Pleasechecktheexaminationdetailsbelowbeforeenteringyourcandidateinformation

Candidatesurname	Othernames

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Thursday 8 June 2023

Morning (Time: 2 hours)

Paper reference **4PM1/02**

Further Pure Mathematics
PAPER 2

Calculators may be used.

TotalMarks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

P72866A

N:1/1/1/1/1/

P 7 2 8 6 6 A 0 1 3 6

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\quad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\quad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\quad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Given that $\frac{a + 2\sqrt{5}}{3 - \sqrt{5}} = \frac{11 + b\sqrt{5}}{2}$ where a is an integer and b is prime,

find the value of a and the value of b
Show your working clearly.

(5)

(Total for Question 1 is 5 marks)

2 The nth term of a convergent geometric series is $8^{(1-2n)}$

Find the sum to infinity of the series.

Give your answer in the form $\frac{p}{q}$ where p and q are integers to be found.

(6)

Question 2 continued

(Total for Question 2 is 6 marks)

3

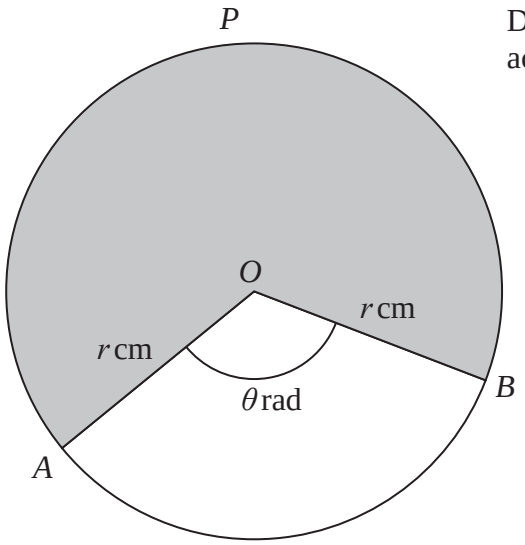


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a circle, centre O, with radius r cm.

The points A, P and B lie on the circle.

The obtuse angle $AOB = \theta$ radians.

The area of the sector $APBO$, shown shaded, is 372.4 cm^2 and the length of the arc APB is 53.2 cm.

Find, to 3 significant figures where appropriate, the value of

(i) r

(ii) θ

(6)

Question 3 continued

(Total for Question 3 is 6 marks)

4

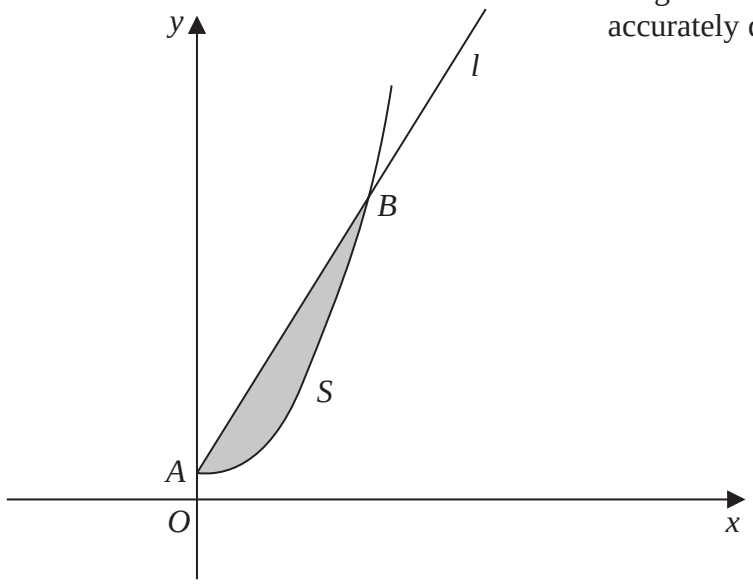


Diagram **NOT** accurately drawn

Figure 2

The curve S with equation $y = \frac{x^2}{4} + 2$ where $x \geqslant 0$ and the line l with equation $2y - x - 4 = 0$ where $x \geqslant 0$ intersect at the points A and B, as shown in Figure 2.

(a) (i) Show that the coordinates of point A are $(0, 2)$

(ii) Find the coordinates of the point B

(4)

The finite region bounded by S and l, shown shaded in Figure 2, is rotated through 2π radians about the y-axis.

(b) Use algebraic integration to find the volume of the solid generated.
Give your answer in terms of π

(4)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 8 marks)

5 (a) On the grid opposite draw the line with equation

(i) $y = 2x + 5$ (ii) $4y = x - 8$ (iii) $5y + 3x = 30$

(3)

(b) Show, by shading, the region R defined by the inequalities

$y \leqslant 2x + 5$ $4y \geqslant x - 8$ $5y + 3x \leqslant 30$

(1)

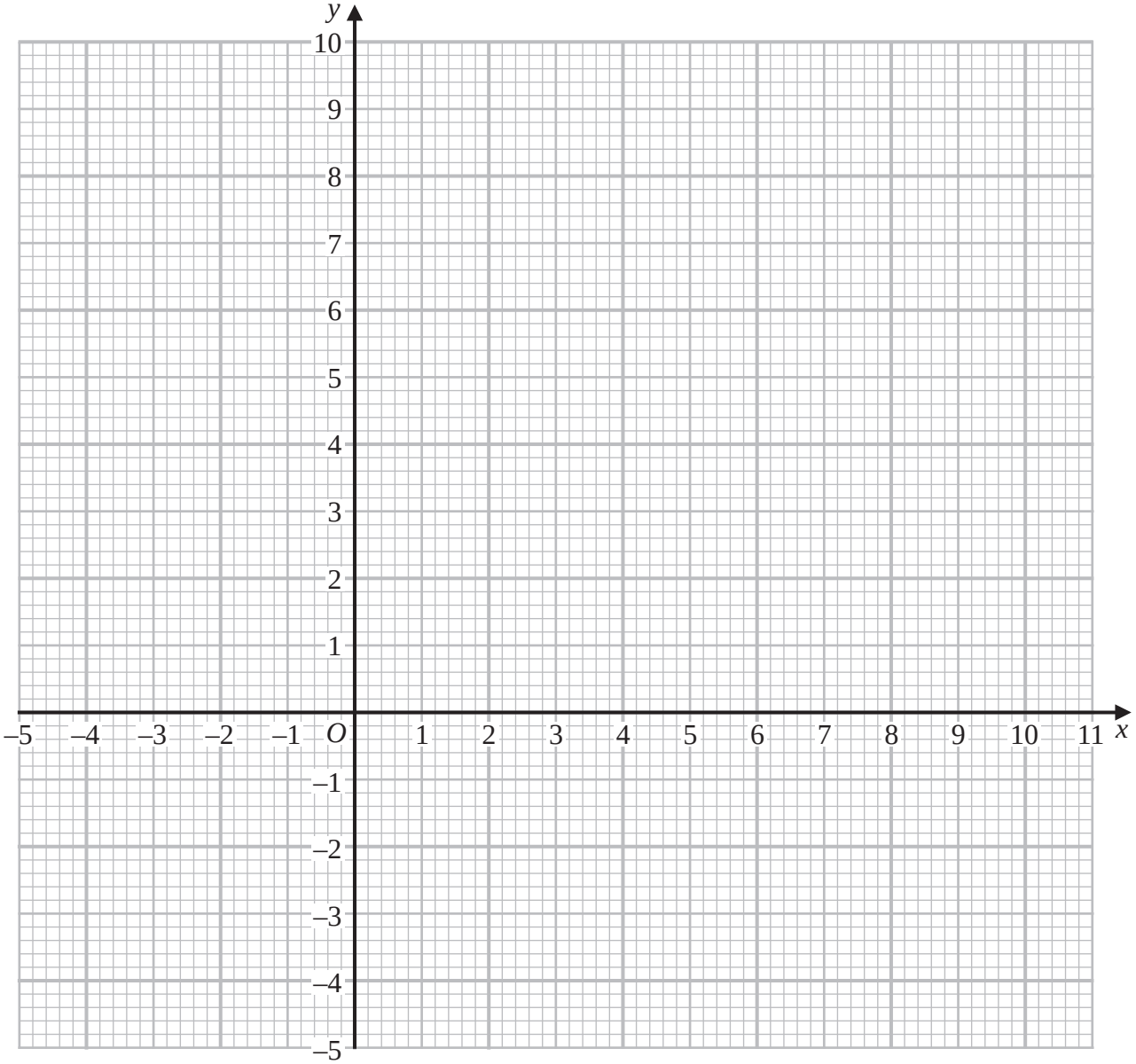
For all points in R with coordinates (x, y)

$$P = 2x - 5y$$

(c) Using your graph, find the least value of P

(3)

Question 5 continued

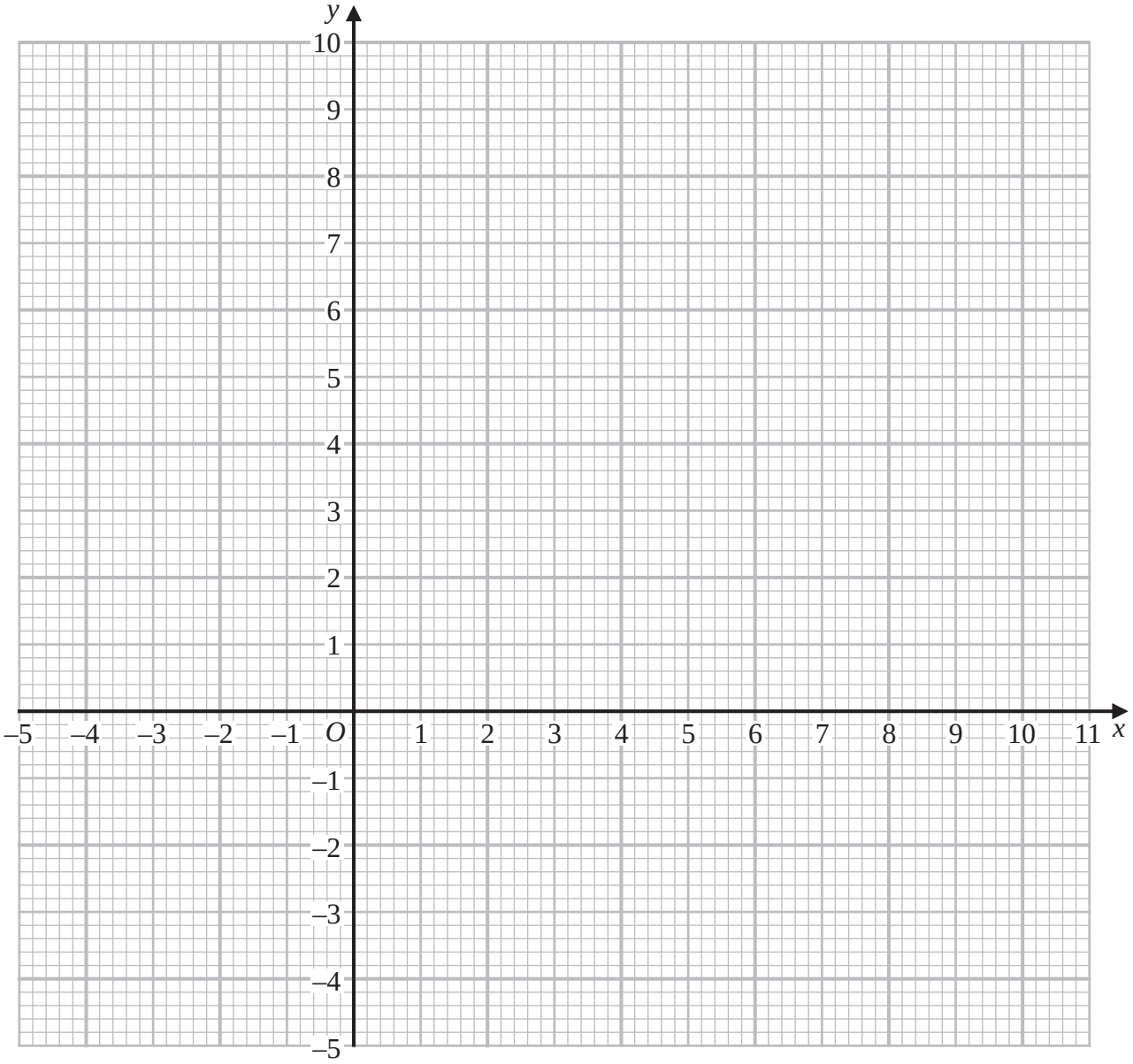


Turn over for a spare grid if you need to redraw your graph.

Question 5 continued

Question 5 continued

Only use this grid if you need to redraw your graph.



(Total for Question 5 is 7 marks)

6

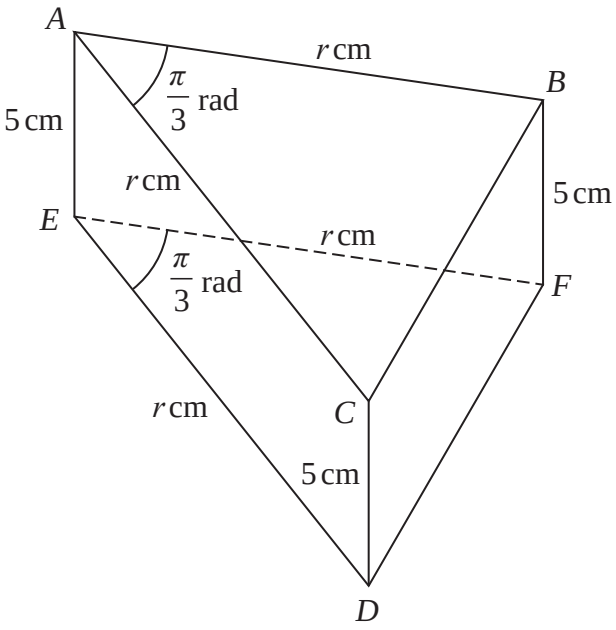


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a right triangular prism *ABCDEF*. A cross section *ABC* of the prism is a triangle in which $AB = AC = r$ cm and $\angle CAB = \frac{\pi}{3}$ radians.

In the prism

$$AE = BF = CD = 5\,\text{cm} \qquad ED = EF = r\,\text{cm} \text{ and } \angle DEF = \frac{\pi}{3} \text{ radians}$$

(a) Show that the volume of the prism is $\frac{5\sqrt{3}}{4} r^2$ cm^3

(1)

The volume of the prism is increasing in such a way that the size of $\angle CAB$ and the size of $\angle DEF$ remain constant and the length of *AE*, the length of *BF* and the length of *CD* remain constant.
The lengths of *AB*, *AC*, *ED* and *EF* are each increasing at a constant rate of 0.2 cm/s

(b) Find the exact rate of increase, in cm^3/s, of the volume of the prism when the area of the rectangular face *BCDF* is 60 cm^2

(5)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 6 marks)

7 (a) Expand $\left(1+\frac{x}{3}\right)^{-3}$ in ascending powers of x up to and including the term in x^3

Where appropriate express each coefficient as an exact fraction in its lowest terms.

(3)

(b) Write down the range of values of x for which your expression is valid.

(1)

(c) Express $(3+x)^{-3}$ in the form $P(1+Qx)^{-3}$ where P and Q are rational numbers whose values should be stated.

(2)

$$f(x) = \frac{(1+4x)}{(3+x)^3}$$

(d) Obtain a series expansion for $f(x)$ in ascending powers of x up to and including the term in x^2

(2)

(e) Hence, using algebraic integration, obtain an estimate of $\int_0^{0.2} f(x)\,dx$

Give your answer to 5 significant figures.

(3)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 11 marks)

8 The points A and B have coordinates $(-6, 8)$ and $(12, 2)$ respectively.

(a) Find an equation of the straight line passing through A and B in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

(b) Find the exact length of AB

(2)

The point X with coordinates (m, n) lies on AB such that $AX : XB = 1 : 2$

(c) Find the value of m and the value of n

(2)

The line L passes through the point X and is perpendicular to AB
The point C with coordinates (p, q) lies on L where $p > 0$ and $q > 0$

Given that AB is a diameter of a circle and C also lies on the circumference of the circle,

(d) find

(i) the exact value of p

(ii) the exact value of q

(7)

(e) Find the exact area of triangle ABC

(3)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 17 marks)

9

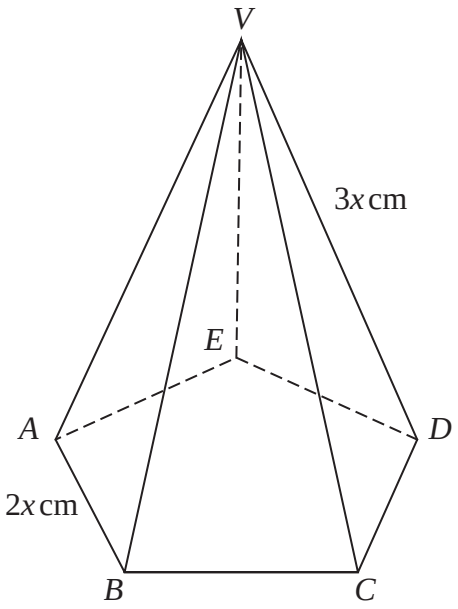


Diagram **NOT** accurately drawn

Figure 4

Figure 4 shows a right pyramid with vertex V and base $ABCDE$ which is a regular pentagon.

$$AB = BC = CD = DE = EA = 2x \text{ cm}$$
$$VA = VB = VC = VD = VE = 3x \text{ cm}$$

Find, in degrees to one decimal place, the size of the angle between the plane VBC and the base $ABCDE$

(6)

Question 9 continued

(Total for Question 9 is 6 marks)

10 The curve C with equation $y = \dfrac{6 - 3x}{x - 4}$ where $x \neq 4$, crosses the x-axis at the point P and the y-axis at the point Q

(a) Find the coordinates of

(i) P (ii) Q

(2)

(b) Write down an equation of the asymptote to C which is

(i) parallel to the y-axis (ii) parallel to the x-axis

(2)

(c) Sketch C showing clearly the asymptotes and the coordinates of the points P and Q

(3)

The line L is the normal to C at the point on C where $x = 2$

(d) Find an equation of L

(6)

The line L intersects C again at the point R

(e) Find the x coordinate of R

(3)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 16 marks)

11 The roots of a quadratic equation E are α and β where $\alpha > \beta > 0$

Given that $\alpha - \beta = 2\sqrt{6}$ and $\alpha^2 + \beta^2 = 30$

(a) show that

(i) $\alpha\beta = 3$

(4)

(ii) $\alpha + \beta = 6$

(2)

(b) Without solving E

(i) find the value of $\alpha^4 + \beta^4$

(2)

(ii) find the exact value of $\alpha^4 - \beta^4$

(2)

Given that $\alpha^4 = P + Q\sqrt{6}$ where P and Q are positive integers,

(c) find the value of P and the value of Q

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 11 continued

Question 11 continued

(Total for Question 11 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS