Mark Scheme (Results)

November 2024

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1) Paper 02

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission
  - cas – Correct answer scores full marks (unless from obvious incorrect working)
  - wr working required

- **No working**

  If no working is shown then correct answers normally score full marks

  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

  If there is a wrong answer indicated on the answer line always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.

  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.

  If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used.

  If there is no answer on the answer line then check the working for an obvious answer.

- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.

It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking
(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:
$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to $x=....$
$(ax^2+bx+c)=(mx+p)(nx+q)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=....$

2. Formula:
Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:
$x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=....$

Method marks for differentiation and integration:

1. Differentiation
Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

2. Integration:
Power of at least one term increased by 1. $(x^n\to x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the is rule may allow the mark to be awarded before the final answer is given.

Question number	Scheme	Marks
1 (a)	$\frac{1}{2}2^2\theta = \frac{\pi}{2} \Rightarrow \theta =$	M1
	$\theta = \frac{\pi}{4}$	A1 [2]
(b)	$2\times"\frac{\pi}{4}"\left(=\frac{\pi}{2}\right)$	M1
	$"\frac{\pi}{2}"+2+2$ or $"\frac{\pi}{2}"+2\times 2$ or $"\frac{\pi}{2}"+4$	M1
	$4+\frac{\pi}{2}$ oe	A1 [3]
		Total 5 marks

Part	Mark	Additional Guidance
(a)	M1	Correct use of formula and rearrangement to give θ. Allow one error in rearrangement. Allow work in degrees for this mark.
	A1	Correct exact value in radians. **isw**
(b)	M1	Correct use of formula for length of arc with their value for θ
	M1	Their arc length + 2 + 2
	A1	Correct exact value – allow work in degrees throughout. **Do not isw** Note: 5.6 is the correct estimated value, which implies M1M1A0

Question number	Scheme	Marks
2	$w(w+2)$ or $w+w+2+w+w+2$ oe	B1
	$w^2+2w>8$ or $4w+4<30$ oe	B1
	$w<\frac{13}{2}$ oe	B1
	$w^2+2w-8\{=0\} \Rightarrow (w-2)(w+4)\{=0\} \Rightarrow w=...$	M1
	cv: $w=2$	A1
	$2<w<\frac{13}{2}$ oe	B1ft [6]
	ALT Works in terms of the length where $l=w+2$ or $w=l-2$	
	$l(l-2)$ or $l+l-2+l+l-2$ oe	B1
	$l^2-2l>8$ or $4l-4<30$ oe	B1
	$\left(l<\frac{34}{4}\right) \Rightarrow w<\frac{13}{2}$ oe	B1
	$l^2-2l-8\{=0\} \Rightarrow (l-4)(l+2)\{=0\} \Rightarrow l=...$	M1
	$l=4$	A1
	$\left(4<l<\frac{17}{2}\right) \Rightarrow 2<w<\frac{13}{2}$ o.e. in terms of *w* only.	B1ft [6]
		Total 6 marks

Mark	**Additional Guidance** **Allow working in other variables, such as *x* for all marks except the final A mark**
B1	For either correct expression shown.
B1	For either correct inequality shown (can be implied by a fully correct answer).
B1	For $w < \frac{13}{2}$, allow = or any inequality
M1	Correct method to solve their 3TQ and finds at least one critical value of *w* (see general guidance for minimally acceptable attempt to solve).
A1 (M1 on ePen)	The correct positive cv solution ($w = 2$ or $w > 2$ or $w < 2$) from a correct 3TQ inequality or 3TQ equation, could be implied by their correct final answer, ignore the negative critical value even if it is incorrect.
B1ft (A1 on ePen)	Correctly combine their solutions from their linear and quadratic inequalities. Follows through their $\frac{13}{2}$ with their 2, both *w* values must be positive. Must be in terms of *w*.
ALT works in terms of *l*	
B1	For either correct expression shown
B1	For either correct inequality shown (can be implied by a fully correct answer)
B1	For $\frac{13}{2}$ in terms of *w* only UNLESS the final inequality is given as $2 < w < \frac{13}{2}$ in which case score B1 here.
M1	Correct method to solve their 3TQ equation or inequality and finds at least one critical value of *w* (see general guidance for minimally acceptable attempt to solve).
A1 (M1 on ePen)	For a value of $l = 4$ coming from a correct 3TQ Ignore the negative value even if it is incorrect.
B1ft (A1 on ePen)	Correctly combine their solutions from their linear and quadratic inequalities. Follows through their $\frac{13}{2}$ with their 2, both *w* values must be positive. Must be in terms of *w*

Question number	Scheme	Marks
3	$4\times 2(2x-1)^3 e^{3x} + 3e^{3x}(2x-1)^4$ *oe*	M1A1A1
	$4\times 2(2(1)-1)^3 e^3 + 3e^3(2(1)-1)^4$	dM1
	$11e^3$ cao	A1
		Total 5 marks

Part	Mark	Additional Guidance
	M1	Use of product rule to give an expression of the form $pe^{3x}(2x-1)^3 + qe^{3x}(2x-1)^4$. There must be a + sign between terms. $p > 0$, $q > 0$ If they have expanded $(2x-1)^4$ first, look for the form $e^{3x}(ax^3 + bx^2 + cx + d) + qe^{3x}(2x-1)^4$, where $q > 0$, and a, b, c, d are nonzero constants FYI, the correct expanded form of $(2x-1)^4$ is $16x^4 - 32x^3 + 24x^2 - 8x + 1$, and its correct derivative is $64x^3 - 96x^2 + 48x - 8$, so you might see the correct $\frac{dy}{dx}$ as $\frac{dy}{dx} = e^{3x}(64x^3 - 96x^2 + 48x - 8) + 3e^{3x}(2x-1)^4$
	A1	Either term correct.
	A1	Both terms correct.
	dM1	Substitution of $x = 1$ into their derivative. If no explicit substitution seen, provided their derivative is correct, we award this mark if awrt 221 seen. Dependent on previous method mark
	A1	Correct exact value, cao

<table>
<tr><th>Question number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>4 (a)</td><td>
<table>
<tr><td>x</td><td>0.25</td><td>0.5</td><td>1</td><td>1.5</td><td>2</td><td>2.5</td><td>3</td></tr>
<tr><td>y</td><td>−0.55</td><td>−0.20</td><td>−0.30</td><td>−0.60</td><td>−0.96</td><td>−1.35</td><td>−1.77</td></tr>
</table>
</td><td>B1

B1
[2]</td></tr>
<tr><td>(b)</td><td>Points plotted.

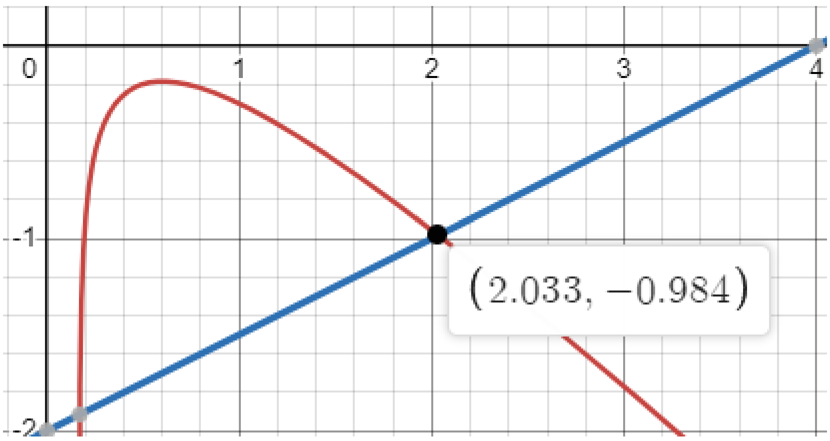
Smooth curve.</td><td>B1ft

B1ft
[2]</td></tr>
<tr><td>(c)</td><td>$\frac{3x-4}{2}=\log_{10}(6x-1)$

$\frac{x}{2}-2=\log_{10}(6x-1)-x$

Draws the line $y=\frac{x}{2}-2$

2.0 / 2.1


</td><td>M1

M1

M1

A1
[4]</td></tr>
<tr><td colspan="3" align="right">Total 8 marks</td></tr>
</table>

Part	Mark	Additional Guidance
(a)	B1	For 2 values correct. 2dp only.
	B1	For all 4 values correct. 2dp only.
(b)	B1ft	For all 7 points plotted within half a square ft their values from the table.
	B1ft	For points joined with a smooth curve ft all their 7 points plotted, their points do not have be plotted correctly.
(c)	M1	Correctly takes logs to give the expression shown. Accept $\frac{3x-4}{2}=\log(6x-1)$
	M1	Rearranges to give an equation of the form $\pm kx-2=\log_{10}(6x-1)-x$ (their left-hand side can be unsimplified equivalent)
	M1	Draws a line of the form $y=\pm kx-2$ oe, correctly.
	A1	Accept answer given as 2.0 or 2.1
ALT (c)	M1	Correctly rearranges $y=\log_{10}\left(6x-1\right)-x$ to $10^{y+x}=6x-1$
	M1	Equates $y+x$ to $\frac{3x-4}{2}$, and obtains $y=\pm kx-2$ (can be unsimplified)
	M1	Draws a line of the form $y=\pm kx-2$ oe, correctly. There must be at least one intersection point with the curve.
	A1	Accept answer given only as either 2.0 or 2.1, Do not allow 2

Question number	Scheme	Marks
5	$\left\{\frac{dV}{dh}\right\}18h^2$	M1
	$\frac{dV}{dt}=(\pm)36$	B1
	$384=6h^3 \Rightarrow h^3=64$ or $h=4$	B1
	$\left\{\frac{dh}{dt}=\frac{dh}{dV}\times\frac{dV}{dt}\right\}$	
	$\left\{\frac{dh}{dt}\right\}=\frac{1}{18\times 4^2}\times(\pm)36$ oe	M1
	$=-\frac{1}{8}$ oe	A1
		Total 5 marks

Part	Mark	Additional Guidance
	M1	Correct differentiation, $18h^2$ seen award the mark
	B1	For $\frac{dV}{dt}=-36$ or 36 correctly stated explicitly or used implicitly in a chain rule.
	B1	For $h^3=64$ or $h=4$ or $h=\sqrt[3]{\frac{384}{6}}$
	M1	For correctly substituting their derivative and their numbers into a correct chain rule, implied or stated.
	A1	For $-\frac{1}{8}$ oe e.g. stating the height is decreasing at the rate of $\frac{1}{8}$ is also correct

Question number	Scheme	Marks
6 (i)	$5(2\log_b 3 + \log_b 3) = 3$	M1
	$\left(3\log_b 3 = \frac{3}{5}\right) \Rightarrow \log_b 3 = \frac{1}{5}$	M1
	$b^{\frac{1}{5}} = 3$	M1
	$b = 243$ or 3^5	A1
	ALT	
	$5\log_b 27 = 3$	M1
	$\log_b 27 = \frac{3}{5}$	M1
	$b^{\frac{3}{5}} = 27$	M1
	$b = 243$ or $b = 27^{\frac{5}{3}}$	A1 [4]
(ii)	$3\log_3 x + 3\frac{\log_3 27}{\log_3 x} =, 28$	M1,B1
	$3(\log_3 x)^2 + 9 = 28\log_3 x$	dM1
	$3(\log_3 x)^2 - 28\log_3 x + 9(=0) \Rightarrow (3\log_3 x - 1)(\log_3 x - 9)(=0)$	ddM1
	$(\log_3 x =)\frac{1}{3}$ and $(\log_3 x =)9$	A1
	$x = 3^{\frac{1}{3}}, 3^9$ oe exact form	M1 A1 [7]
		Total 11 marks

Part	Mark	Additional Guidance
(i)	M1	Rewrites 9 as 3^2 and correctly uses the power log rule as shown.
	M1	Collects terms as rearranges to give $\log_b p = r$, allow 1 error in rearrangement.
	M1	Correctly converts their log equation to an exponential.
	A1	Cao
ALT	M1	Uses the log addition law correctly to combine into a single log.
	M1	Collects terms as rearranges to give $\log_b p = r$, allow 1 error in rearrangement.
	M1	Rearranges to the form $b^{\frac{p}{q}} = r$, allow 1 error in rearrangement.
	A1	Cao
(ii)	B1	Replacement of $8\log_4 128 = 28$, wherever we see
	M1	For a correct change of the same base of logs on the lhs of the equation e.g. $3\log_3 x + 3\frac{\log_3 27}{\log_3 x} = 28$ or $\frac{3}{\log_x 3} + 3\log_x 27 = 28$ Allow slips on the coefficients
	dM1	For forming a 3-term quadratic in any form e.g. $3(\log_3 x)^2 + 9 = 28\log_3 x$ or $3 + 9(\log_x 3)^2 = 28(\log_x 3)$
	ddM1	For solving their 3-term quadratic using a valid method – see general guidance. (Correct answers from a correct 3TQ scores this mark)
	A1	For $(\log_3 x =)\frac{1}{3}$ and $(\log_3 x =)9$ or $(\log_x 3) = 3$ and $(\log_x 3) = \frac{1}{9}$
	M1	For correctly rearranging either of their equations, removes logs to find $x =$
	A1	For both correct values oe exact form. E.g. $x = 19683, x = \sqrt[3]{3}$ **isw**

Question number	Scheme	Marks
7 (a)	$64\left(\frac{1}{4}\right)^3 - 64\left(\frac{1}{4}\right)^2 + 3 = 0$ so $(4x-1)$ is a factor	M1 A1 [2]
(b)	$4x-1\overline{)64x^3-64x^2\{+0x\}+3}$ with quotient $16x^2-12x+\text{"}-3\text{"}$ $64x^3-16x^2$ $-48x^2$ or $64x^3 - 64x^2 + 3 = (4x-1)(16x^2-12x+C)$	M1
	$x = \frac{--12 \pm \sqrt{(-12)^2 - 4(16)(-3)}}{2\times16}$	dM1
	$x = \frac{1}{4}$ oe	B1
	$x = \frac{3 \pm \sqrt{21}}{8}$ oe	A1 [4]
(c)	$ar^2 = 9$ or $\frac{a}{1-r} = 192$ oe	B1
	$(192-192r)r^2 = 9$ oe	M1
	$192r^2 - 192r^3 = 9 \Rightarrow 64r^3 - 64r^2 + 3 = 0$	A1*cso [3]
(d)	$r = \frac{1}{4}$	B1 [1]
(e)	$a\left(\frac{1}{4}\right)^2 = 9$ or $\frac{a}{1-\frac{1}{4}} = 192$ $\Rightarrow a = ...$	M1
	$a = 144$	A1*cso [2]
(f)	$\frac{144\left(1-0.25^n\right)}{1-0.25} > 191.9$	M1
	$1 - 0.25^n > \frac{1919}{1920}$ $0.25^n < 0.00052083...$	dM1
	$n > \frac{\log 0.00052083}{\log 0.25}$	M1
	$n > 5.45... \Rightarrow n = 6$	A1 [4]
		Total 16 marks

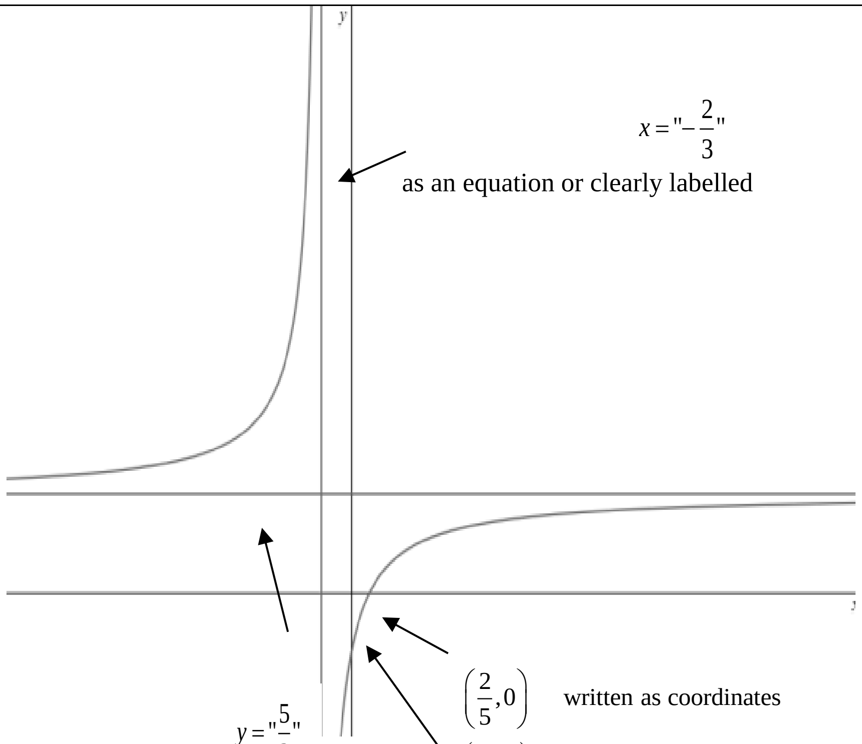
Part	Mark	Additional Guidance
(a)	M1	For $64\left(\pm\frac{1}{4}\right)^3-64\left(\pm\frac{1}{4}\right)^2+3$
	A1	For = 0 and a conclusion stated, no errors seen. Accept minimal conclusion.
(b)	M1	Attempts long division. Minimally acceptable attempt is the division and correct working as written in the scheme. Must get a 3TQ factor. If comparing coefficients, a correct equation/comparison must be written followed by an attempt to find *C*. Must get a 3TQ factor.
	dM1	For a fully correct method to solve their 3TQ to find *x*, correct solutions from a correct 3TQ implies this mark.
	B1	For correct root $\frac{1}{4}$
	A1	For correct roots $\frac{3\pm\sqrt{21}}{8}$
(c)	B1	For either correct equation.
	M1	For both correct equations and any correct step to eliminate a
	A1*cso	For a fully correct solution, no errors, with at least one correct intermediate line leading to $64r^3-64r^2+3=0$
(d)	B1 **(A1 on ePen)**	For the correct value of *r*
(e)	M1	For correct substitution of their value for *r* in either of their equations to find *a*
	A1	For $a = 144$
(f)	M1	**Allow working in equation or any inequality for all M marks** Correct use of sum to n terms formula with *a* = 144 or their *a* and their *r* to set up an equation or inequality
	dM1	Attempts to rearrange and obtains $\pm(\text{"their } r\text{"})^n\ \ldots p$
	M1	Correct use of logs and attempt to evaluate the log. (Can be implied by awrt 5.5 only)
	A1	*n* = 6 (must come from solving inequalities with no errors in processing the inequalities. If from solving equations only, award A0)

Question number	Scheme	Marks
8 (a) (i)	$(\overrightarrow{AB}) = \overrightarrow{AO} + \overrightarrow{OB}$ $(= -4\mathbf{a} - 5\mathbf{b} + 8\mathbf{a} - \mathbf{b}) = 4\mathbf{a} - 6\mathbf{b}$	M1A1 [2]
(ii)	$\left\{ \lvert \overrightarrow{AB} \rvert = \sqrt{52} \right\} = 2\sqrt{13}$	B1 [1]
(b)	$\lvert \overrightarrow{OA} \rvert = \sqrt{4^2 + 5^2} \left(= \sqrt{41}\right)$ *or* $\lvert \overrightarrow{OB} \rvert = \sqrt{8^2 + 1^2} \left(= \sqrt{65}\right)$	M1
	$\left(\sqrt{52}\right)^2 = \left(\sqrt{41}\right)^2 + \left(\sqrt{65}\right)^2 - 2\times\sqrt{41}\times\sqrt{65}\cos(AOB)$	M1
	$\left\{\text{angle } AOB = \cos^{-1}\left(\dfrac{41+65-52}{2\times\sqrt{41}\times\sqrt{65}}\right) = 58.465...\right\}$	
	Area triangle $AOB = \dfrac{1}{2}\times"\sqrt{41}"\times"\sqrt{65}"\times\sin"58.465..."$	M1
	$= 22$ awrt	A1 [4]
ALT 1	Area $= \dfrac{1}{2}\begin{vmatrix} 0 & 4 & 8 & 0 \\ 0 & 5 & -1 & 0 \end{vmatrix}$ oe	M1
	$= \dfrac{1}{2}\lvert(0\times5 + 4\times(-1) + 8\times0) - (0\times4 + 5\times8 + (-1)\times0)\rvert$ oe	M1
	$\left\{= \dfrac{1}{2}\lvert-44\rvert\right\} = 22$	M1A1 [4]
ALT 2	Use Heron's formula	
	$\lvert \overrightarrow{OA} \rvert = \sqrt{4^2 + 5^2} \left(= \sqrt{41}\right)$ *or* $\lvert \overrightarrow{OB} \rvert = \sqrt{8^2 + 1^2} \left(= \sqrt{65}\right)$	M1
	$p = \dfrac{"\sqrt{41}" + "\sqrt{65}" + "\sqrt{52}"}{2}$	M1
	Area $= \sqrt{p(p - "\sqrt{41}")\times(p - "\sqrt{65}")\times(p - "\sqrt{52}")}$	M1
	$= 22$ exact	A1 [4]
(c)	$\overrightarrow{OC} = \lambda\,\overrightarrow{OD}\,\{= \lambda(15\mathbf{a} + 10\mathbf{b})\}$	M1
	$\overrightarrow{OC} = \overrightarrow{OA} + \mu\,\overrightarrow{AB}\,\{= 4\mathbf{a} + 5\mathbf{b} + \mu(4\mathbf{a} - 6\mathbf{b}) = (4 + 4\mu)\mathbf{a} + (5 - 6\mu)\mathbf{b}\}$	M1
	$4 + 4\mu = 15\lambda$ *and* $5 - 6\mu = 10\lambda \Rightarrow \mu = ...$ *or* $\lambda = ...$	ddM1
	$\mu = \dfrac{7}{26}$ *or* $\lambda = \dfrac{22}{65}$	A1
	$\overrightarrow{OC} = \dfrac{66}{13}\mathbf{a} + \dfrac{44}{13}\mathbf{b}$ or $5\dfrac{1}{13}\mathbf{a} + 3\dfrac{5}{13}\mathbf{b}$ cao	A1 [5]
		Total 12 marks

Part	Mark	Additional Guidance
(a)(i)	M1	For a correct vector path to find $\overrightarrow{AB}$ or $\overrightarrow{BA}$ or for $\pm(-4\mathbf{a}-5\mathbf{b}+8\mathbf{a}-\mathbf{b})$
	A1	For the correct vector $4\mathbf{a}-6\mathbf{b}$
(ii)	B1	For the correct exact simplest magnitude $2\sqrt{13}$
(b)	M1	For the correct simplified or unsimplified $\lvert\overrightarrow{OA}\rvert$ or $\lvert\overrightarrow{OB}\rvert$
	M1	For correct substitution into the cosine rule. e.g. $\left(\sqrt{65}\right)^2=\left(\sqrt{41}\right)^2+\left(\sqrt{52}\right)^2-2\times\sqrt{41}\times\sqrt{52}\cos(OAB)$ or $\left(\sqrt{41}\right)^2=\left(\sqrt{52}\right)^2+\left(\sqrt{65}\right)^2-2\times\sqrt{52}\times\sqrt{65}\cos(ABO)$
	M1	For correct substitution into the area of a triangle formula, use their values. e.g. $\frac{1}{2}\times"\sqrt{41}"\times"\sqrt{52}"\times\sin"72.349..."$ or $\frac{1}{2}\times"\sqrt{65}"\times"\sqrt{52}"\times\sin"49.184..."$
	A1	awrt 22
ALT1	M1	For a correct statement for the area, must have $\frac{1}{2}$ (Start with any point, go clockwise or anticlockwise, but must repeat the first point at the end.)
	M1	For a correct evaluation of their determinant, ignore $\frac{1}{2}$
	M1	$\frac{1}{2}$ multiplies their determinant value, correct answer implies this mark.
	A1	Correct **exact** value 22, **not awrt**
ALT2	M1	For the correct simplified or unsimplified $\lvert\overrightarrow{OA}\rvert$ or $\lvert\overrightarrow{OB}\rvert$
	M1	For finding half of the perimeter
	M1	Substitute the perimeter and 3 lengths to a correct Heron's formula
	A1	Correct **exact** value 22, **not awrt**
(c)	M1	For $\overrightarrow{OC}=\lambda\overrightarrow{OD}$ (can be implied in their later work)
	M1	For correct vector statement $\overrightarrow{OC}=\overrightarrow{OA}+\mu\overrightarrow{AB}$ or $\overrightarrow{OC}=\overrightarrow{OB}+\mu\overrightarrow{BA}$ oe Can be implied for later substitution of their vectors **Please note**: $\overrightarrow{OC}=\overrightarrow{OA}+\mu\overrightarrow{BA}$ or $\overrightarrow{OC}=\overrightarrow{OB}+\mu\overrightarrow{AB}$ is also correct, provided they have found a negative value of μ
	ddM1	Equating components and reaching a value for either of their parameters. Dependent on both previous method marks
	A1	Either correct value of their parameters. They do not need to work out both values. Note: $\overrightarrow{OC}=\overrightarrow{OA}+\mu\overrightarrow{AB}\Rightarrow\mu=\frac{7}{26}$, $\overrightarrow{OC}=\overrightarrow{OB}+\mu\overrightarrow{BA}\Rightarrow\mu=\frac{19}{26}$
	A1	Correct vector in simplified expression

Question number	Scheme	Marks
9 (a)	$\cos 2\theta = \{\cos\theta\cos\theta - \sin\theta\sin\theta\} = \cos^2\theta - \sin^2\theta = \cos^2\theta - (1-\cos^2\theta)$ $= 2\cos^2\theta - 1$ *	M1 A1*cso [2]
(b)	$\int_{\frac{\pi}{3}}^{\frac{3\pi}{4}} \cos 2\theta \{\mathrm{d}\theta\}$	M1
	$\left[\frac{\sin 2\theta}{2}\right]_{\frac{\pi}{3}}^{\frac{3\pi}{4}} = \frac{\sin 2\left(\frac{3\pi}{4}\right)}{2} - \frac{\sin 2\left(\frac{\pi}{3}\right)}{2} = -\frac{2+\sqrt{3}}{4}$	M1M1A1 [4]
(c)	$2\cos^2\theta - 1 = -\cos\theta$	M1
	$2\cos^2\theta + \cos\theta - 1 = 0 \Rightarrow (2\cos\theta - 1)(\cos\theta + 1) = 0 \Rightarrow \cos\theta = ...$	M1
	$\theta = \frac{\pi}{3}$	A1
	$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} -\cos\theta \,\mathrm{d}\theta$	M1
	$= [-\sin\theta]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$	M1
	$= -\sin\frac{\pi}{2} - \left(-\sin\frac{\pi}{3}\right) \quad \left\{= \frac{-2+\sqrt{3}}{2}\right\}$	M1
	$\frac{2+\sqrt{3}}{4} - \frac{2-\sqrt{3}}{2}$	ddM1
	$\frac{-2+3\sqrt{3}}{4}$	A1 [8]
ALT for last 5 marks	$\int_{"\frac{\pi}{3}"}^{\frac{\pi}{2}} (-\cos\theta - \cos 2\theta)\{d\theta\}\{+\}\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} (-\cos 2\theta)\{d\theta\}$	M1
	$= \left[-\sin\theta - \frac{\sin 2\theta}{2}\right]_{"\frac{\pi}{3}"}^{\frac{\pi}{2}} \{+\} \left[-\frac{\sin 2\theta}{2}\right]_{\frac{\pi}{2}}^{\frac{3\pi}{4}}$	M1
	$= \left(-\sin\frac{\pi}{2} - \frac{\sin 2\times\frac{\pi}{2}}{2}\right) - \left(-\sin\frac{\pi}{3} - \frac{\sin 2\times\frac{\pi}{3}}{2}\right)\{+\}\left(-\frac{\sin 2\times\frac{3\pi}{4}}{2}\right) - \left(-\frac{\sin 2\times\frac{\pi}{2}}{2}\right)$	M1
	$= \frac{-4+3\sqrt{3}}{4} + \frac{1}{2}$	ddM1
	$= \frac{-2+3\sqrt{3}}{4}$	A1
		Total 14 marks

Part	Mark	Additional Guidance
(a)	M1	For $\cos^2\theta - \sin^2\theta = \cos^2\theta - \left(1-\cos^2\theta\right)$ or $\cos^2\theta - \sin^2\theta = \cos^2\theta + \cos^2\theta - 1$
	A1*cso	Obtains the correct result with no errors seen. Must have $\cos 2\theta$ on the lhs in their working, does not have to be in the final line provided there is a chain of reasoning.
(b)	M1	For replacement of $2\cos^2\theta - 1$ with $\cos 2\theta$,ignore limits
	M1	For an attempt to integrate to an expression of the form $\pm\frac{\sin 2\theta}{2}$
	M1	Correct substitution of the **given** limits to a **changed** expression in terms of $\sin 2\theta$ and subtracts the correct way around. A correct value from a correctly integrated expression scores this mark, no need to see explicit substitution
	A1	Correct answer. isw
(c)	M1	Correctly equates the two curves.
	M1	Rearranges their 3TQ = 0 and attempts to solve (see general guidance). Can be implied by their correct $\cos\theta$ values, $\frac{1}{2}$ **and** –1 from a correct 3TQ, if their 3TQ is not correct, a clear method must be shown.
	A1	Correct angle found. Ignore extra angles.
	M1	**Main**: For stating the correct integral with "correct" limits of their $\frac{\pi}{3}$ and $\frac{\pi}{2}$ $\pm\int_{"\frac{\pi}{3}"}^{\frac{\pi}{2}} -\cos\theta \, d\theta$ and $\pm\int_{\frac{\pi}{3}}^{\frac{3\pi}{4}} (2\cos^2\theta - 1) d\theta$ (the later integral can be implied by later work) (their $\frac{\pi}{3}$ must be positive and < $\frac{\pi}{2}$) **ALT**: Correct integrals $\pm\int_{"\frac{\pi}{3}"}^{\frac{\pi}{2}} (-\cos\theta - \cos 2\theta)\{d\theta\}$ and $\pm\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} (-\cos 2\theta)\{d\theta\}$ oe
	M1 **A1 on Epen**	For a correct integration of **all** their integrals in $\cos 2\theta$ and/or $\cos\theta$ only. Ignore limits
	M1	For correct **explicit** substitution of their limits into **either changed** expression, subtracts the correct way around – substitution of 0 must be seen unless substitution into their changed function gives 0 Can be implied by $\pm\left(\frac{-2+3\sqrt{3}}{4}\right)$ from correct working
	ddM1	Correct strategy finding the required area: **Main**: For subtracting the modulus of their 2 integrals. oe. Either way around. **ALT**: Add the values of two positive area "$\frac{-4+3\sqrt{3}}{4}$" and "$\frac{1}{2}$" oe Dependent on previous **three** method marks
	A1	Correct answer, simplest form and exact area of R

Question number	Scheme	Marks
10 (a)	(i) $\left(\frac{2}{5}, 0\right)$ (ii) $(0, -1)$	B1 B1 [2]
(b) (i) (ii)	$y = \frac{5}{3}$ $x = -\frac{2}{3}$	B1 B1 [2]
(c)		B1 B1ft B1ft [3]
(d)	$\left\{y = \frac{1}{4}x + \frac{7}{4}\right\} \Rightarrow \frac{1}{4}$ $\left\{\frac{dy}{dx} =\right\} \frac{5(3x+2) - 3(5x-2)}{(3x+2)^2} \left\{= \frac{16}{(3x+2)^2}\right\}$ $"\frac{1}{4}" = "\frac{16}{(3x+2)^2}" \Rightarrow x = \ldots$ $x = 2$ and $y\left\{= \frac{5 \times "2" - 2}{3 \times "2" + 2}\right\} = 1$ $y - "1" = "-4"(x - "2")$ $(0, 9)$ and $\left(\frac{9}{4}, 0\right)$ $\{\lvert DE \rvert\} = \sqrt{9^2 + \left(\frac{9}{4}\right)^2} = \frac{9\sqrt{17}}{4}$	B1 M1 dM1A1 M1 A1 M1 dM1 A1ft M1A1 [11]
		Total 18 marks

Part	Mark	Additional Guidance
(a)(i)	B1	$\left(\frac{2}{5}, 0\right)$ Accept $x = \frac{2}{5}, \{y = 0\}$
(a)(ii)	B1	$(0, -1)$ Accept $\{x = 0\}, y = -1$, if not labelled, we score by their order
(b)(i)	B1	Correct equation $y = \frac{5}{3}$
(b)(ii)	B1	Correct equation $x = -\frac{2}{3}$
(c)	B1	Two branches drawn in the correct two "quadrants" created by the two asymptotes. Mark intention, allow poor curves, but do not allow the curve to bend back on itself or touch any asymptotes.
	B1ft	Two clearly labelled intersections with the axes, ft their (a), at least one section of their curve must pass through one of these intersections.
	B1ft	Two clearly marked asymptotes, ft their (b), labelled as described, there must be at least one section of the curve present, not crossing the curve.
(d)	B1	$\frac{1}{4}$ seen any where in part (d)
	M1	Attempt to differentiate y **Quotient rule**: Numerator must be the difference of two terms (either way round) of the form $P(3x+2) - Q(5x-2), \quad P, Q > 0$. Denominator must be correct. **Product rule**: obtains form $P(3x+2)^{-1} - Q(5x-2)(3x+2)^{-2}, \ P, Q > 0$
	dM1	**Quotient rule**: Either term in the numerator correct **Product rule**: Either term correct Dependent on previous method mark.
	A1	Correct $\frac{dy}{dx}$. May be unsimplified.
	M1	Sets their $\frac{1}{4}$ equal to their $\frac{dy}{dx}$ and attempts to solve for x
	A1	$x = 2$ *and* $y = 1$
	M1	Finds the gradient of the normal using $\frac{-1}{their \frac{1}{4}}$
	dM1	A complete and correct method to find the equation of the normal. If using $y = mx + c$, c must be found. Dependent on previous method mark.
	A1ft	(0,9) and $\left(\frac{9}{4}, 0\right)$, accept $y = 9$, $x = \frac{9}{4}$ Ft their normal equation.
	M1	Correct use of the formula for the length of a line segment, using their values from their points of D and E, may not come from the normal.
	A1	Correct exact length in simplest form

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