Pearson Edexcel

Mark Scheme (Results)

Summer 2025

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1) Paper 01R

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks – can only be awarded when relevant M marks have been gained
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - cso – correct solution only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**
  If no working is shown, then correct answers may score full marks
  If no working is shown, then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question: e.g. uses 252 instead of 255; follow through their working and deduct 2A marks from any gained provided the work has not been simplified. (Do not deduct any M marks gained.)
  If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used
  Examiners should send any instance of a suspected misread to review (but see above for simple misreads).

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: e.g. incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect e.g. algebra.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking
(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:
   $\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right)$ where $\left|pq\right|=\left|c\right|$ leading to $x=\ldots$
   $\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right)$ where $\left|pq\right|=\left|c\right|$ and $\left|mn\right|=\left|a\right|$ leading to $x=\ldots$

2. Formula:
   Attempt to use the **correct** formula (shown explicitly or implied by working) with values for a, b and c, leading to $x=\ldots$

3. Completing the square:
   $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=\ldots$

Method marks for differentiation and integration:

1. Differentiation
   Power of at least one term decreased by 1. $(x^n\to x^{n-1})$
2. Integration:
   Power of at least one term increased by 1. $(x^n\to x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either**
quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values
or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks". General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this e.g. in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - i.e. giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question number	Scheme	Marks
1	$w=\dfrac{50}{8-\sqrt{6}}\times\dfrac{8+\sqrt{6}}{8+\sqrt{6}}$	M1
	$=\dfrac{400+50\sqrt{6}}{58}$	M1
	$=\dfrac{200+25\sqrt{6}}{29}$	A1 [3]
	Total 3 marks	

Part	Mark	Additional Guidance
1	M1	For making w the subject, and applying a multiplication to rationalise the denominator $\dfrac{50}{8-\sqrt{6}}\times\dfrac{8+\sqrt{6}}{8+\sqrt{6}}$
	M1	For carrying out the multiplication of the numerator and the denominator to reach single integer values in both, allow one processing error.
	A1	For correctly manipulating the fraction to reach the correct answer.

Question number	Scheme	Marks
2	$a+2d=8 \qquad a+4d=20$	B1
	$2d=12 \Rightarrow d=$	M1
	$(a=)-4 \qquad (d=)6$	A1
	$S_n=\frac{n}{2}(2\times\text{"}-4\text{"}+(n-1)\text{"}6\text{"})(>220)$	M1
	$\left(\frac{n}{2}(-8+6n-6)>220\right)$ or $3n^2-7>220$ oe	M1
	Solution using 3TQ:	
	$6n^2-14n-440>0$ or $3n^2-7n-220>0$ oe	
	$n=\frac{7\pm\sqrt{(-7)^2-4(3)(-220)}}{2\times3} \Rightarrow n=(9.80925,-7.47592)$	M1
	$n=10$	A1 [7]
	Solution using trial and improvement:	
	$n=9$: $3\times(9)^2-7\times(9)=180<220$ $n=10$: $3\times(10)^2-7\times(10)=230>220$	[M1]
	Hence minimum value is $n=10$	[A1]
		Total 7 marks

Part	Mark	Additional Guidance
2	B1	For both correct equations
	M1	For a complete attempt to solve their 2 linear simultaneous equations, allow one processing error.
	A1	For both correct: $(a=)-4$ and $(d=)6$. Values may be implied by subsequent working.
	ALT for first 3 marks: Uses the third term	
	B1	Let the 3rd term be T $20-T=d \quad T-8=d$
	M1	Solves the above equations simultaneously by any method to find d **AND** finds a value for a
	A1	For both correct $a = -4$ and $d = 6$. Values may be implied by subsequent working
	M1	Correct substitution of their values for a and d into the correct formula for S_n (doesn't need to be placed > 220 at this point)
	M1	For the correct inequality shown with their values oe
	M1	For a full and correct method to solve their 3TQ leading to at least one value for n. Award this mark for the positive root [9.809…] without explicit working seen [use of calculator] following a correct 3TQ seen. If the 3TQ is incorrect and there is no evidence of explicit working to solve their 3TQ do not award this mark.
	A1	For choosing only positive root, and rounding to $n = 10$ If a candidate offers a positive and negative value without rejecting the negative [-7.47…] do not award this mark.
	ALT: uses T & I Final two marks	
	M1	Attempts $n = 10$ **AND** $n = 9$ When $n = 10$ $S = 230$ and when $n = 9$ $S = 180$ **These values must be seen**
	A1	For choosing $n = 10$

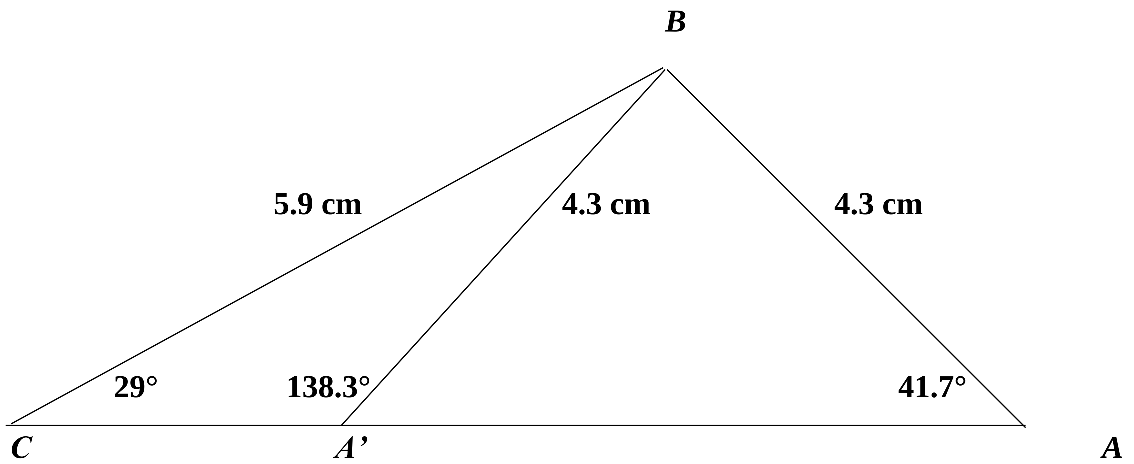
Question number	Scheme	Marks
3 (a)	$\left(v = 2t^2 - 19t + 35\right)$	
	$\left(\frac{\mathrm{d}v}{\mathrm{d}t} =\right) 4t - 19$	M1
	$4t - 19 = 0$	M1
	$t = \frac{19}{4}$	A1 [3]
(b)	$\left(v = 0 = 2t^2 - 19t + 35\right)$	
	$0 = (2t-5)(t-7)$	M1
	$T_1 = \frac{5}{2},\ T_2 = 7$	A1, A1 [3]
(c)	$\left(\int\left(2t^2 - 19t + 35\right)\mathrm{d}t\right) = \frac{2}{3}t^3 - \frac{19}{2}t^2 + 35t(+C)$ oe	M1 A1
	OR	
	$\int_{'2.5'}^{'7'}\left(2t^2 - 19t + 35\right)\ \mathrm{d}t = \left[\frac{2}{3}t^3 - \frac{19}{2}t^2 + 35t\right]_{'2.5'}^{'7'}$	
	$\left(\frac{2}{3}(7)^3 - \frac{19}{2}(7)^2 + 35(7)\right) - \left(\frac{2}{3}\left(\frac{5}{2}\right)^3 - \frac{19}{2}\left(\frac{5}{2}\right)^2 + 35\left(\frac{5}{2}\right)\right) = -\frac{243}{8}$	M1
	$d = \frac{243}{8}$	A1 [4]
		Total 10 marks

Part	Mark	Additional Guidance
3 (a)	M1	For a minimally acceptable attempt to differentiate, see general guidance.
	M1	For placing their changed expression for $a = 0$ and an attempt to solve, allow 1 processing error.
	A1	For $t = \frac{19}{4}$
(b)	M1	For setting $v = 0$ and attempting to solve, see general guidance for definition of a valid attempt to solve.
	A1	For one correct value of either T_1 or T_2
	A1	For both correct values
(c)	**Note:** **The question states 'Show your working clearly' No integration seen – award M0A0M0A0**	
	M1	For an attempt to integrate, see general guidance.
	A1	For a fully correct integration (constant of integration need not be shown).
	M1	For correct substitution of **their** T_1 and T_2 **into a changed expression AND subtracts their values correctly.** If these are not the correct values, then must see each limit correctly substituted into their expression at least once, If the correct values are used, but their final answer is not correct, explicit substitution must be seen for the award of this mark. **Note:** Mark may be implied by a correct answer given **following correct integration**
	A1	Correct answer **which must be a positive value.**

Question number	Scheme	Marks
4 (a)	$\dfrac{\sin "A"}{5.9} = \dfrac{\sin 29}{4.3}$ oe	M1
	$(A =)\sin^{-1}\left(\dfrac{\sin 29}{4.3} \times 5.9\right)$ or $\sin^{-1}("0.66520.......")$ oe	dM1
	$A = 41.7$	A1
	$A = 180 - "41.69796.........." = 138.3$	A1
		[4]
(b)	$180 - (29 + "138.30203")$ $(=12.69796........)$	M1
	$(AC)^2 = 5.9^2 + 4.3^2 - 2 \times 5.9 \times 4.3\cos(180 - (29 + "138.30203"))$ or $AC = \sqrt{5.9^2 + 4.3^2 - 2 \times 5.9 \times 4.3\cos(180 - (29 + "138.30203"))}$	M1
	$= 1.95(\text{cm})$	A1
		[3]
ALT (b)	$180 - (29 + "138.30203")$ $(=12.69796........)$	[M1]
	$\dfrac{\sin 12.7}{AC} = \dfrac{\sin 29}{4.3}$	
	$AC = 0.2198 \times \dfrac{4.3}{0.4848}$	[M1]
	$= 1.95$ (cm)	[A1]
		[3]
		Total 7 marks

Part	Mark	Additional Guidance
4 (a)	M1	For correct substitution into the sine rule, any form.
	dM1	For rearrangement to give A = , allow one processing error
	A1	For awrt 41.7
	A1	For awrt 138.3
(b)	M1	For calculating angle using $180-(29+\text{"}138.30203\text{"})$ They must use their 'larger' angle. If they have only one angle to use then this mark is M0
	M1	For correct substitution into the cosine rule, any form, allow any angle calculated from their acute or obtuse value from part (a) applied in 180 – (29 + "their angle")
	A1	For awrt 1.95 (cm)
ALT (b)	M1	For calculating angle using $180-(29+\text{"}138.30203\text{"})$ They must use their 'larger' angle. If they have only one angle to use then this mark is M0
	M1	For correct substitution into the sine rule, any form, allow any angle calculated from their acute or obtuse value from part (a) applied in 180 – (29 + "their angle")
	A1	For awrt 1.95 (cm)

Useful Sketch



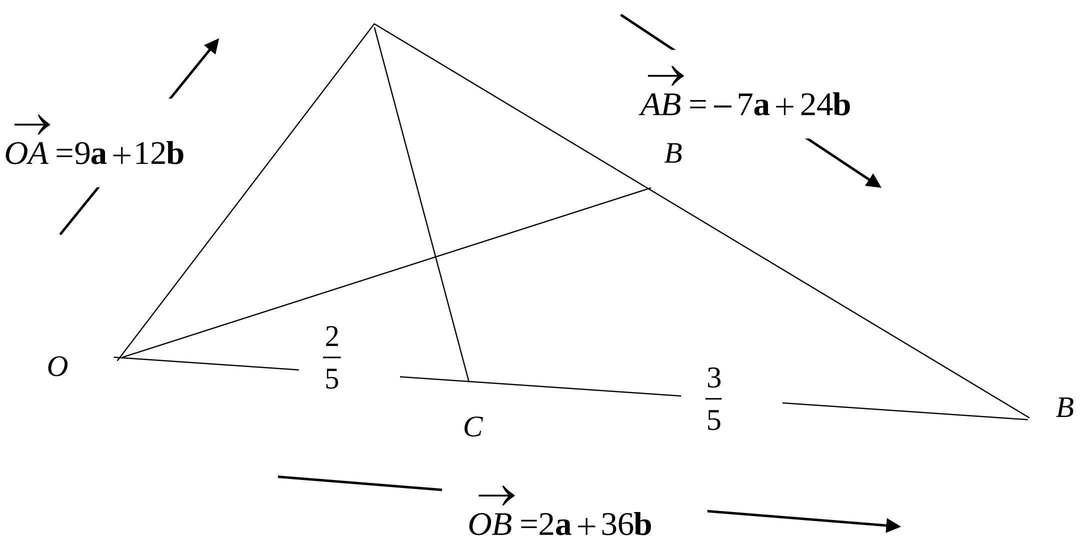
Question number	Scheme	Marks
5	$(P =)\ 6x+6$ $(A =)\ x(2x+3)$	B1 B1
	$6x+6>10$ or $x(2x+3)<35$	M1
	$x>\frac{2}{3}$ oe	A1
	$\left(2x^2+3x-35<0\right)$ $(2x-7)(x+5)(<0)$	M1
	(critical values are) $x=\frac{7}{2}$, $(x=-5)$	A1
	$\frac{2}{3}<x<\frac{7}{2}$	A1ft [7]
		Total 7 marks

Part	Mark	Additional Guidance
5 (a)	B1	For either correct expression.
	B1	For both correct expressions.
	M1	**Either** their linear expression > 10 (seen or implied in later work) **or** their quadratic expression < 35 (seen or implied in later work).
	A1	For the correct linear inequality.
	M1	For forming a 3TQ = 0 or 3TQ < 0 and for a minimally acceptable attempt at its solution leading to two values of x
	A1	For the value of $\frac{7}{2}$, and no other positive value.
	A1ft	For the answer shown, allow ft marks, so long as they have come from solution of a linear and 3 term quadratic. If the linear solution is out of range and cannot be incorporated into the final answer this is A0

Question number	Scheme	Marks
6 (a)	$\left(\overrightarrow{AB}\right) = \overrightarrow{AO} + \overrightarrow{OB} = -9\mathbf{a} - 12\mathbf{b} + 2\mathbf{a} + p\mathbf{b}$	M1
	$(-7)^2 + (p-12)^2 = 25^2$	M1
	$(p-12)^2 = 576$	M1
	$p = 12 \pm \sqrt{576}$	
	$p = -12,\ 36*$	A1* [4]
(b)	$\sqrt{12^2 + 2^2} = \rightarrow \left(\sqrt{148} = 2\sqrt{37}\right)$	M1
	$\pm \dfrac{1}{\sqrt{37}}(\mathbf{a} - 6\mathbf{b})$ **or** $\pm \dfrac{\sqrt{37}}{37}(\mathbf{a} - 6\mathbf{b})$	A1 [2]
(c)	$(\overrightarrow{OC} =)\dfrac{2}{5}(2\mathbf{a} + 36\mathbf{b})$	M1
	$(\overrightarrow{CA}) = -\dfrac{2}{5}(2\mathbf{a} + 36\mathbf{b}) + 9\mathbf{a} + 12\mathbf{b}$	M1
	$= \dfrac{41}{5}\mathbf{a} - \dfrac{12}{5}\mathbf{b}$	A1 [3]
		Total 9 marks

Part	Mark	Additional Guidance
6 (a)	M1	For a correct vector path to find $\pm\overrightarrow{AB}$ or $-9\mathbf{a} - 12\mathbf{b} + 2\mathbf{a} + p\mathbf{b}$ or $9\mathbf{a} + 12\mathbf{b} - 2\mathbf{a} - p\mathbf{b}$
	M1	For using Pythagoras' Theorem with th[illegible] $\overrightarrow{AB}$
	M1	For any complete and minimally acceptable attempt to solve for p See General Guidance.
	A1*	For both correct values of p – no errors seen. **Note:** This is a show question and the answers are given
(b)	M1	For a corr[illegible]od to find the magnitude.
	A1	For a corr[illegible] vector as shown.
(c)	M1	For a corr[illegible]r fc[illegible]
	M1	For the correct vector for $\overrightarrow{CA}$ using their $\overrightarrow{CO} + \overrightarrow{OA}$
	A1	For $\frac{41}{5}\mathbf{a} - \frac{12}{5}\mathbf{b}$
	ALT: Uses the vector path $\overrightarrow{CA} = \overrightarrow{CB} + \overrightarrow{BA}$	
	M1	For a correct vector for $\overrightarrow{CB} = \frac{3}{5}(2\mathbf{a} + 36\mathbf{b})$ or $\overrightarrow{BA} = 7\mathbf{a} - 24\mathbf{b}$
	M1	For the correct vector for $\overrightarrow{CA}$ using their $\overrightarrow{CB}$ and $\overrightarrow{BA}$ $\overrightarrow{CA} = \frac{3}{5}(2\mathbf{a} + 36\mathbf{b}) + (7\mathbf{a} - 24\mathbf{b})$
	A1	$\overrightarrow{CA} = \frac{41}{5}\mathbf{a} - \frac{12}{5}\mathbf{b}$

Useful Sketch



<table>
<tr><th>Question number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>7 (a)</td><td>

x	1.3	1.5	2	2.5	3	3.5
y	**−5.17**	−3.79	−2.11	**−1.39**	**−1.11**	**−1.08**

</td><td>B1 B1
[2]</td></tr>
<tr><td>(b)</td><td>Points plotted.

Smooth curve.
</td><td>B1ft

B1ft
[2]</td></tr>
<tr><td>(c)</td><td>General principle for marking
M1 – Deal with 9 = 3^2 or apply addition law and change $\log_3 9 = 2$
M1 – Deal with logs
M1 – Apply power law
A1 – Correct line

$\frac{3^{\frac{4}{3}x}}{3^2} = (x^2 - x)^3$
$3^{\frac{4}{3}x-2} = (x^2 - x)^3$
$\frac{4}{3}x - 2 = \log_3 (x^2 - x)^3$
$\frac{4}{3}x - 2 = 3\log_3 (x^2 - x)$
$-\frac{2}{3}x - 2 = 3\log_3 (x^2 - x) - 2x$</td><td>M1

M1

M1

A1</td></tr>
</table>

	OR $3^{\frac{4}{3}x} = 9\left(x^2 - x\right)^3$ $\log_3 3^{\frac{4}{3}x} = \log_3 9\left(x^2 - x\right)^3 \Rightarrow \frac{4}{3}x = \log_3 9\left(x^2 - x\right)^3$ $\frac{4}{3}x = \log_3 9 + \log_3\left(x^2 - x\right)^3 \Rightarrow \frac{4}{3}x = 2 + \log_3\left(x^2 - x\right)^3$ $\frac{4}{3}x - 2 = \log_3\left(x^2 - x\right)^3 \Rightarrow \frac{4}{3}x - 2 = 3\log_3\left(x^2 - x\right)$ $-\frac{2}{3}x - 2 = 3\log_3\left(x^2 - x\right) - 2x$ Draws the line $y = \frac{-2}{3}x - 2$ (x =) 1.6 or 1.7	2nd M1 1st M1 3rd M1 A1 M1 A1 [6]
ALT (c)	Set $3\log_3\left(x^2 - x\right) - 2x = Ax + B$ $\log_3\left(x^2 - x\right)^3 = Ax + 2x + B$ $\Rightarrow \left(x^2 - x\right)^3 = 3^{(A+2)x+B}$ Drawing on given equation $9\left(x^2 - x\right)^3 = 9 \times 3^{(A+2)x+B} = 3^{(A+2)x+B+2}$ Equating exponents $\frac{4}{3}x = (A+2)\text{x} \qquad 0 = B + 2$ $A = \frac{4}{3} - 2 = -\frac{2}{3} \qquad B = -2$ Hence $y = \frac{-2}{3}x - 2$ Draws the line $y = \frac{-2}{3}x - 2$ (x =) 1.6 or 1.7	M1 M1 M1 A1 M1 A1*cso [6]
		Total 10 marks

Part	Mark	Additional Guidance
7 (a)	B1	For 1 value correct to 2dp
	B1	For all 4 values correct.
(b)	B1ft	For points plotted within half a square ft their values from the table.
	B1ft	For points joined with a smooth curve ft their points plotted.
(c)	**General principles using this method** M1 – Deal with 9 or apply addition law and change $\log_3 9 = 2$ M1 – Deal with logs M1 – Apply power law A1 – Correct line	
	M1	Correct rearrangement and use of index rule to give the equation shown. [Deals correctly with $9 = 3^2$]
	M1	Correctly takes logs with their expression to eliminate the exponential.
	M1	Applies the power law for logs correctly with their expression
	A1	For the equation seen oe
	M1	Draws a line of the form $y = \frac{a}{b}x - 2$ Check the intercept on the y-axis or otherwise (0, –2) (1.5, –3) (3, –4)
	A1*cso	1.6 or 1.7, 1 decimal place only.
ALT (c)	M1	For equating the given formula for y with a generic linear equation and setting out to find the coefficients and uses power law correctly.
	M1	Correctly takes exponential from their expression to eliminate the.log
	M1	Substitutes from their expression into the given equality.
	A1	For finding the coefficients of the linear equation oe
	M1	Draws a line of the form $y = \frac{a}{b}x - 2$ Check the intercept on the y-axis or otherwise (0, –2) (1.5, –3) (3, –4)
	--------- A1*cso	1.6 or 1.7, 1 decimal place only.

Question number	Scheme	Marks
8 (a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)4x-4$	M1
	Gradient of $l=8$	B1
	Gradient of normal = $-\frac{1}{8}$	B1
	$4x-4=-\frac{1}{8}\Rightarrow x=\frac{31}{32}$	M1A1 [5]
(b) (i)	$(\alpha+\beta)^3=\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3=\alpha^3+\beta^3+3\alpha\beta(\alpha+\beta)$	B1*cso [1]
(ii)	$\alpha^4+\beta^4=\left(\alpha^2+\beta^2\right)^2-2\alpha^2\beta^2=\left((\alpha+\beta)^2-2\alpha\beta\right)^2-2(\alpha\beta)^2$	B1*cso [1]
(c)	$\alpha+\beta=2 \quad \alpha\beta=\frac{9}{2}$ oe	B1
	$\left(\alpha^3-\beta+\beta^3-\alpha=\alpha^3+\beta^3-(\alpha+\beta)\right)$	
	$=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)-(\alpha+\beta)$	M1
	(sum of roots =)$2^3-3\left(\frac{9}{2}\right)(2)-2(=-21)$	A1
	$\left[\left(\alpha^3-\beta\right)\left(\beta^3-\alpha\right)=\right]\alpha^3\beta^3-\alpha^4-\beta^4+\alpha\beta$	M1
	$=(\alpha\beta)^3-\left(\alpha^4+\beta^4\right)+\alpha\beta$	
	Now, $\left(\alpha^4+\beta^4\right)=\left((2)^2-2\times\frac{9}{2}\right)^2-2\times\left(\frac{9}{2}\right)^2=\left(-\frac{31}{2}\right)$	A1ft
	(product of roots =) $=\left(\frac{9}{2}\right)^3-\left(-\frac{31}{2}\right)+\frac{9}{2}\left(=\frac{889}{8}\right)$	A1
	$x^2-\text{"}-21\text{"}x+\text{"}\frac{889}{8}\text{"}$	M1
	$8x^2+168x+889=0$	A1 [8]
		Total 15 marks

Part	Mark	Additional Guidance
8 (a)	M1	For a correct differentiation.
	B1	For 8, identified as the gradient or used to calculate gradient of normal
	B1	For $-\frac{1}{8}$ stated as gradient of normal or used in subsequent work
	M1	For correctly solving their $ax+b=-\frac{1}{8}$
	A1	For $\frac{31}{32}$
(b) (i)	B1*cso	Complete and full algebra to show the given identity
(ii)	B1*cso	Complete and full algebra to show the given identity
(c)	B1	Correct values for $\alpha+\beta$ **and** $\alpha\beta$. Award if values not seen explicitly but embedded in the sum/product calculations for the new equation.
	M1	**Sum:** Using the result from part (b) and correctly manipulating to prepare for substitution of $\alpha+\beta$ **and** $\alpha\beta$
	A1	For -21
	M1	**Product:** For the correct algebraic expansion shown.
	A1ft	Correct substitution of their values into $\alpha^4+\beta^4$ within the product of roots
	A1	For $\frac{889}{8}$
	M1	Use $x^2-(\text{their sum of their roots})x+\text{their product of roots}$ "= 0" may be missing.
	A1	Correct equation oe with integer coefficients.

Question number	Scheme	Marks
9 (a)	$f'(x) = 4\times 2x(1+x^2)^3 = 8x(1+x^2)^3$ *	M1 A1* cso [2]
(b)	Find the gradient of the tangent to C at A:	
	$\frac{dy}{dx} = \frac{3\cos 3x(1+x^2)^4 - 8x(1+x^2)^3\sin 3x}{(1+x^2)^8}$	M1A1A1
	$= \frac{(1+x^2)^3[3(1+x^2)\cos 3x - 8x\sin 3x]}{(1+x^2)^8}$	
	$= \frac{3\cos 3x + 3x^2\cos 3x - 8x\sin 3x}{(1+x^2)^5}$	
	$\frac{3\cos\left(3\times\frac{2\pi}{3}\right)+3\left(\frac{2\pi}{3}\right)^2\cos\left(3\times\frac{2\pi}{3}\right)-8\times\frac{2\pi}{3}\sin\left(3\times\frac{2\pi}{3}\right)}{\left(1+\left(\frac{2\pi}{3}\right)^2\right)^5}$	M1
	$= \frac{3\left(1+\left(\frac{2\pi}{3}\right)^2\right)}{\left(1+\left(\frac{2\pi}{3}\right)^2\right)^5} = \frac{3}{\left(1+\left(\frac{2\pi}{3}\right)^2\right)^4}$	A1
	Find the negative reciprocal:	
	$= -\frac{1}{3}\left(1+\left(\frac{2\pi}{3}\right)^2\right)^4$	M1A1* cso [7]
ALT (b)	Find the gradient of the tangent to C at A:	
	Using product rule from $y = \sin 3x\left(1+x^2\right)^{-4}$	M1
	$\frac{dy}{dx} = \sin 3x\left(-8x\left(1+x^2\right)^{-5}\right) + 3\cos 3x\left(1+x^2\right)^{-4}$	A1A1
	$\frac{dy}{dx}\left(\frac{2\pi}{3}\right) = \sin\left(3\times\frac{2\pi}{3}\right)\left(-8\times\frac{2\pi}{3}\left(1+\left(\frac{2\pi}{3}\right)^2\right)^{-5}\right) + 3\times\cos\left(3\times\frac{2\pi}{3}\right)\left(1+\left(\frac{2\pi}{3}\right)^2\right)^{-4}$	M1
	$= \frac{3}{\left(1+\left(\frac{2\pi}{3}\right)^2\right)^4}$ oe	A1
	Find the negative reciprocal:	
	$= -\frac{1}{3}\left(1+\left(\frac{2\pi}{3}\right)^2\right)^4$	M1A1* cso [7]
		Total 9 marks

Part	Mark	Additional Guidance
9 (a)	M1	For showing an intermediate equation in which are shown multiplication by the original index, multiplication by the derivative of the bracketed term and a reduction in the index of the bracketed term. Should be in the form $ax(1+x^2)^3$
	A1*cso	For a fully correct solution, minimum steps shown, no omissions or error. Accept $"\frac{dy}{dx}="$ or $"\frac{d}{dx}="$
(b)	M1	For a minimally acceptable attempt to differentiate using the quotient rule • $\sin 3x \Rightarrow k\cos 3x$ $k \neq 0,1$ where k is an integer • $\left(1+x^2\right)^4$ as given in (a) • Allow + /- in the numerator • Denominator must be correct
	A1	For both terms in the numerator, in either order
	A1	For the correct expression shown oe **NB: This mark appears as an M mark in ePen**
	M1	For substitution of $x=\frac{2\pi}{3}$ into any changed expression representing the gradient of the tangent. **NB: This mark appears as an A mark in ePen**
	A1	For the correct expression shown. This may be implied by further correct work. **NB: This mark appears as an M mark in ePen**
	M1	For attempting to find the negative reciprocal of their expression.
	A1cso	For the correct expression shown
ALT (b)	M1	For a minimally acceptable attempt to differentiate using the product rule • $\sin 3x \Rightarrow k\cos 3x$ $k \neq 0,1$ where k is an integer • $\left(1+x^2\right)^4$ as given in (a) • Allow + /- between the terms
	A1	For one term correct
	A1	For the correct expression shown oe **NB: This mark appears as an M mark in ePen**
	M1	For substitution of $x=\frac{2\pi}{3}$ into any changed expression representing the gradient of the tangent. **NB: This mark appears as an A mark in ePen**
	A1	For the correct expression shown This may be implied by further correct work **NB: This mark appears as an M mark in ePen**
	M1	For attempting to find the negative reciprocal of their expression.
	A1cso	For the correct expression shown

Question number	Scheme	Marks
10 (a)(i)	$\cos(A+A)=\cos A\cos A-\sin A\sin A\to\cos 2A=\cos^2 A-\sin^2 A$ $\cos 2A=1-\sin^2 A-\sin^2 A\Rightarrow\cos 2A=1-2\sin^2 A$ * $\cos 2A-1=-2\sin^2 A$ or $1-\cos 2A=2\sin^2 A$ $\sin^2 A=\dfrac{1-\cos 2A}{2}$ *	M1 M1 A1*cso [3]
/ALT (a)(i)	Starting from r.h.s of equation: $\cos(A+A)=\cos A\cos A-\sin A\sin A\to\cos 2A=\cos^2 A-\sin^2 A$ Gives $\dfrac{1-\cos 2A}{2}=\dfrac{1-(\cos^2 A-\sin^2 A)}{2}$ $=\dfrac{\sin^2 A+\sin^2 A}{2}=\sin^2$ as required.	 M1 M1 A1*cso [3]
(ii)	$\left(\cos 2A=\cos^2 A-\sin^2 A\right.$ or $\left.\cos 2A=1-2\sin^2 A\right)$ $\cos 2A=1-\sin^2 A-\sin^2 A$ or $\cos 2A=1-\ 2(1-\cos^2 A)$ $\to\cos 2A=2\cos^2 A-1$ $\cos 2A+1=2\cos^2 A$ $\cos^2 A=\dfrac{\cos 2A+1}{2}$ *	 M1 A1*cso [2]
ALT (ii)	Starting from r.h.s of equation: $\cos 2A=\cos^2 A-\sin^2 A$ Gives $\dfrac{\cos 2A+1}{2}=\dfrac{\cos^2 A-\sin^2 A+1}{2}$ $=\dfrac{\cos^2 A+\cos^2 A}{2}=\cos^2 A$ as required.	 M1 A1*cso [2]
(b)	$\cos 2x=\sin x$ $1-2\sin^2 x=\sin x\to 0=2\sin^2 x+\sin x-1$ $0=(2\sin x-1)(\sin x+1)$ $\sin x=\dfrac{1}{2},(-1)\to x=$ $\left(\dfrac{\pi}{6},\dfrac{1}{2}\right)$	M1 M1 M1 A1 A1 [5]

(c)	$\pi\int_0^{"\frac{\pi}{6}"}\left(\cos^2 2x - \sin^2 x\right)dx$	B1ft
	$(\pi)\int_0^{"\frac{\pi}{6}"}\left(\frac{\cos 4x+1}{2}\right)-\left(\frac{1-\cos 2x}{2}\right)\ dx$	M1
	$\left[(\pi)\int_0^{"\frac{\pi}{6}"}\left(\frac{\cos 4x}{2}+\frac{\cos 2x}{2}\right)dx\right]$	
	$(\pi)\left[\frac{\sin 4x}{8}+\frac{\sin 2x}{4}\right]_0^{"\frac{\pi}{6}"}$	M1
	$(\pi)\left(\frac{\sin 4\left(\frac{\pi}{6}\right)}{8}+\frac{\sin 2\left(\frac{\pi}{6}\right)}{4}\right)$	M1
	$\frac{3\sqrt{3}}{16}\pi$	A1 [5]
		Total 15 marks

Part	Mark	Additional Guidance
10 (a)(i)	M1	Correct use of the addition formula as shown leading to $\cos 2A = \cos^2 A - \sin^2 A$
	M1	Correct substitution of $1-\sin^2 A$ to get $\cos 2A = 1-2\sin^2 A$
	A1*cso	Fully correct solution with the minimum steps shown, no errors.
ALT (a)(i)	M1	Correct use of the addition formula as shown leading to $\cos 2A = \cos^2 A - \sin^2 A$
	M1	Correct substitution into the right hand side of the equation.
	A1*cso	Rearrangement of resultant expression to reach left hand side of original equation., without errors..
(ii)	M1	Replacement of $\sin^2 A$ with $1-\cos^2 A$ in an expression from their part (i)
	A1*cso	Rearrangement of a correct equation to attain the given result, minimum steps shown, no errors.
ALT (ii)	M1	Replacement pf cos2*A* with expression from (i)
	A1*cso	Rearrangement of resultant expression to reach left hand side of original equation, without errors.
(b)	M1	Correctly equates the 2 expressions
	M1	Correctly uses the identity for cos2*x* and rearranges to an equation of the form $0 = a\sin^2 x + b\sin x + c \quad a,b,c \neq 0$
	M1	Uses a full method to solve their equation of the form $0 = a\sin^2 x + b\sin x + c \quad a,b,c \neq 0$ to find the inverse sine to arrive at a value for *x*. See general guidance for the solution of a 3TQ.
	A1	For the correct value of *x*
	A1	For correct coordinates, allow omission of brackets or *x* = and *y* = . **SC**: Allow A1 where candidate works in degrees and gets x = 30 degrees
(c)	B1ft	For the integral shown, ft their limits only.
	M1	An attempt to replace $\cos^2 2x$ **and** $\sin^2 x$ with an identity from part (a). At least one replacement must be fully correct.
	M1	At least one term correctly integrated.
	M1	Substitution of their limits clearly seen into a changed expression. This can be implied by a correct answer. Where the expression is not correct, or their limits are not correct, the substitution must be seen into at least half of their terms.
	A1	For the correct answer. Withold this mark if the candidate works in degrees rather than radians

Question number	Scheme	Marks
11	$\frac{3}{r}=\frac{4}{h}$ oe $\rightarrow$ $r=\frac{3h}{4}$ oe	M1 A1
	$(V=)\frac{1}{3}\pi\left(\frac{3h}{4}\right)^2 h$ $\left(V=\frac{3\pi h^3}{16}\right)$ oe	M1
	$\left(\frac{\mathrm{d}V}{\mathrm{d}h}=\right)\frac{9\pi h^2}{16}$ oe	M1 A1
	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}.\frac{\mathrm{d}h}{\mathrm{d}V}$ oe	M1
	$\left(\frac{\mathrm{d}h}{\mathrm{d}t}=\right)9\pi\times\frac{16}{9\pi(2)^2}$ oe	dddM1
	$\left(\frac{\mathrm{d}h}{\mathrm{d}t}=\right)4$	A1 [8]
	OR	
	$\frac{3}{r}=\frac{4}{h}$ oe $\Rightarrow$ $h=\frac{4r}{3}$ oe	M1A1
	$(V=)\frac{1}{3}\pi r^2 h$ $\left(V=\frac{4\pi r^3}{9}\right)$ oe	M1
	$\left(\frac{\mathrm{d}V}{\mathrm{d}r}=\right)\frac{4\pi r^2}{3}$ AND $\frac{\mathrm{d}h}{\mathrm{d}r}=\frac{4}{3}$	M1A1
	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}.\frac{\mathrm{d}r}{\mathrm{d}V}.\frac{\mathrm{d}h}{\mathrm{d}r}$ oe	M1
	$\left(\frac{\mathrm{d}h}{\mathrm{d}t}=\right)9\pi\times\frac{1}{\frac{4\pi\left(\frac{3}{2}\right)^2}{3}}\times\frac{4}{3}$ oe $\left[h=2,\quad r=\frac{3}{2}\right]$	M1
	$\left(\frac{\mathrm{d}h}{\mathrm{d}t}=\right)4$	A1 [8]
		Total 8 marks

Mark	Additional Guidance
M1	For any correct equation using similar triangles to link together r and h
A1	For $r=\frac{3h}{4}$. This may be implied in working.
M1	For using their expression for r in terms of h to get an expression for V of the form $a\pi h^3$
M1	For a minimally acceptable attempt to differentiate their expression for V to get an equation of the form $b\pi h^2$
A1	For $\left(\frac{dV}{dh}=\right)\frac{9\pi h^2}{16}$ oe
M1	For a correct, relevant chain rule. This may be implied in working.
dddM1	Correct substitution into chain rule, dependent on the previous 3 method marks.
A1	Correct value.
ALT	
M1	For any correct equation using similar triangles to link together r and h
A1	For $h=\frac{4r}{3}$
M1	For using their expression for r in terms of h to get an expression for V of the form $a\pi r^3$
M1	For a minimally acceptable attempt to differentiate their expression for V to get an equation of the form $b\pi r^2$
A1	For $\left(\frac{dV}{dr}=\right)\frac{4\pi r^2}{3}$ AND $\frac{dh}{dr}=\frac{4}{3}$
M1	For a correct, relevant chain rule. This may be implied in working
dddM1	Correct substitution into chain rule, dep on previous three marks
A1	Correct value

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