Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

Centre Number | Candidate Number

# Pearson Edexcel International GCSE

## Friday 15th November 2024

Morning (Time: 2 hours)

Paper reference **4PM1/02**

# Further Pure Mathematics
## PAPER 2

**Calculators may be used.**

Total Marks

### Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

### Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

*Turn over* ▶

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P 7 3 5 8 7 A 0 1 3 2

Pearson

# International GCSE in Further Pure Mathematics Formulae sheet

## Mensuration

**Surface area of sphere** = $4\pi r^2$

**Curved surface area of cone** = $\pi r \times$ slant height

**Volume of sphere** = $\frac{4}{3}\pi r^3$

## Series

### Arithmetic series

Sum to $n$ terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

### Geometric series

Sum to $n$ terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

### Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

## Calculus

### Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

## Trigonometry

### Cosine rule

In triangle $ABC$: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\quad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\quad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\quad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

## Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

**Answer all TEN questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

**1**

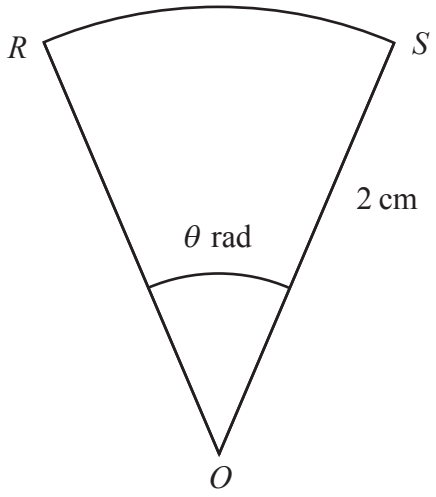



Diagram **NOT** accurately drawn

**Figure 1**

Figure 1 shows sector $ROS$ of a circle with centre $O$ and radius 2 cm
The size of angle $ROS$ is $\theta$ radians.

The area of sector $ROS$ is $\frac{\pi}{2}$ cm$^2$

(a) Find the exact value of $\theta$

**(2)**

The perimeter of sector $ROS$ is $P$ cm

(b) Find the exact value of $P$

**(3)**

**(Total for Question 1 is 5 marks)**

**2** The length of rectangle $R$ is 2 cm greater than its width.

The area of $R$ is greater than $8\,cm^2$ and the perimeter of $R$ is less than 30 cm.

Given that the width of $R$ is $w$ cm,

find the set of possible values of $w$
Give your answer in the form $a < w < b$ where $a$ and $b$ are rational numbers.

**(6)**

**Question 2 continued**

**(Total for Question 2 is 6 marks)**

**3** The curve $C$ has equation $y = e^{3x}(2x-1)^4$

Using calculus, find the exact value of the gradient of the tangent to $C$ when $x = 1$

**(5)**

**Question 3 continued**

**(Total for Question 3 is 5 marks)**

**4** (a) Complete the table of values for $y = \log_{10}(6x-1) - x$ giving your answers to 2 decimal places.

| $x$ | 0.25 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|---|---|
| $y$ | | –0.20 | –0.30 | –0.60 | | | |

**(2)**

(b) On the grid opposite, draw the graph of $y = \log_{10}(6x-1) - x$ for $0.25 \leqslant x \leqslant 3$

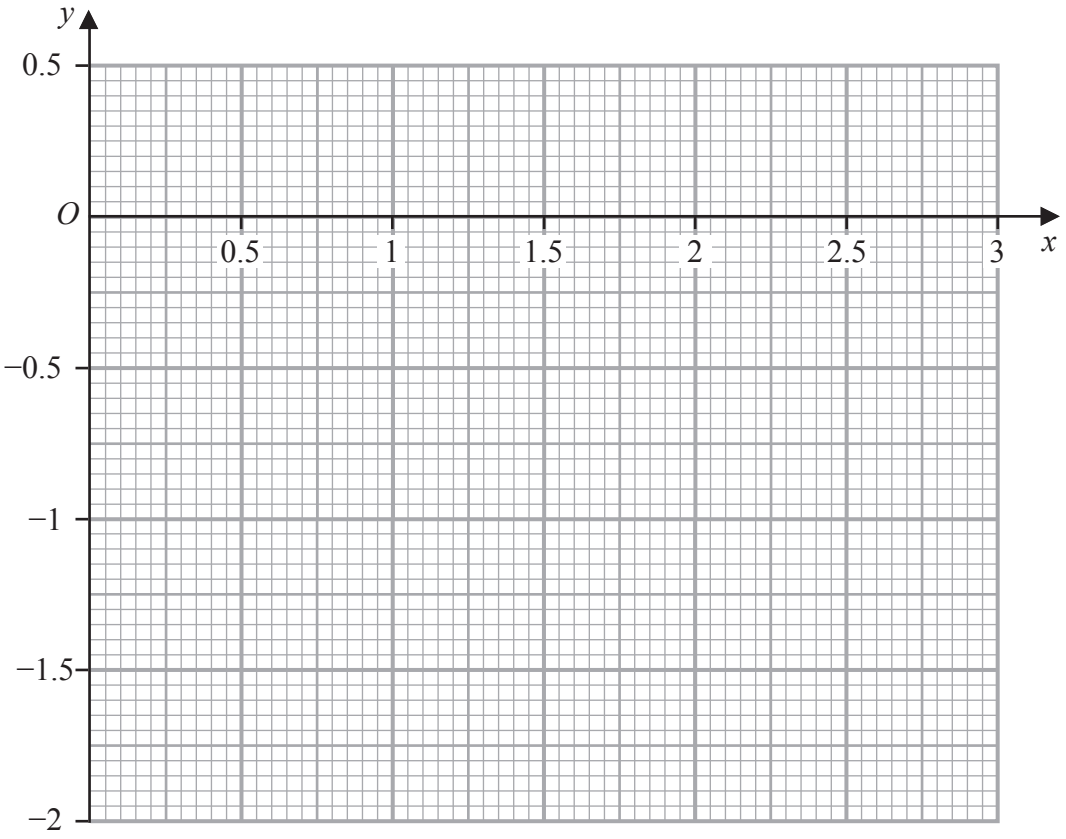
**(2)**

(c) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation

$$10^{\frac{3x-4}{2}} = 6x - 1 \quad \text{in the interval } 0.25 \leqslant x \leqslant 3$$

**(4)**

**Question 4 continued**

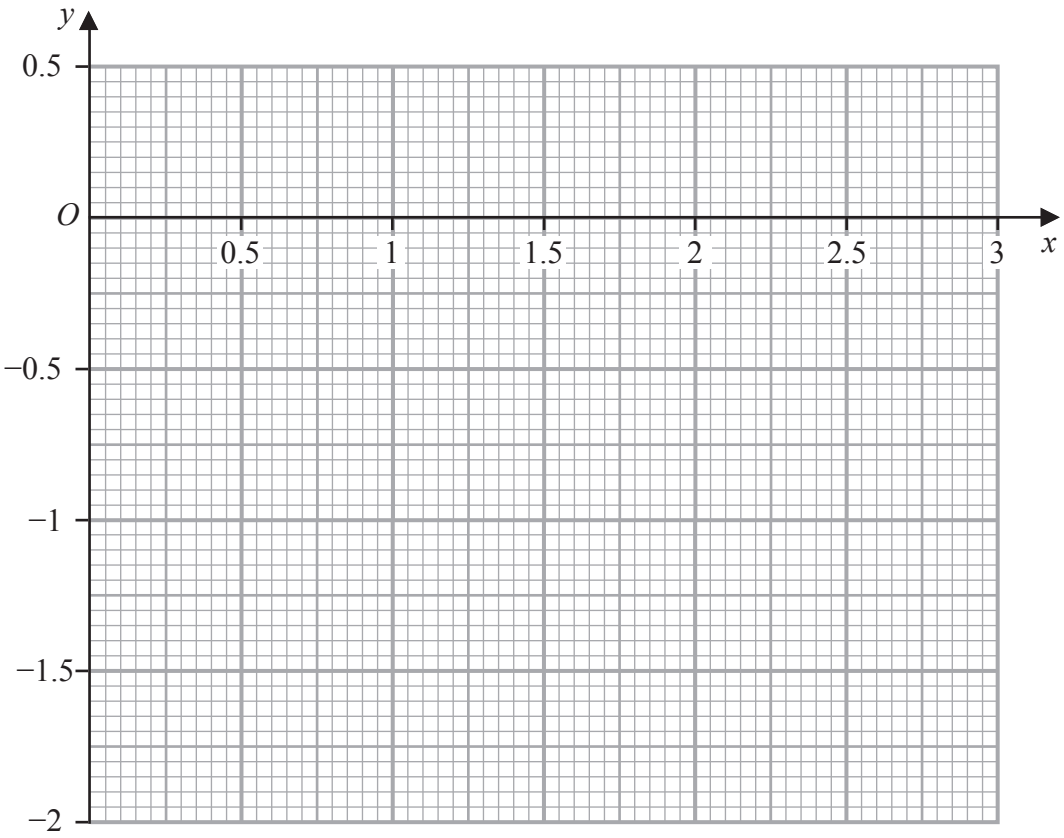



**Turn over for a spare grid if you need to redraw your graph.**

**Question 4 continued**

**Question 4 continued**

**Only use this grid if you need to redraw your graph.**




**(Total for Question 4 is 8 marks)**

**5** The height of liquid in a vessel $P$ is $h$
The volume, $V$, of the liquid in $P$ is given by $V = 6h^3$
Liquid is leaking from $P$ at a constant rate of $36\,\text{cm}^3/\text{s}$

Find the exact rate of change, in cm/s, of $h$ when $V = 384\ \text{cm}^3$

**(5)**

**Question 5 continued**

**(Total for Question 5 is 5 marks)**

**6** (i) Solve the equation $5\left(\log_b 9 + \log_b 3\right) = 3$

**(4)**

(ii) Solve the equation $3\log_3 x + 3\log_x 27 = 8\log_4 128$
Give your answers in exact form.

**(7)**

**Question 6 continued**

**(Total for Question 6 is 11 marks)**

**7** (a) Use the factor theorem to show that $(4x-1)$ is a factor of

$$f(x) = 64x^3 - 64x^2 + 3$$

**(2)**

(b) Hence, or otherwise, find the exact roots of the equation

$$f(x) = 0$$

**(4)**

A geometric series $G$ has first term $a$ and common ratio $r$
The third term of $G$ is 9 and the sum to infinity of $G$ is 192

(c) Show that $64r^3 - 64r^2 + 3 = 0$

**(3)**

Given that $r$ is a rational number

(d) write down the value of $r$

**(1)**

(e) show that $a = 144$

**(2)**

The sum to $n$ terms of $G$ is $S_n$

(f) Using logarithms, find the least value of $n$ such that $S_n > 191.9$

**(4)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**(Total for Question 7 is 16 marks)**

**8**

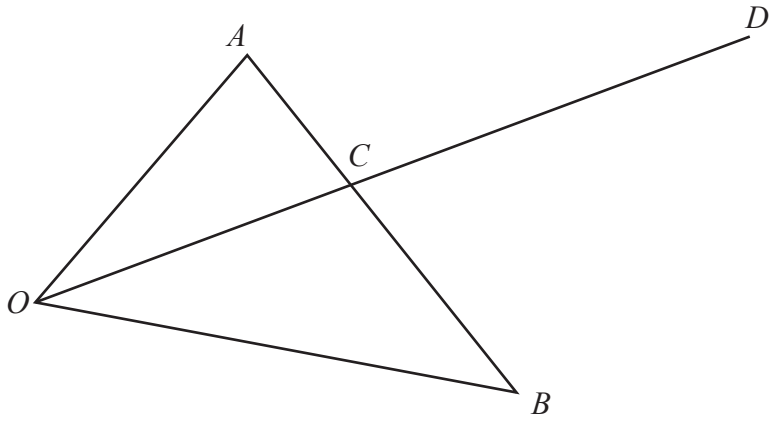



Diagram **NOT** accurately drawn

**Figure 2**

Figure 2 shows triangle $AOB$

$\overrightarrow{OA} = 4\mathbf{a} + 5\mathbf{b}$ $\overrightarrow{OB} = 8\mathbf{a} - \mathbf{b}$ $\overrightarrow{OD} = 15\mathbf{a} + 10\mathbf{b}$ where $|\mathbf{a}| = |\mathbf{b}| = 1$

(a) (i) Find $\overrightarrow{AB}$ in terms of **a** and **b**

(ii) Find, in its simplest form, the exact value of $|\overrightarrow{AB}|$

**(3)**

(b) Find the area of triangle $AOB$

**(4)**

The point $C$ lies on $AB$ and $OD$ such that $O$, $C$ and $D$ are collinear.

(c) Use a vector method to find vector $\overrightarrow{OC}$ as a simplified expression in terms of **a** and **b**

**(5)**

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**Question 8 continued**

**Question 8 continued**

**Question 8 continued**

**(Total for Question 8 is 12 marks)**

**9** (a) Using a formula given on page 2, show that

$$\cos 2\theta = 2\cos^2\theta - 1$$

**(2)**

(b) Hence show that

$$\int_{\frac{\pi}{3}}^{\frac{3\pi}{4}} \left(2\cos^2\theta - 1\right) \mathrm{d}\theta = -\frac{a+\sqrt{b}}{c}$$

where *a, b* and *c* are integers to be found.

**(4)**

Diagram **NOT** accurately drawn

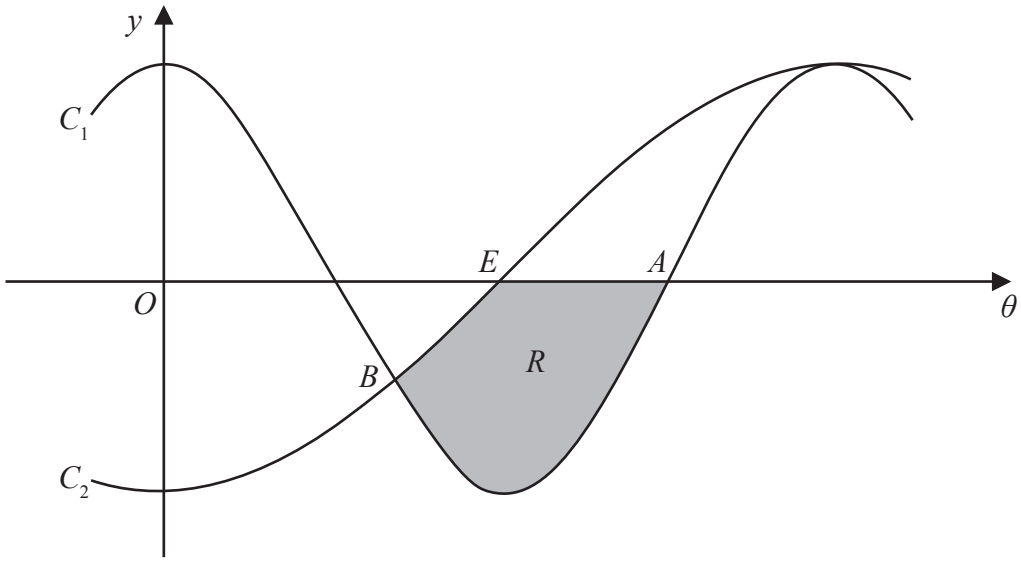



**Figure 3**

Figure 3 shows part of the curve $C_1$ with equation $y = 2\cos^2\theta - 1$ and part of the curve $C_2$ with equation $y = -\cos\theta$

Point $B$ is the intersection of $C_1$ and $C_2$ as shown in Figure 3

Point $A\left(\frac{3\pi}{4}, 0\right)$ is the intersection of $C_1$ with the $\theta$-axis as shown in Figure 3

Point $E\left(\frac{\pi}{2}, 0\right)$ is the intersection of $C_2$ with the $\theta$-axis as shown in Figure 3

The finite region $R$, shown shaded in Figure 3, is bounded by the $\theta$-axis, $C_1$ and $C_2$

(c) Use calculus to find, in its simplest form, the exact area of $R$

**(8)**

**Question 9 continued**

**Question 9 continued**

**Question 9 continued**

**(Total for Question 9 is 14 marks)**

**10** A curve $C$ has equation

$$y = \frac{5x-2}{3x+2} \qquad x \neq -\frac{2}{3}$$

(a) Find the coordinates of the point where $C$ intersects the

(i) $x$-axis
(ii) $y$-axis

**(2)**

(b) Write down an equation of the asymptote to $C$ that is

(i) parallel to the $x$-axis
(ii) parallel to the $y$-axis

**(2)**

(c) Sketch $C$ on the opposite page.
Show and label the asymptotes and the coordinates of the points where $C$ crosses the coordinate axes.

**(3)**

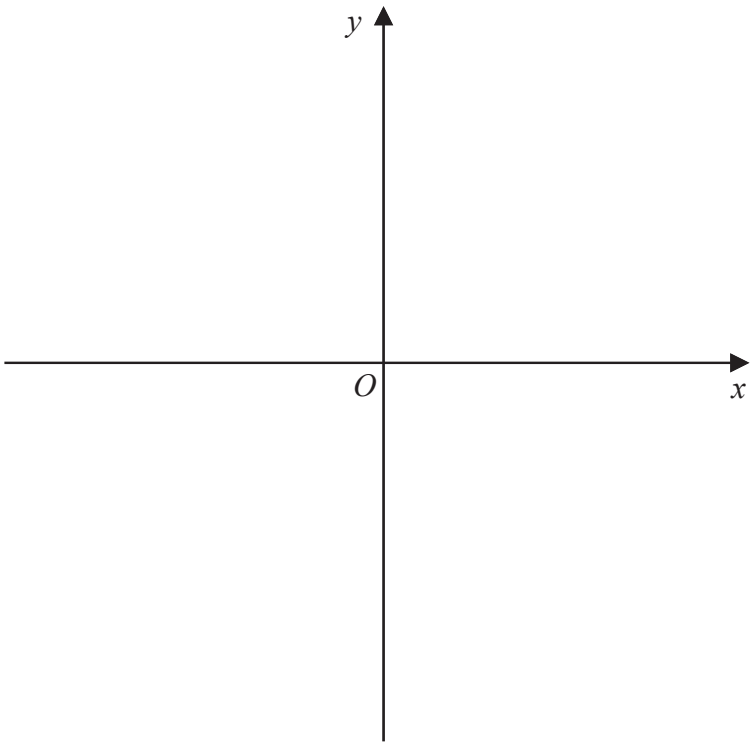
Point $A$ lies on $C$ such that the gradient of $C$ at $A$ is parallel to the line with equation $4y - x = 7$

The normal to $C$ at $A$ intersects the $x$-axis at point $D$ and the $y$-axis at point $E$
Given that the $x$ coordinate of $A$ is positive,

(d) find, in its simplified form, the exact length of line $DE$

**(11)**

**Question 10 continued**



**Question 10 continued**

**Question 10 continued**

**Question 10 continued**

**(Total for Question 10 is 18 marks)**

**TOTAL FOR PAPER IS 100 MARKS**