GradeMax

Pearson Edexcel

Mark Scheme (Results)

Summer 2023

Pearson Edexcel International GCSE In Mathematics B (4MB1) Paper 02R

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.
  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission
  - cas – Correct answer scores full marks (unless from obvious incorrect working)
  - wr – working required

- **No working**
  If no working is shown then correct answers normally score full marks

If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

  If the final answer is wrong always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.

  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.

  If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used.

  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.

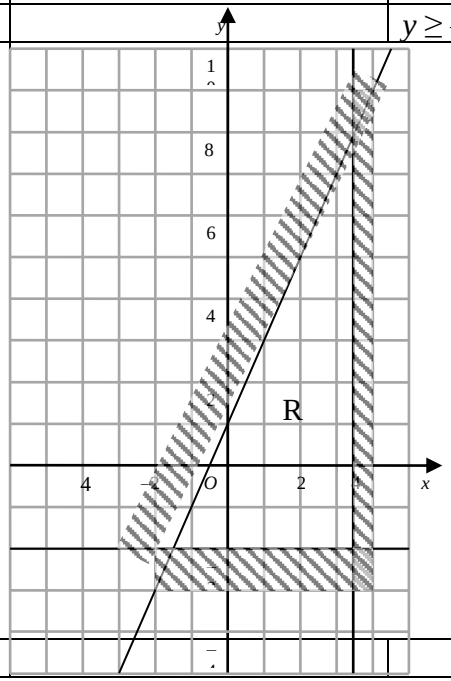
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

Qu		Working		Answer	Mark	Notes
1	(a)	$200 \times 8 \times 0.60$			2	M1 correct method.
				[£] 960		A1 cao (cas)
	(b)	$(600 - 200) \times 0.6$ [= 240]			2	M1 correct method to find 60% of remaining melons.
		[“240” × 1.40]		[£] 336		A1 cao (cas)
	(c)	eg 90×0.0097 [= 0.873 or 87.3p] or $600 \times 90 \times 0.0097$ [= 523.8(0)]	eg “960” ÷ 0.0097[=98969(.0..)] or “336” ÷ 0.0097[=34639(.1..)] or “960 + 336” ÷ 0.0097[=133608(.2..)] or 700 ÷ 0.0097[=72164(.9..)]		4	M1 Conversion of any value in Rupees to £ or any value in £ to Rupees. ft their answer to part (a) and (b) if appropriate.
		eg “960” + “336” – “523.8(0)” – 700 [=1296 – 1223.8(0) =72.2(0)]	eg “98969”+ “34639” – “72164” – 600 × 90 [=133608(.2…) – 126164(.9…) = 7443.29…]			M1 dep on the previous method mark. Method to find profit, that uses the cost of the melons, the sales calculated in parts (a) and (b) and the exchange rate. The transport cost can be omitted but must not be used with an inconsistent currency. May be seen within calculation for % profit. ft their answer to part (a) and their answer to part (b) but working must be shown. Note: use of inconsistent currencies is not permitted for the award of this mark.
		eg $\frac{"72.2(0)"}{"523.80"+700}\times 100$ or $\frac{"72.2(0)"}{"523.80"+700}[=0.059]$ or $\frac{"960"+"336"}{"523.80"+700}[=1.059]$ or $\frac{"960"+"336"}{"523.80"+700}\times 100[=105.9]$	eg $\frac{"7443.29"}{“72164”+600\times 90}\times 100$ or $\frac{"7443.29"}{“72164”+600\times 90}[=0.059]$ or $\frac{”98969”+“34639”}{“72164”+600\times 90}[=1.059]$ or $\frac{”98969”+“34639”}{“72164”+600\times 90}\times 100[=105.9]$			M1 dep on the previous method mark. Method to find percentage profit that uses the cost of the melons, the sales calculated in parts (a) and (b), the transport cost and the exchange rate. ft their answer to part (a) and their answer to part (b) but working must be shown. Note: use of inconsistent currencies is not permitted for the award of this mark.
				5.9 (%)		A1 AWRT 5.9 (cas) SC B2 for a profit of 147.4(%)
						Total 8 marks

Qu		Working	Answer	Mark	Notes
2	(a)		*p*, *t*	1	B1 cao (cas) (elements can be stated in either order) Note: do not accept *p*, *t* seen in the correct region on a Venn diagram.
	(b)		{*p*, *q*, *t*} {*p*, *s*, *t*} {*p*, *r*, *u*} {*r*, *t*, *u*}	3	B3 (cas)for all 4 correct sets listed with no incorrect sets listed. B2 for at least 3 correct sets listed with at most 2 incorrect sets listed. **or** for 2 correct sets listed with no incorrect sets listed. B1 for at least 1 correct set listed with at most 4 incorrect sets listed. Note: Elements can be stated in either order Sets need not be in brackets but must be clearly distinguishable. Treat a number in the set as an element of the set. Ignore any repeats of elements or sets.
					Total 4 marks

Qu		Working	Answer	Mark	Notes
3	(a)	$4x \leq 11 + 3$		2	M1 for adding 3 to both sides or dividing throughout by 4 (in an inequality or equation) as a first step **or** Showing 3.5 oe as the critical value (can be written as $x = 3.5$ oe)
			$x \leq \frac{7}{2}$		A1 (cas) accept equivalent fraction or decimal (eg. $\frac{14}{4}$ or 3.5) do not isw $x = 3.5$
	(b)	$y + 8 \geq 3 \times 2$ or $\frac{y}{2} \geq 3 - 4$		2	M1 for a correct start by either removing the fraction by multiplying throughout by 2 or isolating the y term by splitting the fraction and subtracting 4 (in an inequality or equation)
			$y \geq -2$		A1 (cas) do not isw $y = -2$
	(c)			4	B1ft Vertical line through 3.5 on x axis ft answer to (a) B1ft Horizontal line through –2 on y axis ft answer to (b) B1 $y = 2x + 1$ drawn through point (0, 1) and (2,5) B1cao dep on all previous three marks awarded in this part. Correct region indicated with either this region shaded or labelled as **R** or the outside of the region shaded (lines must be sufficient in length to define the region). Note: Condone any mix of solid and/or dashed lines Max score of B2 if more than 3 lines are drawn.
					Total 8 marks

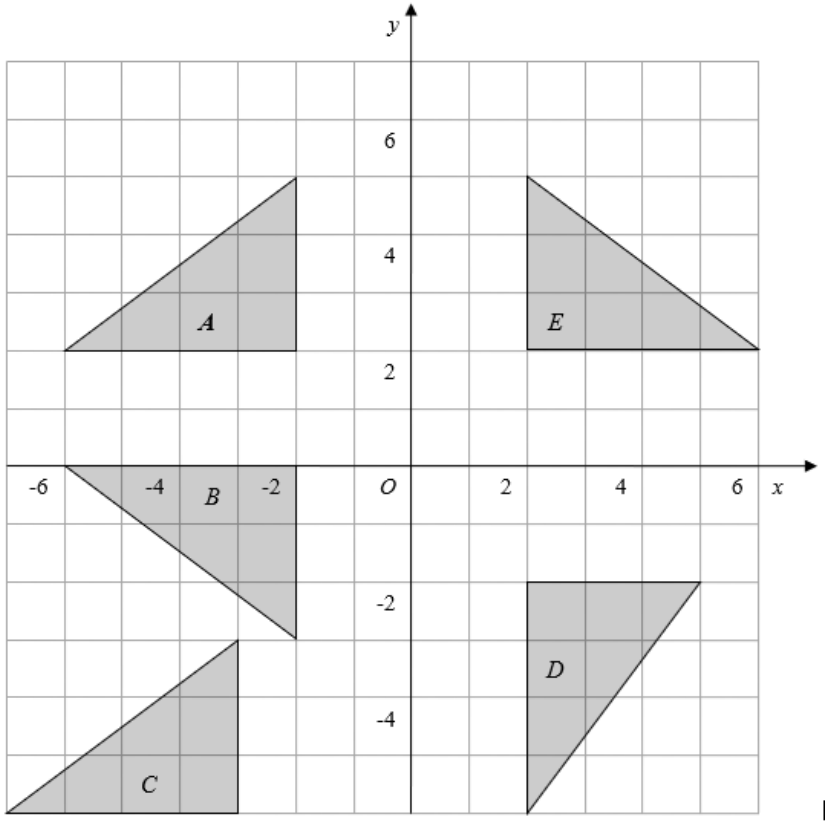
Qu		Working	Answer	Mark	Notes
4	(a)		420 000	1	B1cao
	(b)	252×10^{-96} or 25.2×10^{-95} or 2.52×10^{n} or $k\times10^{-94}$		2	M1 n is any integer or $1 \le k < 10$ Allow $\frac{126}{5}$ for 25.2 or for a correct answer in any form
			2.52×10^{-94}		A1 cao (cas)
	(c)	0.7×10^{105} or 7×10^{n} or $k\times10^{104}$		2	M1 n is any integer or $1 \le k < 10$ Allow $\frac{7}{10}$ for 0.7 or for a correct answer in any form
			7×10^{104}		A1 cao (cas)
	(d)		$y, \sqrt{y}, \sqrt{x}, x$	2	B2 Fully correct (cas) B1 order reversed ie $x, \sqrt{x}, \sqrt{y}, y$ or both y values before both x values eg $y, \sqrt{y}, x, \sqrt{x}$ or $\sqrt{y}$ after y and $\sqrt{x}$ before x eg $\sqrt{x}, y, \sqrt{y}, x$ Can be written numerically eg $\sqrt{y}$ as $\sqrt{6\times10^{-100}}$ or 2.45×10^{-50} $\sqrt{x}$ as $\sqrt{4.2\times10^{5}}$ or 648(.074) Accept y as 0 and $\sqrt{y}$ as $\sqrt{0}$ but do not accept 0 for $\sqrt{y}$ in their ordered list
					Total 7 marks

Qu		Working	Answer	Mark	Notes
5	(a) (i)	eg $\sqrt{8^2-2.5^2}+\sqrt{3.5^2-2.5^2}$ $\left[=\frac{\sqrt{231}}{2}+\sqrt{6}=7.60+2.45\right]$ or $\cos\angle ABD=\frac{2.5}{8}$ **and** $\cos\angle DBC=\frac{2.5}{3.5}$ **and** $AC^2=8^2+"3.5^2-2\times8\times3.5\cos("71.8"+"44.4")$		3	M2 Fully correct method to find *AC* (condone incorrect labelling) If not M2 then M1 for (condone incorrect labelling) Use of Pythagoras or trigonometry on either triangle *ABD* or *BCD*, to find an equation in *AD* or *CD* or correct method to find *AD* or *CD* eg $(AD=)\sqrt{8^2-2.5^2}$ $(=\frac{\sqrt{231}}{2}=7.60)$ or $\cos\angle DBC=\frac{2.5}{3.5}$ **and** $DC=3.5\sin\angle DBC$ oe or $\sin\angle DBC=\frac{2.5}{3.5}$ **and** $DC=3.5\cos\angle DBC$ oe eg $(DC=)\sqrt{3.5^2-2.5^2}$ $(=\sqrt{6}=2.45)$ or $\cos\angle ABD=\frac{2.5}{8}$ **and** $DC=8\sin\angle ABD$ oe or $\sin\angle BAD=\frac{2.5}{8}$ **and** $DC=8\cos\angle BAD$ oe
			10(.0) or 10.1		A1 awrt 10.0 or 10.1 Allow an exact value eg $\frac{\sqrt{231}+2\sqrt{6}}{2}$ isw

Qu		Working	Answer	Mark	Notes
	(a) (ii)	eg $\sin \angle BAD = \frac{2.5}{8}$ or $\frac{\sin A}{2.5} = \frac{\sin 90}{8}$ oe eg $\sin A = \frac{2.5 \sin 90}{8}$ or $\cos \angle BAD = \frac{"7.60"}{8}$ (=0.94 or 0.95) or $\tan \angle BAD = \frac{2.5}{"7.60"}$ (=0.32 or 0.33) or $2.5^2 = 8^2 + "7.60"^2 - 2 \times 8 \times "7.60" \cos A$ oe		2	M1 trig function of $\angle BAD$ or method to find $\angle BAD$ ft their *AD* from part a(i) (either from a correct method or be clearly labelled) Note: if find $\angle ABD$ then must do 90 – "$\angle ABD$" for M1
			18.2		A1 (cas) awrt 18.2
	(b)	eg $\sin "18.2" = \frac{DE}{"7.60"}$ [(DE=) "7.6" sin "18.2"] or $\sin(90 - "18.2") = \frac{DE}{2.5}$ [(DE=) 2.5 sin (90 – "18.2")] or $\cos(90 - "18.2") = \frac{DE}{"7.60"}$ [(DE=) "7.6" $\cos(90 - "18.2")$ or $\cos "18.2" = \frac{DE}{2.5}$ [(DE=) 2.5 cos "18.2"] or $\frac{1}{2} \times 8 \times DE = \frac{1}{2} \times$ "7.60" $\times 2.5$ oe $\left[(DE =) \frac{\frac{1}{2} \times "7.60" \times 2.5}{\frac{1}{2} \times 8} \right]$ or $\frac{2.5}{DE} = \frac{8}{"7.60"}$ oe $\left[(DE =) \frac{2.5 \times "7.60"}{8} \right]$		2	M1 Forming an equation in *DE*. Using trigonometry or Areas of triangles (Area of triangle *ABD* = 9.499177596) or Similar Triangles ft their $\angle BAD$ and *AD* (either from a correct method or be clearly labelled) Note: if find *AE* or *EB* first then proceed to find *DE* for M1
			2.37		A1 (cas) awrt 2.37 or 2.38 or for an exact value of $\frac{5\sqrt{231}}{32}$
					Total 7 marks

Qu		Working	Answer	Mark	Notes
6	(a)		Reflection (in line) $y = 1$	2	B1 for reflection (Allow reflaction, refraction, reflation) B0 if multiple transformations listed. Multiple transformations are when **more than one** of reflection, rotation (turn), translation (move), enlargement(stretch/squash) is stated eg centre (0, 0), SF 1, 180, a vector or matrix do not imply multiple transformations B1 for $y = 1$
	(b)		Correct triangle C drawn	2	B2 Fully correct triangle B1 For 2 of 3 points correct and joined to make a triangle or for 3 correct points stated (could be written as a matrix) or plotted correctly but not joined to make a triangle Correct coordinate (–3, –6), (–7 , –6) and (–3 , –3) Award 2 marks for correct triangle drawn and labelled, irrespective of working in working space.
	(c)	$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}-2 & -2 & -6\\2 & 5 & 2\end{pmatrix}$ oe		3	M1 For intention to multiply the correct way, can be implied by writing in the correct order or one correct point stated **or** (1 point correctly plotted) Points can be in any order
		$\begin{pmatrix}2 & 5 & 2\\-2 & -2 & -6\end{pmatrix}$ oe			M1 Correct result for their matrix multiplication (or 2 points correctly plotted) Points can be in any order
			Correct triangle D drawn		A1 (cas) cao Correct coordinates (2 , –2), (5 , –2) and (2 , –6) Award 3 marks for correct triangle drawn and labelled, irrespective of working in working space.
	(d)			2	M1 Triangle *B* rotated by 180° about any centre **or** for 2 of 3 points correct and joined to make a triangle or for 3 correct points stated (could be written as a matrix) or plotted correctly but not joined to make a triangle
			Correct triangle E drawn		A1(cas) cao Correct coordinates (2 , 2), (6 , 2) and (2 , 5) Award 2 marks for correct triangle drawn and labelled, irrespective of working in working space.
	(e)		$\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$	2	B2 (cas) Correct matrix given B1 for stating transformation as reflection in y axis/$x = 0$ or matrix with 3 correct elements (in the correct positions)

Qu	Working	Answer	Mark	Notes



Total 11 marks

Qu		Working	Answer	Mark	Notes
7	(a)		$\begin{pmatrix}6.5\\0.5\end{pmatrix}$	1	B1 cao isw if find the magnitude
	(b)	$\left(\overrightarrow{AE}=\right)\frac{3}{5}\times\begin{pmatrix}15\\5\end{pmatrix}\left[=\begin{pmatrix}9\\3\end{pmatrix}\right]$ oe $\left(\overrightarrow{EC}=\right)\frac{2}{5}\times\begin{pmatrix}15\\5\end{pmatrix}\left[=\begin{pmatrix}6\\2\end{pmatrix}\right]$ oe		3	M1 correct use of ratio on $\overrightarrow{AC}$ to find either $\overrightarrow{AE}$ or $\overrightarrow{EC}$
		$\left(\overrightarrow{OA}=\overrightarrow{OE}+\overrightarrow{EA}=\right)\begin{pmatrix}6\\4\end{pmatrix}-$ "$\begin{pmatrix}9\\3\end{pmatrix}$" oe or $\left(\overrightarrow{OA}=\overrightarrow{OE}+\overrightarrow{EC}+\overrightarrow{CA}=\right)\begin{pmatrix}6\\4\end{pmatrix}+$ "$\begin{pmatrix}6\\2\end{pmatrix}$"$-\begin{pmatrix}15\\5\end{pmatrix}$ oe			M1 correct method to find position vector of A follow through their $\overrightarrow{AE}$ or $\overrightarrow{EC}$ If following through their $\overrightarrow{AE}$ or $\overrightarrow{EC}$ then this must be from a correct method or be clearly labelled as $\overrightarrow{AE}$ or $\overrightarrow{EC}$
			(–3, 1)		A1 cao (cas) Allow $\begin{pmatrix}-3\\1\end{pmatrix}$ SCB2 for one correct value SCB1 $\begin{pmatrix}0.375\\2.125\end{pmatrix}$ oe

Qu		Working		Answer	Mark	Notes
	(c)	$\left(\overrightarrow{OB}=\right)$ "$\begin{pmatrix}-3\\1\end{pmatrix}$" $+\begin{pmatrix}8.5\\4.5\end{pmatrix}\left[=\begin{pmatrix}5.5\\5.5\end{pmatrix}\right]$	$\left(\overrightarrow{BE}=\right)\begin{pmatrix}-8.5\\-4.5\end{pmatrix}+$ "$\begin{pmatrix}3\\-1\end{pmatrix}$"$+\begin{pmatrix}6\\4\end{pmatrix}\left[=\begin{pmatrix}0.5\\-1.5\end{pmatrix}\right]$		3	M1 for a correct start to find $\overrightarrow{OB}$ or $\overrightarrow{BE}$ (may be embedded within working or on diagram) follow through their $\overrightarrow{OA}$
		$\left[\overrightarrow{OD}=\overrightarrow{OE}+\left(\overrightarrow{BO}+\overrightarrow{OE}\right)=\right]$ $\begin{pmatrix}6\\4\end{pmatrix}+\left($"$\begin{pmatrix}-5.5\\-5.5\end{pmatrix}$"$+\begin{pmatrix}6\\4\end{pmatrix}\right)\left[=\begin{pmatrix}6.5\\2.5\end{pmatrix}\right]$ or $\left[\overrightarrow{OD}=\overrightarrow{OB}+2\times\left(\overrightarrow{BO}+\overrightarrow{OE}\right)=\right]$ "$\begin{pmatrix}5.5\\5.5\end{pmatrix}$"$+2\times\left($"$\begin{pmatrix}-5.5\\-5.5\end{pmatrix}$"$+\begin{pmatrix}6\\4\end{pmatrix}\right)\left[=\begin{pmatrix}6.5\\2.5\end{pmatrix}\right]$ or may go via A eg $\left[\overrightarrow{OD}=\overrightarrow{OA}+\left(\overrightarrow{AO}+\overrightarrow{OE}\right)+\left(\overrightarrow{BO}+\overrightarrow{OE}\right)=\right]$ "$\begin{pmatrix}-3\\1\end{pmatrix}$"$+\left($"$\begin{pmatrix}3\\-1\end{pmatrix}$"$+\begin{pmatrix}6\\4\end{pmatrix}\right)+\left($"$\begin{pmatrix}-5.5\\-5.5\end{pmatrix}$"$+\begin{pmatrix}6\\4\end{pmatrix}\right)\left[=\begin{pmatrix}6.5\\2.5\end{pmatrix}\right]$ or $\left[\overrightarrow{OD}=\overrightarrow{OA}+\overrightarrow{AB}+2\times\left(\overrightarrow{BO}+\overrightarrow{OE}\right)=\right]$ "$\begin{pmatrix}-3\\1\end{pmatrix}$"$+\begin{pmatrix}8.5\\4.5\end{pmatrix}+2\times\left($"$\begin{pmatrix}-5.5\\-5.5\end{pmatrix}$"$+\begin{pmatrix}6\\4\end{pmatrix}\right)\left[=\begin{pmatrix}6.5\\2.5\end{pmatrix}\right]$ or $\frac{x_D+5.5}{2}=6$ and $\frac{y_D+5.5}{2}=4$	$\left[\overrightarrow{OD}=\overrightarrow{OE}+\overrightarrow{BE}=\right]$ $\begin{pmatrix}6\\4\end{pmatrix}+$"$\begin{pmatrix}0.5\\-1.5\end{pmatrix}$"$\left[=\begin{pmatrix}6.5\\2.5\end{pmatrix}\right]$ or $\left[\overrightarrow{OD}=\left(\overrightarrow{OE}+\overrightarrow{EB}\right)+2\times\overrightarrow{BE}=\right]$ $\left(\left(\begin{pmatrix}6\\4\end{pmatrix}+$"$\begin{pmatrix}-0.5\\1.5\end{pmatrix}$"$\right)+2\times$"$\begin{pmatrix}0.5\\-1.5\end{pmatrix}$"$\left[=\begin{pmatrix}6.5\\2.5\end{pmatrix}\right]\right)$ or may go via A eg $\left[\overrightarrow{OD}=\overrightarrow{OA}+\left(\overrightarrow{AO}+\overrightarrow{OE}\right)+\overrightarrow{BE}=\right]$ "$\begin{pmatrix}-3\\1\end{pmatrix}$"$+\left($"$\begin{pmatrix}3\\-1\end{pmatrix}$"$+\begin{pmatrix}6\\4\end{pmatrix}\right)+$"$\begin{pmatrix}0.5\\-1.5\end{pmatrix}$"$\left[=\begin{pmatrix}6.5\\2.5\end{pmatrix}\right]$ or $\left[\overrightarrow{OD}=\overrightarrow{OA}+\overrightarrow{AB}+2\times\overrightarrow{BE}=\right]$ "$\begin{pmatrix}-3\\1\end{pmatrix}$"$+\begin{pmatrix}8.5\\4.5\end{pmatrix}+2\times$"$\begin{pmatrix}0.5\\-1.5\end{pmatrix}$"$\left[=\begin{pmatrix}6.5\\2.5\end{pmatrix}\right]$			M1 dep on the previous mark fully correct method to find position vector of D follow through their $\overrightarrow{OA}$ follow through their $\overrightarrow{OB}$ or $\overrightarrow{BE}$ If following through their $\overrightarrow{OB}$ or $\overrightarrow{BE}$ then this must be from a correct method or use midpoint formula
				$(6.5, 2.5)$		A1 cao (cas) AO for $\begin{pmatrix}6.5\\2.5\end{pmatrix}$ SCB2 for one correct value
						Total 7 marks

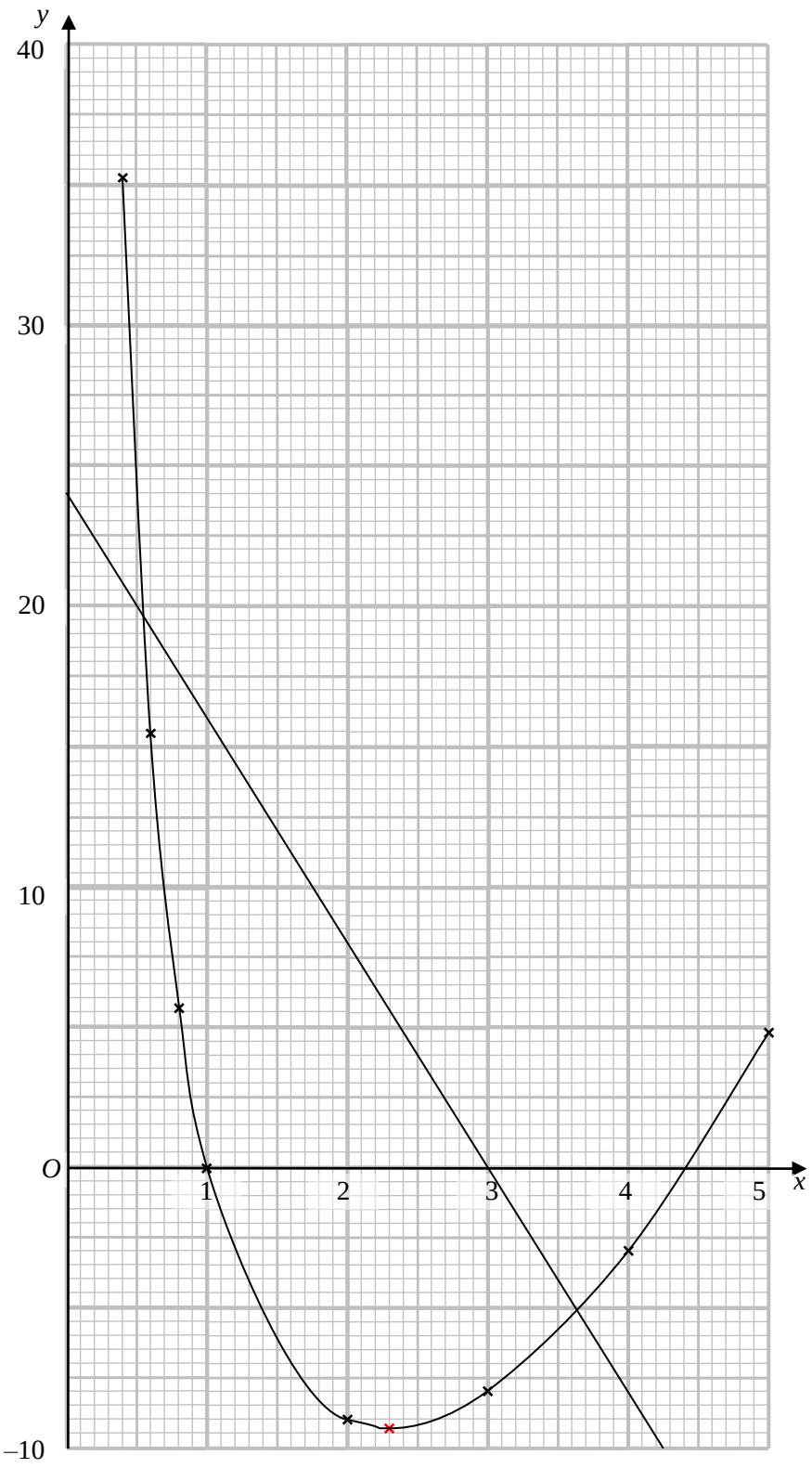
These are some alternative methods that can be used for Question 7b and 7c

Qu	Working		Answer	Mark	Notes
ALT 1 Ratio					
(b)	$\left[\frac{AE}{AC}=\frac{3}{5}\right]$ $\frac{6-x}{15}=\frac{3}{5}$ or $\frac{4-y}{5}=\frac{3}{5}$	[3:5] [3:5] $[-x+6:15]$ $[4-y:5]$ $-x+6=9$ or $4-y=3$			M1 for writing an equation in terms of x or y
	$x=-3$ or $y=1$	$x=-3$ or $y=1$			M1dep for solving either equation to find either the x coordinate or the y coordinate
			(–3, 1)		A1 cao
ALT 1 Eqn of line and distance					
(c)	Distance DE $(x-6)^2+(y-4)^2=2.5$ oe Distance AD $(x+3)^2+(y-1)^2=92.5$ oe	Line ED $y=-3x+22$			M1 for equation of line ED or distance of AD or DE
	eg $(x-6)^2+(-3x+22-4)^2=2.5$ $(x+3)^2+(-3x+22-1)^2=92.5$				M1dep for a correct equation for the x or y coordinate of D
			(6.5, 2.5)		A1 cao
ALT 2 Pythagoras and Trig					
(c)	$\tan^{-1} BAG=\frac{4.5}{8.5}\left[\Rightarrow BAG=27.8(9...)\right]$				M1 for angle BAG (where AGB is a right angle)
	$\left[AB=\right]\sqrt{8.5^2+4.5^2}\left[=\frac{\sqrt{370}}{2}\right]$ **and** $\tan^{-1}\left(\frac{4.5}{8.5}\right)-2\times\left(\tan^{-1}\left(\frac{4.5}{8.5}\right)-\tan^{-1}\left(\frac{3}{9}\right)\right)\left[\Rightarrow DAH=8.97(2...)\right]$				M1dep for distance AB **and** for angle DAH (where angle AHD is a right angle)
			(6.5, 2.5)		A1 cao

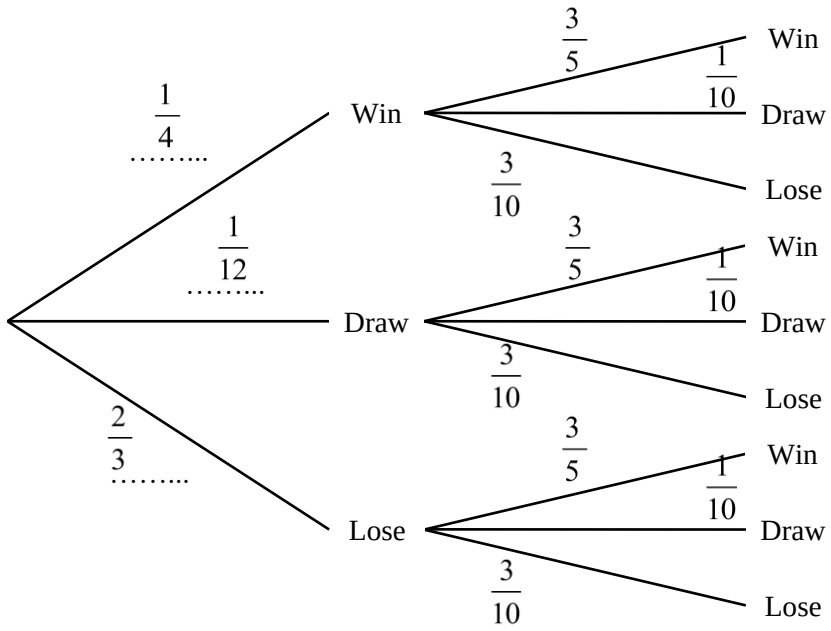
Qu		Working	Answer	Mark	Notes
8	(a)		1	1	B1 cao allow f $\neq 1$, $y \neq 1$ or suitable interval notation Condone f = 1, f:x=1, y = 1 but do not allow $x \neq 1$or $x = 1$. If they give the correct answer ignore any reference to $x = 0$ or $x \neq 0$ eg. $(-\infty,1) \cup (1,\infty)$
	(b)	$16\left(1-\frac{1}{x}\right)=x+8$		4	M1 Set 16 f(x) = x + 8 to give a correct equation in one unknown. Only condone missing brackets if they're recovered
		$16x-16=x^2+8x$			M1 multiply out brackets and multiply through by x (condone one sign error for this mark)
		$x^2-8x+16=0 \Rightarrow (x-4)^2=0$			M1 indep method to solve their three term quadratic. If factorising must expand to give at least two correct terms for their quadratic. If using quadratic formula then must be correct substitution (condone missing brackets around the b in b^2) If using completing the square, allow one error
			4		A1 cao (cas) (ignore any given y value)

Qu		Working	Answer	Mark	Notes
	(c)	$1-\frac{1}{1-\frac{1}{x}}$ or $\frac{\frac{x-1}{x}-1}{\frac{x-1}{x}}$ or $1-\frac{1}{\frac{x-1}{x}}$		6	M1 form a correct expression for ff (x)
		$\frac{1-\frac{1}{x}-1}{1-\frac{1}{x}}$ or $\frac{-\frac{1}{x}}{1-\frac{1}{x}}$ or $\frac{\frac{x-1-x}{x}}{\frac{x-1}{x}}$ or $\frac{-x}{x^2-x}$ or $\frac{-1}{x}\times\frac{x}{x-1}$ or $\frac{x-1-x}{x-1}$ or $\frac{x-1}{x-1}-\frac{x}{x-1}$ or $1-\frac{x}{x-1}$			M1 correct expression as a single fraction with $1-\frac{1}{x}$ as the denominator or correct expression as a single fraction with $\frac{x-1}{x}$ as denominator with a single fraction on numerator and denominator or correct expression with no nested fractions but not $-\frac{1}{x-1}$ or $\frac{1}{1-x}$ implies previous method mark with no errors seen up to this point
		$-\frac{1}{x-1}$ or $\frac{1}{1-x}$			A1 (wr) correct simplified result for ff(x) must gain both M marks with no errors seen when finding composite function
		$-\frac{1}{x}=y-1$ or $xy-x=-1$ or $\frac{1}{x}=1-y$ or $x-xy=1$	$xy-y=-1$ or $y-xy=1$		M1 using y = f(x) and rearrange so that all x terms isolated on one side in a correct equation. x and y may be exchanged in these expressions (then all y terms isolated on one side) M2 for $-\frac{1}{y}=x-1$ or $\frac{1}{y}=1-x$
		$-\frac{1}{y-1}=x$ or $x(y-1)=-1$ or $\frac{1}{1-y}=x$ or $x(1-y)=1$	$y(x-1)=-1$ or $y(1-x)=1$		M1 for taking x out as a common factor within an equation (dep on two terms in x) which must be seen and not implied or correct expression for x in terms of y. x and y may be exchanged in these expressions (for taking y out as a common factor within an equation (dep on two terms in y) which must be seen and not implied.
		$-\frac{1}{x-1}$ or $\frac{1}{1-x}$			A1 (wr) correct simplified result for $f^{-1}(x)$ must be in terms of x with no errors seen when finding inverse function. Dep on the third and fourth method marks being awarded. If previous A marked gained must either state this is equal to previous result or be an identical expression. isw for A1 but not for A2
	(d)		1	1	B1 ft follow through their answer to part (a) or their inverse in (c) provided this is a rational function with a linear denominator.
	(e)		2	1	B1 cao
					Total 13 marks

Qu		Working	Answer	Mark	Notes
9	(a)	$\left[y = x^2\right] + 24x^{-1}\left[-25\right]$		5	M1 expressing $\frac{24}{x}$ as $24x^{-1}$, may be implied by correct derivative of this term($-24x^{-2}$)
		$\left[\frac{dy}{dx} =\right] 2x - 24x^{-2}$			M1 at least 1 (non-constant) term differentiated correctly
		$2x - 24x^{-2} = 0 \Rightarrow x = ...$			M1 dep on previous M mark set derivative = 0 and attempt to solve for x
		$\left[y =\right] \sqrt[3]{12}^2 + \frac{24}{\sqrt[3]{12}} - 25$			M1 dep on previous M mark use their x to find y (an answer of -9.3 can imply this mark)
			(2.3, –9.3)		A1 (awrt 2.3, awrt -9.3) (allow $x = \sqrt[3]{12}$)
	(b)	<table><tr><td>x</td><td>0.4</td><td>0.6</td><td>0.8</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>35.2</td><td>15.4</td><td>5.6</td><td>0</td><td>–9</td><td>–8</td><td>–3</td><td>4.8</td></tr></table>		2	B2 –1eeoo penalise not rounded to 1dp only once.
	(c)	Stationary point from part (a) plotted		4	B1 ft follow through answer to part (a) (ft is dep on at least 3 method marks awarded in part (a) and not being one of the points from the table)
		At least 5 points correctly plotted from their table			M1 ft their answer to part (b)
		Smooth curve through at least 5 of their points			M1 ft clearly plotted points
		Correct curve			A1 a fully correct curve passing through the points on the overlay (tolerance is one small square), ignoring the stationary point. This implies the method marks
	(d)	$x^2 + 8x - 49 + \frac{24}{x} = 0$ or $x^2 + \frac{24}{x} - 25 = mx + c$ and $x^3 + 24 - 25x = mx^2 + cx$		4	M1 divide through by x or for writing curve equation equal to $mx+c$ (where m and c can be any letters) and multiplying throughout by x
		$[x^2 + \frac{24}{x} - 25 =] 24 - 8x$			M1 dep
		Line drawn through (0, 24) with a negative gradient			M1 for a line drawn through (0, 24) with a negative gradient or correct line drawn ($y = 24 - 8x$) and intersects the curve twice, this implies M3 Note: half square tolerance allowed for the line
			0.55, 3.65		A1 allow value 0.45 - 0.65 and 3.5 - 3.8 follow through their graph. Must follow from a correct line drawn (and intersects curve twice) (half square tolerance allowed for the line) Note: A0 if give answers as coordinates or give y values as well as x values
					Total 15 marks


y
40
30
20
10
O
1
2
3
4
5
x
–10

Qu		Working	Answer	Mark	Notes
10	(a)	$1-\frac{1}{4}-\frac{1}{12}$	$1-\frac{1}{4}-\frac{1}{12}=\frac{2}{3}$	1	B1 Correct answer with correct method to find P(lose). Do not allow $\frac{1}{4}+\frac{1}{12}+\frac{2}{3}=1$
	(b)			2	B1 $\frac{1}{4}$, $\frac{1}{12}$ and $\frac{2}{3}$ all placed correctly B1 All 9 of the values $\frac{3}{5}$, $\frac{1}{10}$ and $\frac{3}{10}$ correctly placed on the second branches.
	(c)	$"\frac{1}{12}"\times"\frac{1}{10}"$		2	M1 ft their tree diagram. Fully correct method to find required probability (do not isw) Note: $2\times\left("\frac{1}{12}"\times"\frac{1}{10}"\right)$ gains M0
			$\frac{1}{120}$		A1 (cas) Correct probability
	(d)	$"\frac{1}{12}"\times"\frac{3}{10}"\left[=\frac{1}{40}\right]$ or $"\frac{2}{3}"\times"\frac{1}{10}"\left[=\frac{1}{15}\right]$		5	M1 ft their tree diagram Calculate probability of draw then lose or lose then draw (can be embedded within another calculation where probabilities are added)
		eg $"\frac{1}{12}"\times"\frac{3}{10}"+"\frac{2}{3}"\times"\frac{1}{10}"\left[=\frac{11}{120}\right]$			M1 ft their tree diagram. Implies previous method mark. Calculate the probability of draw and lose in either order
		$\frac{p}{"\left(\frac{11}{120}\right)"}$ or $\frac{"\left(\frac{1}{15}\right)"}{p}$			M1 Divide any probability by their probability for draw and lose in either order or their probability of lose then draw divided by any probability $"\frac{11}{120}"$ or $"\frac{1}{15}"$ must come from a correct method or their $"\frac{11}{120}"$ must be from the sum of two products that are clearly labelled as being draw and lose or their $"\frac{1}{15}"$ must come from the product of two probabilities that is clearly labelled as lose then draw Note: The value of *p* can be any probability.
		$\frac{"\left(\frac{1}{15}\right)"}{"\left(\frac{11}{120}\right)"}$			M1 dep on previous method mark. For dividing their probability of lose then draw by their probability of draw and lose in either order. $"\frac{11}{120}"$ or $"\frac{1}{15}"$ must come from a correct method or be correctly labelled
			$\frac{8}{11}$		A1 cao (cas) equivalent fractions eg. $\frac{240}{330}$ or awrt 0.727
					Total 10 marks


1/4
Win
3/5
Win
1/10
Draw
3/10
Lose
1/12
Draw
3/5
Win
1/10
Draw
3/10
Lose
2/3
Lose
3/5
Win
1/10
Draw
3/10
Lose

An alternative method for Q10d

Qu		Working	Answer	Mark	Notes
ALT 1 Ratio					
		$\frac{1}{12}\times\frac{3}{10}\left[=\frac{1}{40}\right]$ or $\frac{2}{3}\times\frac{1}{10}\left[=\frac{1}{15}\right]$			M1 ft their tree diagram Calculate probability of draw then lose or lose then draw
		$\frac{1}{12}\times\frac{3}{10}\left[=\frac{1}{40}\right]:\frac{2}{3}\times\frac{1}{10}\left[=\frac{1}{15}\right]$ $[3:8]$			M1 ft their tree diagram. Implies previous method mark. Write down the ratio of draw then lose: lose then draw (or vice versa) This ratio may then (but does not have to) be simplified
		$\frac{a}{"3+8"}$ or $\frac{"8"}{b}$			M1 Divide any value by their sum of ratio parts for draw and lose in either order or their term for lose then draw divided by any value. $"8"$ or $"3+8"$ must come from a correct method or be clearly labelled as draw and lose in either order or as lose then draw
		$\frac{"8"}{"3+8"}$			M1 dep Divide their term for lose then draw by their sum of ratio parts for draw and lose in either order $"8"$ or $"3+8"$ must come from a correct method or be clearly labelled as draw and lose in either order or as lose then draw Dependent on previous method mark
			$\frac{8}{11}$		A1 cao (cas) equivalent fractions eg. $\frac{240}{330}$ or awrt 0.727

Qu		Working	Answer	Mark	Notes
11	(a)	$\left(3\sqrt{5}\right)^2 = x^2 + \left(4\sqrt{3}\right)^2 - 2x \times 4\sqrt{3} \times \cos 15$ oe or $\cos 15 = \frac{x^2 + (4\sqrt{3})^2 - (3\sqrt{5})^2}{2 \times 4\sqrt{3}x}$ oe $\frac{\sin C}{4\sqrt{3}} = \frac{\sin 15}{3\sqrt{5}}$ or $C = 15.5..$ or $B = 180 - 15 - 15.5\ldots =$ $149.495\ldots$ or $b = \frac{\sin 149(.4...) \times 3\sqrt{5}}{\sin 15} = 13.1(56..)$ with $(13.156)^2 - \left(6\sqrt{2} + 2\sqrt{6}\right) \times 13.156 + 3 = 0$		3	M1 correct substitution into cosine rule accept values correct to 3 sf. $(6.71^2 = x^2 + 6.93^2 - 2 \times 6.93 \times 0.966)$ (only condone missing brackets if recovered including on the $3\sqrt{5}$ and $4\sqrt{3}$) or finding the value of x as a decimal (13.1…..) by using the sine rule and substituting this x value into the equation to show that $x^2 - \left(6\sqrt{2} + 2\sqrt{6}\right)x + 3 = 0$ will only gain a maximum score of M1
		$45 = x^2 + 48 - 8\sqrt{3}x \times \frac{\sqrt{6} + \sqrt{2}}{4}$ oe or $\frac{\sqrt{6} + \sqrt{2}}{4} = \frac{x^2 + 48 - 45}{8\sqrt{3}x}$ oe			M1 correct expansion of surds which may be seen in earlier working Must see cos15 replaced with $\frac{\sqrt{6} + \sqrt{2}}{4}$ and either 45 or 48 and in a correct equation. This implies the previous method mark Allow $\frac{\sqrt{6} + \sqrt{2}}{4} = \frac{x^2 + 3}{8\sqrt{3}x}$ provided the 45 or 48 are seen in earlier working
		$x^2 - \left(6\sqrt{2} + 2\sqrt{6}\right)x + 3 = 0$			A1 (wr) No incorrect working seen, must gain both method marks.
	(b)	$[(x-k)^2 =] x^2 - 2kx...$ or $-2k = -\left(6\sqrt{2} + 2\sqrt{6}\right)$ or $-2k = -6\sqrt{2} - 2\sqrt{6}$ or $2k = 6\sqrt{2} + 2\sqrt{6}$ oe or $\left[x^2 - (6\sqrt{2} + 2\sqrt{6})x + 3 =\right] \left(x - \frac{6\sqrt{2} + 2\sqrt{6}}{2}\right)^2 ....$ or equivalent surd eg $\left[x^2 - (6\sqrt{2} + 2\sqrt{6})x + 3 =\right] \left(x - (3\sqrt{2} + \sqrt{6})\right)^2 ....$ or $\left[x^2 - (6\sqrt{2} + 2\sqrt{6})x + 3 =\right] \left(x - 3\sqrt{2} - \sqrt{6}\right)^2 ....$		2	M1 for expansion of $(x-k)^2$ to find the first two terms or comparing of coefficients of x or correct start of completing the square on $x^2 - \left(6\sqrt{2} + 2\sqrt{6}\right)x + 3 = 0$ to find k
			$3\sqrt{2} + \sqrt{6}$		A1 (cas) implies M1 condone $\left(x - (3\sqrt{2} + \sqrt{6})\right)^2 ...$ and isw but A0 for $\frac{6\sqrt{2} + 2\sqrt{6}}{2}$
	(c)	$9 + 6\sqrt{3} + 6\sqrt{3} + 12$		2	M1 expand binomial to give 12 (which can be written as 4×3) with two other terms correct if 4 terms given and one other term correct if 3 terms given
			$21 + 12\sqrt{3}$		A1(wr) fully correct expression with no incorrect working seen.

	(d)	$\left(x-"\left(3\sqrt{2}+\sqrt{6}\right)"\right)^2=\left(3+2\sqrt{3}\right)^2$ oe or $x-"\left(3\sqrt{2}+\sqrt{6}\right)"=(\pm)\left(3+2\sqrt{3}\right)$ oe		3	M1 Use $\left(3+2\sqrt{3}\right)^2$ (ft their result from part(b)) to find x or use $3+2\sqrt{3}$ (ft their result from part(b)) if sqrt (condone missing ±)
		$x=3\sqrt{2}+\sqrt{6}\pm\left(3+2\sqrt{3}\right)$			M1 dep isolate x term (allow +, – or ±)
		$x=3+3\sqrt{2}+2\sqrt{3}+\sqrt{6}$			A1 cao (cas) do not allow both solutions given. Accept equivalent answers in the given form.
					Total 10 marks

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