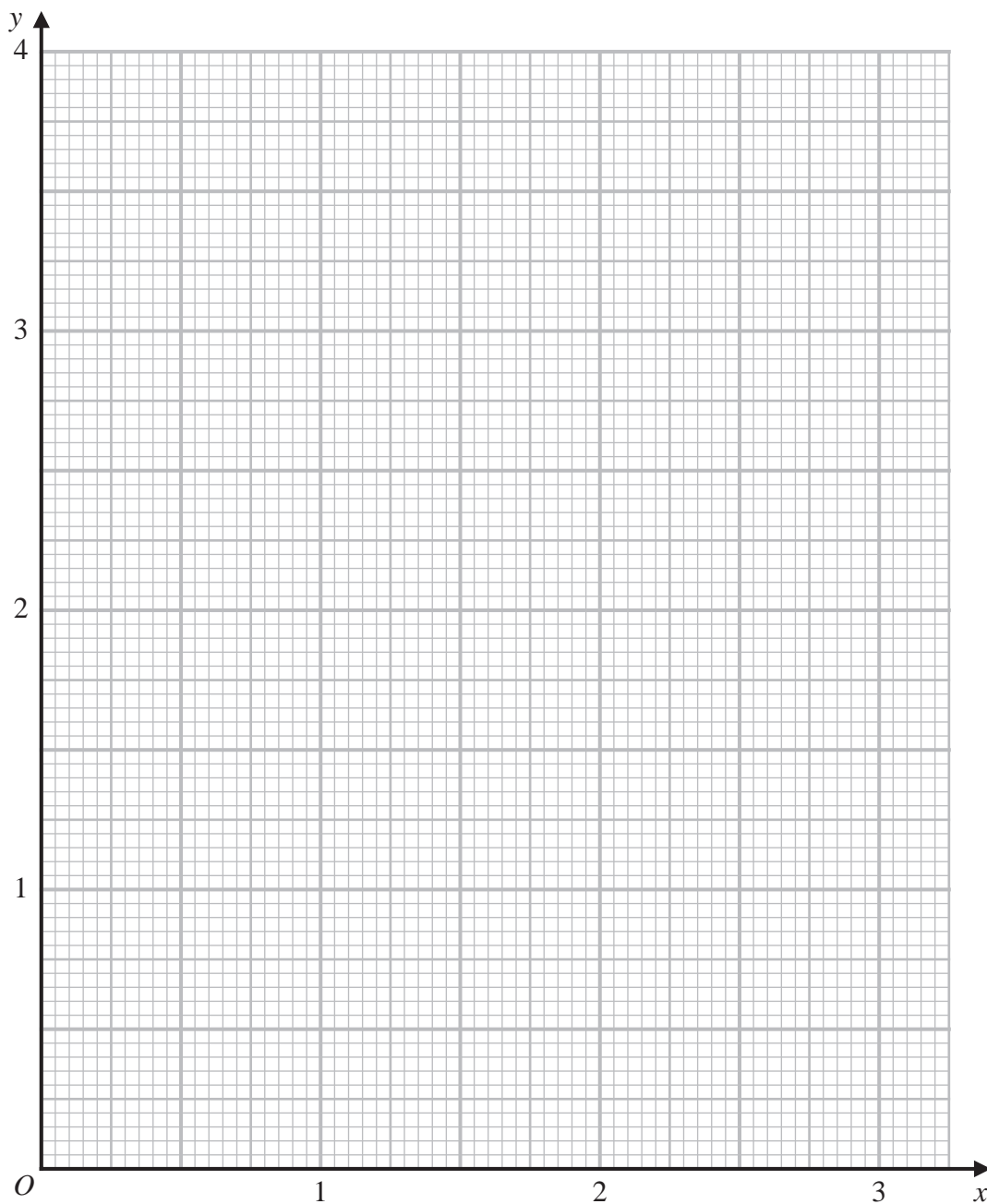
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**(Total for Question 5 is 11 marks)**





**Question 6 continued**



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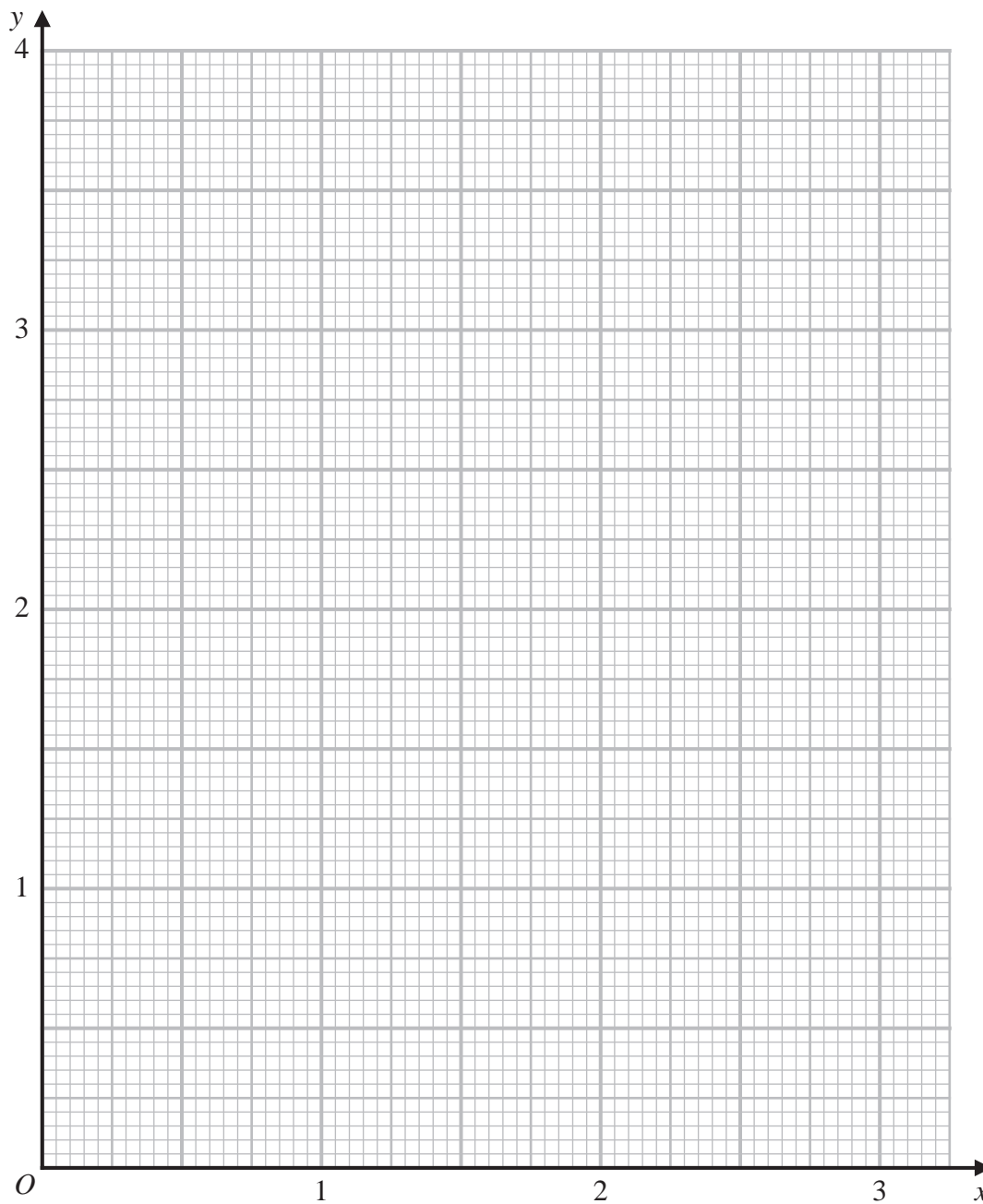
**Turn over for a spare grid if you need to redraw your graph.**





**Question 6 continued**

**Only use this grid if you need to redraw your graph.**



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**(Total for Question 6 is 11 marks)**



7 The point with coordinates  $(4, -104)$  lies on the curve  $C$  with equation  $y = f(x)$

Given that  $f'(x) = 4x^3 - 12x^2 - 19x + 12$

(a) (i) show that  $C$  passes through the origin,

(4)

(ii) show that  $C$  has a maximum at the point on the curve where  $x = 0.5$

(3)

The curve  $C$  has another turning point at  $A$  and another turning point at  $B$ .

Given that the  $x$  coordinate of  $A$  is negative,

(b) (i) find the coordinates of  $A$  and the coordinates of  $B$ ,

(5)

(ii) determine the nature of these turning points.

(3)

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**Question 7 continued**

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**Question 7 continued**

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**(Total for Question 7 is 15 marks)**





**Question 8 continued**

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**(Total for Question 8 is 7 marks)**



9

$$f(x) = 3x^4 + 4x^3 - 36x^2 + 64$$

Given that  $f(x)$  can be written in the form  $(x - 2)^2(ax^2 + bx + c)$

- (a) find the value of  $a$ , the value of  $b$  and the value of  $c$  (4)

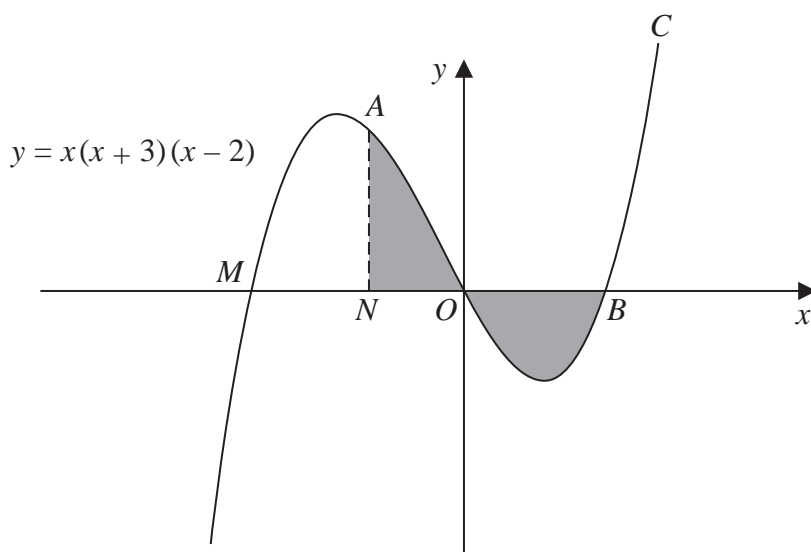


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a sketch of part of the curve  $C$  with equation  $y = x(x + 3)(x - 2)$

The curve  $C$  crosses the  $x$ -axis at the point  $M$ , the origin and the point  $B$ .

The point  $N$  lies on the  $x$ -axis between  $M$  and  $O$ .

The point  $A$  lies on  $C$  such that  $AN$  is parallel to the  $y$ -axis.

The area of the shaded region bounded by the curve and  $OB$  is numerically equal to the area of the shaded region bounded by the curve,  $ON$  and  $NA$ .

Given that the coordinates of  $N$  are  $(n, 0)$ ,

- (b) use algebraic integration to show that  $n$  satisfies the equation

$$(x - 2)^2(3x^2 + 16x + 16) = 0 \tag{7}$$

- (c) Hence find the exact coordinates of  $A$ . (5)

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**Question 9 continued**

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**Question 9 continued**

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**(Total for Question 9 is 16 marks)**



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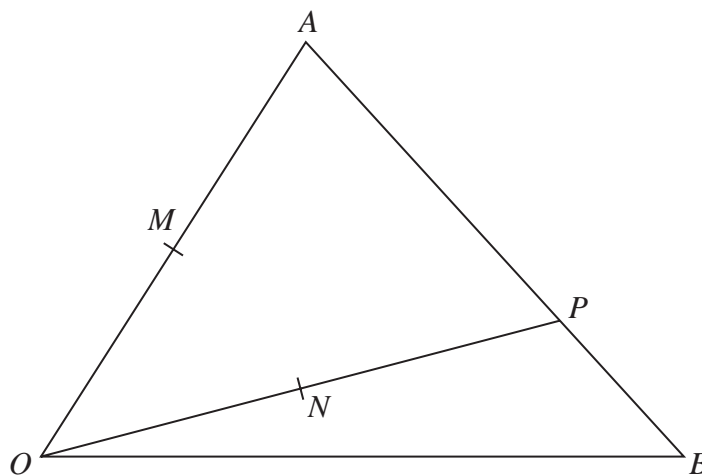


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Figure 4

Figure 4 shows triangle  $OAB$  in which

$$\vec{OA} = \mathbf{a} \text{ and } \vec{OB} = \mathbf{b}$$

The point  $P$  lies on  $AB$  such that  $AP:PB = 3:1$

The point  $M$  is the midpoint of  $OA$  and the point  $N$  is the midpoint of  $OP$ .

(a) Find, as simplified expressions in terms of  $\mathbf{a}$  and  $\mathbf{b}$ , the vector

- (i)  $\vec{OP}$       (ii)  $\vec{MN}$

(4)

The point  $C$  lies on  $OB$  such that  $ANC$  is a straight line.

(b) Using a vector method, find the vector  $\vec{OC}$  as a simplified expression in terms of  $\mathbf{b}$

(6)

Given that  $\frac{\text{area of quadrilateral } AMNP}{\text{area of triangle } OAB} = K$

(c) find the exact value of  $K$

(4)

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**Question 10 continued**

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