Mark Scheme (Results)

Summer 2022

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 02R

Summer 2022
Question Paper Log Number P71642A
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme - not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed out work should be marked **unless** the candidate has replaced it with an alternative response.

- **Types of mark**
    - M marks: method marks
    - A marks: accuracy marks
    - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
    - cao – correct answer only
    - ft – follow through
    - isw – ignore subsequent working
    - SC - special case
    - oe – or equivalent (and appropriate)
    - dep – dependent
    - indep – independent
    - awrt – answer which rounds to
    - eeoo – each error or omission

- **No working**

  If no working is shown then correct answers normally score full marks

  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right),\text{ where } |pq|=|c| \quad \text{leading to } x=....$$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right) \text{ where} |pq|=|c| \text{ and } |mn|=|a| \quad \text{leading to } x=....$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$$x^2+bx+c=0:\ \left(x\pm\tfrac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0 \qquad \text{leading to } x=....$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\rightarrow x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\rightarrow x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication

from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

International GCSE Further Pure Mathematics – Paper 2R mark scheme

Question number	Scheme	Marks
1(a)	$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$ or $6\mathbf{i} + 8\mathbf{j} = \overrightarrow{OB} - (3\mathbf{i} - 2\mathbf{j})$ oe $9\mathbf{i} + 6\mathbf{j}$	M1 A1 [2]
(b)	$\sqrt{6^2 + 8^2}$ *or* 10 (from Pythagorean triple)	B1 [1]
(c)	$(\pm)\frac{1}{"10"}(6\mathbf{i} + 8\mathbf{j})$	M1 A1 [2]
		Total 5 marks

Part	Mark	Additional Guidance
(a)	M1	Correct vector path written, can be implied by correct addition of vectors OR correct vector statement together with correct substitution of the given vectors (where $\overrightarrow{OB}$ is not the subject)
	A1	$9\mathbf{i} + 6\mathbf{j}$
(b)	B1	Need not be simplified
(c)	M1	Correctly uses their magnitude from part (b)
	A1	Correct vector Penalise column vector notation for answer first time only

Question number	Scheme	Marks
2	$(V=)3x^3$	B1
	$\frac{dV}{dx}=9x^2$	M1
	$\left(\frac{dx}{dt}=\right)\frac{dV}{dt}\times\frac{dx}{dV}$ or $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}$ oe	M1
	$\left(\frac{dx}{dt}=\right)\frac{8}{9x^2}$ oe	A1
	$\left(\frac{dx}{dt}=\right)\frac{8}{9\times2^2}$ oe	dM1
	$\frac{2}{9}$ oe	A1 [6]
		Total 6 marks

Part	Mark	Additional Guidance
	B1	Correct simplified expression for Volume
	M1	Minimally acceptable attempt at differentiation– see general guidance (kx^2 where $k \neq 0$ if working from correct V)
	M1	A correct chain rule that could be used to find $\frac{dx}{dt}$ Condone absence of $\frac{dx}{dt}$ unless $\frac{dx}{dt}$ is not the subject.
	A1	As shown oe
	dM1	Substitution of $x = 2$ into their $\frac{dx}{dt}$, dependent on second method mark.
	A1	Correct answer (exact or correct to 2dp or better)
	For all marks condone poor notation e.g. use of dy/dx as long as not ambiguous.	

Question number	Scheme	Marks
3 (a) (i)	$ar^2 = 5$ or $ar^4 = \frac{5}{2}$ or $5r^2 = \frac{5}{2}$	B1
	$\left(\dfrac{ar^4}{ar^2}\right) = \dfrac{\frac{5}{2}}{5}$ or $\dfrac{5}{r^2} = \dfrac{\frac{5}{2}}{r^4}$ oe $\to r$ or $r = \sqrt{\dfrac{\frac{5}{2}}{5}}$	M1
	$r = \dfrac{\sqrt{2}}{2}$ oe	A1
(ii)	$a = 10$	A1 [4]
(b)	$S_\infty = \dfrac{"10"}{1 - "\dfrac{\sqrt{2}}{2}"}$	M1
	$20 + 10\sqrt{2}$	A1 [2]
		Total 6 marks

Part	Mark	Additional Guidance
(a)	Ignore labelling and mark parts (i) and (ii) together.	
(i)	B1	One correct equation as shown. This is an M mark in epen.
	M1	Attempts to solve simultaneously. Must be working with correct equations or with $ar^3 = 5$ and $ar^5 = \frac{5}{2}$ Minimum attempt to correctly divide their equations or rearrange for *a* and equate as shown or to correctly rearrange and eliminate *r*, must achieve a value for *r* or for *a*. OR attempts to solve $5r^2 = \frac{5}{2}$ to obtain r Allow errors in arithmetic but not mathematically incorrect process.
	A1	Value as shown. Allow this mark for correct answer from working with $ar^3 = 5$ and $ar^5 = \frac{5}{2}$ Must reject negative if seen. isw attempt to convert to decimal.
(ii)	A1	Value as shown.
(b)	M1	Correctly substitutes their values for *a* and *r* into the formula provided $\lvert r \rvert < 1$
	A1	Correct value.

Question number	Scheme	Marks
4 (a)	f (±1)=0 **or** f (±2)= –5	M1
	$-1 + 1p + -1q + 7 = 0$ $(p - q + 6 = 0)$ **and** $-8 + 4p + -2q + 7 = -5$ $(4p - 2q - 1 = -5)$	A1
	$4(q - 6) - 2q = -4$ $(2q = 20)$	dM1
	$p = 4$ $q = 10$	A1 A1 [5]
	ALT – polynomial division $(x^3 + px^2 + qx + 7) \div (x + 1) = x^2 + (p - 1)x + q - p + 1$ and comparison of final step of division with 7 to obtain an equation **or** $(x^3 + px^2 + qx + 7) \div (x + 2) =$ $x^2 + (p - 2)x + (q - 2p + 4)$ remainder -5 and comparison of final step of division with obtaining remainder -5 to obtain an equation	M1
	$(x^3 + px^2 + qx + 7) \div (x + 1) = x^2 + (p - 1)x + q - p + 1$ and comparison of final step of division with 7 to identify $q - p + 1 = 7$ **and** $(x^3 + px^2 + qx + 7) \div (x + 2) =$ $x^2 + (p - 2)x + (q - 2p + 4)$ remainder -5 and comparison of final step of division with obtaining remainder -5 to identify $7 - 2(q - 2p + 4) = -5$	A1
	$(p + 6) - 2p = 2$ $(-p = -4)$	dM1
	$p = 4$ $q = 10$	A1 A1
(b)	$x + 1 \overline{) x^3 + "4"x^2 + "10"x + 7}$ quotient $x^2 \; (+3x + 7)$ $x^3 + x^2$ $"3"x^2$ or $x^3 + 4x^2 + 10x + 7 \equiv (x + 1)(x^2 + Ax + B)$	M1
	"3"^{2} – 4(1)("7") = a value	dM1
	–19 and a conclusion drawn e.g. the discriminant is negative so only one real root	A1 [3]
	ALT – use of completing the square $x + 1 \overline{) x^3 + "4"x^2 + "10"x + 7}$ quotient $x^2 \; (+3x + 7)$ $x^3 + x^2$ $"3"x^2$ or $x^3 + 4x^2 + 10x + 7 \equiv (x + 1)(x^2 + Ax + B)$	M1

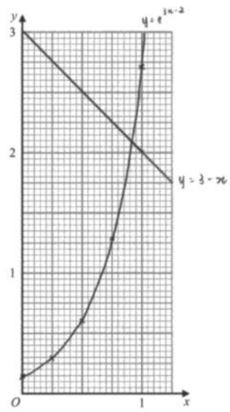
	$\left(x+\frac{3}{2}\right)^2+\frac{19}{4}>0$	dM1
	$\left(x+\frac{3}{2}\right)^2+\frac{19}{4}>0$ and a conclusion drawn e.g. the completed square form is always greater than 0 so only one real root	A1
		Total 8 marks

(a)	M1	Correct substitution of ±2 or ±1 into f (x) and equating to appropriate value.
	A1	Two fully correct equations as shown, powers of -1 and -2 evaluated, need not be simplified Missing brackets must be recovered for the award of this mark.
	dM1	An attempt to solve algebraically. Minimum attempt is to correctly substitute an expression for p or q into the other equation and attempt to solve or to multiply each equation correctly to make the coefficients $\pm p$ or $\pm q$ the same in each equation and attempt to add or subtract the equations (consistent appropriate operation).
	A1	Correct p
	A1	Correct q
ALT – polynomial division		
	M1	Correct method for polynomial division together with comparison of the final step of the division with the required result (no remainder for division by $(x+1)$ and remainder -5 for division by $(x+2)$)
	A1	Two fully correct equations as shown, need not be simplified
	dM1	An attempt to solve algebraically. Minimum attempt is to correctly substitute an expression for p or q into the other equation and attempt to rearrange or to multiply each equation correctly to make the coefficients $\pm p$ or $\pm q$ the same in each equation and attempt to add or subtract the equations.
	A1	Correct p
	A1	Correct q
(b)	M1	Attempts long division. Minimally acceptable attempt is the division and correct working as written in the MS as shown for their p and their q. Or if comparing coefficients, a correct equation/comparison (for their p and their q) must be written followed by an attempt to find A or B. Must get a 3TQ.
	dM1	Correctly substitutes 1, their A and their B into the expression for the discriminant and reaches a value.
	A1	Correct value of -19 (or for stating $9-28<0$) from correct quadratic and draws a conclusion. Allow for showing that there are complex roots for the quadratic together with appropriate conclusion. There must be some conclusion drawn, but it can be as simple as writing '#' or "shown".
ALT – completing the square		
	M1	Attempts long division. Minimally acceptable attempt is the division and correct working as written in the MS as shown for their p and their q. Or if comparing coefficients, a correct equation/comparison (for their p and their q) must be written followed by an attempt to find A or B. Must get a 3TQ.

	dM1	Attempt at completing the square See general guidance for what constitutes an attempt at completing the square
	A1	Correct completed square form and draws a conclusion. There must be some conclusion drawn, but it can be as simple as writing '#' or "shown".

Question number	Scheme	Marks
5 (a)	x: 0, 0.25, 0.5, 0.75, 1 y: 0.14, **0.29**, **0.61**, **1.28**, 2.72	first B1 2 values correct second B1 all 3 values correct [2]
(b)	Each point plotted correctly within the correct small square Smooth curve through the points	B1ft B1ft [2]
(c)	$e^{3x-2} = 3 - x$ $y = 3 - x$ 0.9	M1 M1 A1ft [3]
		Total 7 marks

Part	Mark	Additional Guidance
(a)	B1	SC1 – allow 0.29 and/or 0.61 to be truncated to 0.28 and/or 0.60 with 1.28 correct to gain this mark OR for all three values correct but given to greater than 2 decimal places
	B1	For all 3 values rounded correctly as shown.
(b)	B1ft	ft the correct plotting of their points.
	B1ft	ft a curve "sensibly" plotted through their points, need not have the correct shape. Must pass through all of the points they have plotted. Minimum 4 points.
(c)	M1	Rearranges the equation must be of form $e^{3x\pm2} = 3 - x$
	M1	$y = 3 - x$ drawn correctly on the graph paper. Passing through the points (0,3) and (1,2) as a minimum.
	A1ft	"0.9" must be given to 1 decimal place. Not as part of a coordinate. Follow through an appropriately shaped curve (or line segments) and correct $y = 3 - x$. With answer given to 1 decimal place.



<table>
<tr><th>Question number</th><th colspan="2">Scheme</th><th>Marks</th></tr>
<tr><td rowspan="5">6</td><td colspan="2">Correctly identifies the angle VXO where X is the midpoint of CD and O is the foot of the perpendicular from V

There are other valid triangles that can be used.</td><td>B1</td></tr>
<tr><td colspan="2">$VC = a$ and $CX = \frac{a}{2}$
note could also be 2a and a or could use values where VC is twice the length of CX</td><td>M1</td></tr>
<tr><td>$((VX)^2) = a^2 - \left(\frac{a}{2}\right)^2$</td><td>$((VO)^2) = \sqrt{a^2 - \left(\frac{\sqrt{2}}{2}a\right)^2}$</td><td>dM1</td></tr>
<tr><td>$VX = \frac{\sqrt{3}a}{2}$ oe</td><td>$VO = \frac{\sqrt{2}}{2}a$ oe</td><td>A1</td></tr>
<tr><td>$\cos\theta = \frac{\frac{a}{2}}{\frac{\sqrt{3}a}{2}}$ oe

$\left[\cos\theta = \frac{1}{\sqrt{3}}\text{oe leading to } \theta =\right]$

54.7</td><td>$\tan\theta = \frac{\frac{\sqrt{2}a}{2}}{\frac{a}{2}}$ oe

$\left[\tan\theta = \sqrt{2} \text{ leading to } \theta =\right]$

54.7</td><td>M1

A1</td></tr>
<tr><td colspan="4" align="right">Total 6 marks</td></tr>
</table>

Part	**Mark**	**Additional Guidance**
	B1	Angle identified in written work or on diagram. Allow labelling to be any letters.
	M1	Denotes any side of the pyramid with *a* and any appropriate length on the base $\frac{a}{2}$. This can be in written work or on the diagram. The two sides can be any two sides (including values) which will form a triangle with the required angle and must be used in the work that follows (even if incorrectly). Allow if the candidate denotes any side of the pyramid with *a* and identifies *AO* as $\frac{\sqrt{2}a}{2}$ oe
	dM1	Uses Pythagoras with a minus sign in a correct triangle with correctly labelled sides.
	A1	Correct expression for their correct choice of sides oe.
	M1	Working in triangle *VXO* (or other valid triangle) with their values from previous working, using any appropriate trigonometry.
	A1	Awrt 54.7

Question number	Scheme	Marks
7 (a)	$(3x - 15 =)$ 30 **or** 330 **or** 390	M1
	$x = 15$ $x = 115$ $x = 135$	M1 A1 A1 [4]
(b)	$3\dfrac{\sin y}{\cos y} + 4\sin y = 0 \rightarrow \sin y\left(\dfrac{3}{\cos y} + 4\right) = 0$	M1
	$\sin y = 0$ and $\cos y = -\frac{3}{4}$ $\rightarrow$ $y =$	A1ft
	$y = -180, 0$	A1
	$y = 138.6, -138.6$	A1
	ALT	
	$3\tan y + 4\tan y\cos y = 0 \rightarrow \tan y\,(3 + 4\cos y) = 0$	M1
	$\tan y = 0$ and $\cos y = -\frac{3}{4}$ $\rightarrow$ $y =$	A1ft
	$y = -180, 0$	A1
	$y = 138.6, -138.6$	A1 [4]
(c)	$\cos\theta = 3(1 - \cos^2\theta) - 1 \rightarrow 3\cos^2\theta + \cos\theta - 2 = 0$	M1
	$(3\cos\theta - 2)(\cos\theta + 1)$	M1
	$\theta = -180$	A1
	$\theta = 48.2, -48.2$	A1 [4]
		Total 12 marks

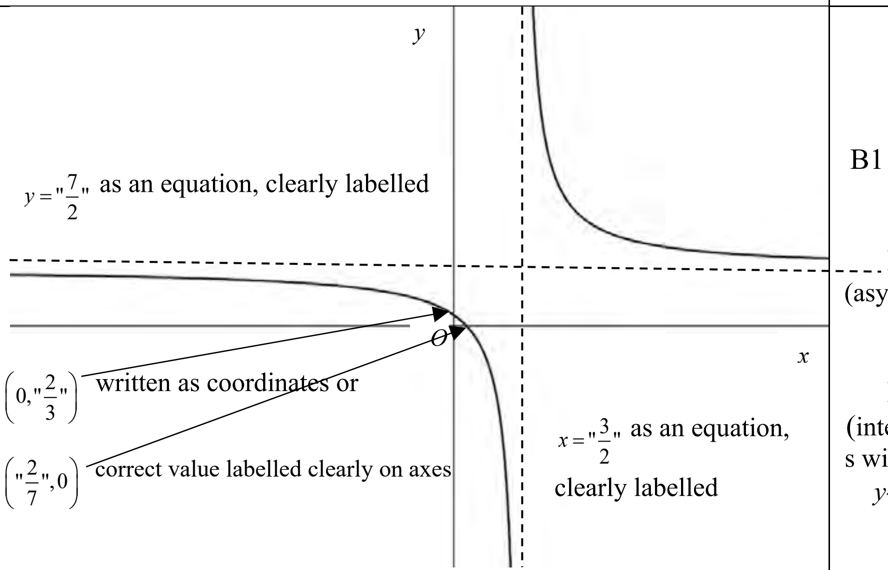
Part	Mark	Additional Guidance
(a)	M1	For any of 30 **or** 330 **or** 390 May be implied by correct answers.
	M1 A1 A1	Solves a linear equation coming from attempt at use of inverse trigonometric function to obtain one value. e.g. solves $3x - 15 = "30"$ First A1 for any correct value, second A1 for all 3 correct values and no others in the range. Ignore values outside the range.
(b)	M1	Correctly replaces the identity for tan y and attempts to deal with $\sin y$. Allow for factorising. Condone dividing through by $\sin y$. Minimally acceptable attempt for factorisation is $A\sin y\left(\dfrac{B}{\cos y}+C\right)$
	A1ft	$\sin y = 0$ and $\cos y = -\dfrac{B}{C}$, follow through their B and C **only.**
	A1	From sine: Both values, ignore extra values out of range, A0 for extra values in range.
	A1	From cosine: Both values, ignore extra values out of range, A0 for extra values in range. y = awrt 138.6, awrt – 138.6
	ALT	
	M1	Correctly replaces the identity for tan y and attempts to deal with $\tan y$. Allow for factorising. Condone dividing through by $\tan y$. Minimally acceptable attempt for factorisation is $A\tan y\,(B + C\cos y)$
	A1ft	$\tan y = 0$ and $\cos y = -\dfrac{B}{C}$, follow through their B and C **only.**
	A1	From tangent: Both values, ignore extra values out of range, A0 for extra values in range.
	A1	From cosine: Both values, ignore extra values out of range, A0 for extra values in range. y = awrt 138.6, awrt – 138.6
(c)	M1	Correctly uses the identity for $\cos^2\theta$ and rearranges to get a 3TQ Minimally acceptable attempt is $\pm 3\cos^2\theta \pm \cos\theta \pm 2 = 0$
	M1	Solves a 3TQ to arrive at 2 distinct values for $\cos\theta$. See general guidance.
	A1	For – 180, ignore extra values out of range, A0 for extra values in range.
	A1	Both values, ignore extra values out of range, A0 for extra values in range. θ = awrt 48.2, awrt – 48.2

Question number	Scheme	Marks
8 (a)(i)	$e^{3x} - 1 = 9 - 9e^{-3x}$	M1
	$(e^{3x})^2 - e^{3x} = 9e^{3x} - 9$	M1
	$\left(e^{3x}\right)^2 - 10e^{3x} + 9 = 0\,*$	A1cso*
(ii)	$(e^{3x} - 1)(e^{3x} - 9)\ [= 0]$ leading to $e^{3x} =$	M1
	$e^{3x} = 9 \quad \rightarrow \quad x = \frac{1}{3}\ln 9\,*$	A1cso* [5]
Note: subscripts on marks in (b) indicate which mark is being awarded on epen (1=1st etc)		
(b)	$\left(\int_0^{\frac{1}{3}\ln 9}(9 - 9e^{-3x})\,dx =\right)$ $9x + 3e^{-3x}$ $\left(\int_0^{\frac{1}{3}\ln 9}(e^{3x} - 1)\,dx =\right)$ $\frac{e^{3x}}{3} - x$	M1₁ A1₂A1₃
	$"9x + 3e^{-3x}" - "\frac{e^{3x}}{3} - x"$ **M1₄** $\left(10 \times \frac{1}{3}\ln 9 - \frac{9}{-3}e^{-3\times\frac{1}{3}\ln 9} - \frac{e^{3\times\frac{1}{3}\ln 9}}{3}\right) - \left(10 \times 0 - \frac{9}{-3}e^{-3\times 0} - \frac{e^{3\times 0}}{3}\right)$ **dM1₅** OR $\left(9 \times \frac{1}{3}\ln 9 + 3e^{-3\times\frac{1}{3}\ln 9}\right) - (9 \times 0 + 3e^{0})$ **and** $\left(\frac{e^{3\times\frac{1}{3}\ln 9}}{3} - \frac{1}{3}\ln 9\right) - \left(\frac{e^{0}}{3} - 0\right)$ **dM1₅** $"3\ln 9 - \frac{8}{3}" - \left("\frac{8}{3} - \frac{1}{3}\ln 9"\right)$ **M1₄**	
	$\frac{10}{3}\ln 9 - \frac{16}{3}$	A1₆
	ALT $\int (9 - 9e^{-3x}) - (e^{3x} - 1)\,dx$	M1₄
	$\left(\int (10 - 9e^{-3x} - e^{3x})\,dx =\right)\ 10x - \frac{9}{-3}e^{-3x} - \frac{e^{3x}}{3}$	M1₁ A1₂ A1₃
	$\left(10 \times \frac{1}{3}\ln 9 - \frac{9}{-3}e^{-3\times\frac{1}{3}\ln 9} - \frac{e^{3\times\frac{1}{3}\ln 9}}{3}\right) - \left(10 \times 0 - \frac{9}{-3}e^{-3\times 0} - \frac{e^{3\times 0}}{3}\right)$	dM1₅
	$\frac{10}{3}\ln 9 - \frac{16}{3}$ oe	A1₆ [6]
		Total 11 marks

Part	Mark	Additional Guidance
Mark parts (i) and (ii) together.		
(a)(i)	M1	For equating the two equations.
	M1	For multiplying through by e^{3x}, minimum of 2 out of 4 correct terms. (presence of $\pm 10e^{3x}$ indicates 2 correct terms).
	A1*cso	Correct solution only, no errors or omissions.
(ii)	M1	Minimally acceptable attempt at solving the equation leading to $e^{3x} =$ See general guidance, if the formula is quoted allow up to two slips in substitution, otherwise the substitution must be correct.
	A1*cso	Correct solution only, no errors or omissions. If 0 also included then this should be rejected.
(b)	$M1_1$	For attempt to integrate one of: $9 - 9e^{-3x}$ or $e^{3x} - 1$ or $\pm[(9 - 9e^{-3x}) - (e^{3x} - 1)]$ Limits may not be present. At least one term correct. Ignore +c if included.
	$A1_2$	For correct integration of one of the exponential terms $\pm 9e^{-3x} \rightarrow \mp \frac{9}{3}e^{-3x}$ **or** $\pm e^{3x} \rightarrow \pm \frac{1}{3}e^{3x}$ Limits need not be present. Ignore +c if included.
	$A1_3$	For correct integration of both curves $9x + 3e^{-3x}$ **and** $\frac{e^{3x}}{3} - x$ or for a **fully correct** integration where the difference between two expressions is found $\pm\left(10x - \frac{9}{-3}e^{-3x} - \frac{e^{3x}}{3}\right)$ or $\pm\left(9x - \frac{9}{-3}e^{-3x} - \frac{e^{3x}}{3} + x\right)$ **Note: this is an M mark in epen**
	$M1_4$	For the difference between the two expressions either before or after integration. Allow subtraction either way around. **Note: this is an A mark in epen**
	$dM1_5$	Substitution of correct limits into their integrated expressions (limits subtracted the correct way around). Dependent on first M scored. If substituting before difference found then must substitute into both integrated expressions. May be implied by awrt 1.99 If integration is not correct then substitution must be shown.
	$A1_6$	For the correct answer oe. Must be exact value.

Question number	Scheme	Marks
9(a)	($3(3-x)^{-3} = \frac{1}{9}\left(1-\frac{x}{3}\right)^{-3}$) $a=\frac{1}{9}$ $b=\frac{1}{3}$	B1 B1 [2]
(b)	$\left(1-\frac{x}{3}\right)^{-3} =$ $\left[1+(-3)\left(-\frac{x}{3}\right)+\frac{(-3)(-4)\left(-\frac{x}{3}\right)^2}{2!}+\frac{(-3)(-4)(-5)\left(-\frac{x}{3}\right)^3}{3!}\right]$	M1 A1ft
	$\frac{1}{9}+\frac{1}{9}x+\frac{2}{27}x^2+\frac{10}{243}x^3$	A1 [3]
(c) (i)	$\frac{24}{125}=\frac{3}{(3-x)^3}$ or $\frac{125}{8}=(3-x)^3 \Rightarrow \frac{5}{2}=3-x$	
	$x=0.5$ oe	B1
	$\frac{1}{9}+\frac{1}{9}(\text{"}0.5\text{"})+\frac{2}{27}(\text{"}0.5\text{"})^2+\frac{10}{243}(\text{"}0.5\text{"})^3$ (=0.19033)	B1ft
(ii)	$\pm\left(\frac{\frac{24}{125}-\text{"}0.19033\text{"}}{\frac{24}{125}}[\times 100]\right)$ oe	M1
	0.87% or −0.87%	A1 [4]
		Total 9 marks

Part	Mark	Additional Guidance
(a)	B1	Correct a, can be left embedded
	B1	Correct b, can be left embedded
(b)	M1	An attempt to use the binomial expansion for their $(1-bx)^{-3}$ The minimally acceptable attempt is as follows: • The power of x must be correct in each term. • The first term is 1 (or "$\frac{1}{9}$" × 1 ...) • The 2!, 3! are correct (may be unevaluated) • Their "$-\frac{x}{3}$" must appear in at least one term of the expansion. a does not need to be present to attain this mark.
	A1ft	Any two (unsimplified) algebraic terms fully correct in their expansion. Follow through their value for b. a does not need to be present to attain this mark.
	A1	Fully simplified correct expression.
Mark parts (i) and (ii) together. If you see $\frac{24}{125} = 8 \times \frac{3}{125}$ leading to $x = -2$ for part (c) then send to review.		
(c) (i)	B1	Correct identification of $x = 0.5$.
	B1ft	Correct use of their value of x in their expansion. If their x and / or their expansion is incorrect then must show the substitution.
(ii)	M1	Uses the correct formula, with their value from part (i) to calculate a percentage error.
	A1	0.87% or −0.87% Awrt 0.87% or awrt −0.87%

Question number	Scheme	Marks
10 (a) (i)	$x = \dfrac{3}{2}$	B1
(ii)	$y = \dfrac{7}{2}$	B1 [2]
(b)	$\left(\dfrac{2}{7}, 0\right)$ $\left(0, \dfrac{2}{3}\right)$	B1 B1 [2]
(c)	$\dfrac{7(2x-3)-2(7x-2)}{(2x-3)^2}$	M1 A1
	$\dfrac{-17}{(2x-3)^2}$ or $\dfrac{-17}{4x^2-12x+9}$	A1
	Correct conclusion	B1 [4]
	ALT – product rule	
	$7(2x-3)^{-1}+(7x-2)(-1)(2)(2x-3)^{-2}$	M1 A1
	$\dfrac{-17}{(2x-3)^2}$ or $\dfrac{-17}{4x^2-12x+9}$	A1
	Correct conclusion	B1
(d)		B1 (curve) B1ft (asymptotes) B1ft (intersections with x- and y-axes) [3]
(e)	$-\dfrac{1}{17} = "\dfrac{-17}{(2x-3)^2}"$	M1
	"$(2x-3)^2 = 17^2$" **or** "$4x^2 - 12x - 280 = 0$" oe	dM1
	$x = 10$ $y = 4$ or $(10, 4)$	A1
	$y -$ "4" $= 17(x -$ "10") **or** "4" $= -17 \times$ "10" $+ c$ leading to $c =$	M1
	$y = 17x - 166$ oe	A1

	"$17x - 166$" $= \dfrac{7x-2}{2x-3}$ $\rightarrow$ $17x^2 - 195x + 250$	M1
	$x = \dfrac{25}{17}$ $y = -141$ or $\left(\frac{25}{17}, -141\right)$	A1 [7]
		Total 18 marks

Part	Mark	Additional Guidance
	If a candidate gives **no response** to (a) and/or (b) but shows the correct answers on the graph we will award the marks. Where answers are given in (a) and/or (b) these should be marked as they stand with no reference to the graph. Ignore labelling of (i) and (ii) and mark (a) together.	
(a)(i)	B1	For $x = \frac{3}{2}$ oe
(a)(ii)	B1	For $y = \frac{7}{2}$ oe
(b)	B1 B1	First B1 for either correct, second B1 for both correct Condone if not given as coordinates e.g. $x = \frac{2}{7}$ and/or $y = \frac{2}{3}$ given
(c)	M1	Attempt the quotient rule. Numerator must be the difference of two terms (either way round) of the form $A.(2x-3) - B.(7x-2)$, A and $B > 1$. Denominator must be of the form $(2x-3)^2$
	A1	Either term on the numerator correct (either way round), dependent on previous method mark.
	A1	Obtains $\frac{-17}{(2x-3)^2}$ or $\frac{-17}{4x^2-12x+9}$
	B1	Correct conclusion based on correct working only, for example, (the numerator is a negative number and) the denominator is always positive and therefore the fraction/gradient is always negative.
	ALT – product rule	
	M1	For an attempt at Product Rule. Must be a sum of two products. Must have the form $c(2x-3)^{-1} + d(7x-2)(2x-3)^{-2}$ for constants c, d.
	A1	Either term correct, dependent on previous method mark.
	A1	Obtains $\frac{-17}{(2x-3)^2}$ or $\frac{-17}{4x^2-12x+9}$
	B1	Correct conclusion based on correct working only, for example, (the numerator is a negative number and) the denominator is always positive and therefore the fraction/gradient is always negative.
(d)	B1	Two branches drawn in the correct two "quadrants" created by the two aymptotes. Mark intention, allow poor curves, but do not allow the curve to bend back on itself or touch any asymptotes. Allow BOD if intention is for curve to run alongside asymptote but there is a slight deviation back on itself.
	B1ft	Two clearly marked asymptotes, ft their (a), labelled as described, there must be one section of the curve present, tending towards these asymptotes.
	B1ft	Two clearly labelled intersections with the axes, ft their (b), at least one section of their curve must pass through one of these intersections. Intersections must be labelled correct way around. If additional intersections seen then B0
(e)	M1	Sets their differentiated function from part (c) = $-\frac{1}{17}$
	dM1	Rearranges to get to an equation of the form shown with no denominators or a 3TQ and solves using an acceptable method to obtain $x = ….$ Dependent on previous method mark
	A1	Correct values for point A (10, 4)
	M1	Uses their values for x and y (from an attempt at working with gradient of the curve) with gradient 17 to find an equation for l (if using $y = mx + c$, must be a complete method arriving at $c =$) If correct $c = -166$.
	A1	Correct equation, any form

	M1	Sets their equation for the normal equal to the curve, makes a **correct** rearrangement to remove any denominator and forms a 3TQ Note this method mark is not dependant.
	A1	Correct exact values for x and y

Question number	Scheme	Marks
11 (a)	$(600 =)2\pi r^2 + 2\pi rh$ oe eg $(300 =)\pi r^2 + \pi rh$	M1
	$h = \frac{300-\pi r^2}{\pi r}$ oe | $\pi rh = 300 - \pi r^2$ oe	A1 cao
	$(V =)\ \pi r^2 "\left(\frac{300-\pi r^2}{\pi r}\right)"$ oe | $V = "(300-\pi r^2)"r$ oe	M1
	$V = 300r - \pi r^3$ *	A1* cso [4]
(b) (i)	$\frac{dV}{dr} = 300 - 3\pi r^2$	M1
	$0 = 300 - 3\pi r^2 \quad \rightarrow \quad r =$	M1
	$r = \sqrt{\frac{100}{\pi}}$ * cso	A1* cso
(ii)	$\frac{d^2V}{dr^2} = -6\pi r$	M1
	$\rightarrow \frac{d^2V}{dr^2} = -6\pi\sqrt{\frac{100}{\pi}}$ or $\frac{d^2V}{dr^2} = -6\pi \times 5.6418958........$	
	When r is positive, $-6\pi r$ is negative (-106.347231…..) and therefore this value of r gives a maximum	A1 [5]
(c)	$(V =)300"\sqrt{\frac{100}{\pi}}" - \pi\left("\sqrt{\frac{100}{\pi}}"\right)^3 (= 1128.(379167))$	M1
	$p^3 = \frac{300"\sqrt{\frac{100}{\pi}}" - \pi\left("\sqrt{\frac{100}{\pi}}"\right)^3}{\frac{4}{3}\pi}$ (=269(.3806…))	dM1
	$p = 6.5$ cm	A1 [3]
		Total 12 marks

Part	Mark	Additional Guidance
(a)	M1	Correct expression for the surface area of the cylinder and an attempt to rearrange to $h =$ or $\pi rh =$ Allow errors in arithmetic but not mathematically incorrect process. πrh may be embedded, e.g. $300 = \pi r^2 + \pi rh$ becoming $300 = \pi r^2 + V$ would score M1A1M1 and may score full marks if correct final result obtained.
	A1	cao
	M1	Substitutes their expression for height or their expression for πrh into a correct expression for the volume.
	A1	cso no errors or omissions, must state $V =$
	Mark parts (i) and (ii) together.	
(b) (i)	M1	Minimally acceptable attempt at differentiation, see general guidance, no power to increase.
	M1	Places their derivative = 0 and attempts to rearrange to find r. Minimally acceptable derivative is of the form $a \pm b\pi r^2$
	A1	Correct value for r, exact value only. Must reject negative value if found, award A0 if not rejected.
(ii)	M1	Minimally acceptable attempt to differentiate their first derivative, see general guidance, no power to increase. Or testing gradients or a sketch.
	A1	Correct evaluation of second derivative or explanation of why second derivative is negative. Conclusion this value of r gives a maximum. No incorrect work.
(c)	M1	Correct substitution of their r into the expression for V.
	dM1	Attempts rearrangement using the formula for volume of a sphere to make p^3 the subject Correct order of operations applied to right hand side. Accept arithmetic slips.
	A1	$p = 6.5$ Accept awrt 6.5

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