Mark Scheme (Results)

Summer 2014

Pearson Edexcel International GCSE in Further Pure Mathematics Paper 1 (4PM0/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.

  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks. Can only be awarded if the relevant method mark(s) has (have) been gained.
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - eeoo – each error or omission
- **No working**

  If no working is shown then correct answers may score full marks

  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

  If there is a wrong answer indicated always check the working and award any marks appropriate from the mark scheme.

  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.

  Any case of suspected misread which does not significantly simplify the question loses two A (or B) marks on that question, but can gain all the M marks. Mark all work on follow through but enter A0 (or B0) for the first two A or B marks gained.

  If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

  If there are multiple attempts shown, then all attempts should be marked and the highest score on a single attempt should be awarded.

- **Follow through marks**

  Follow through marks which involve a single stage calculation can be awarded without working since you can check the answer yourself, but if ambiguous do not award.

  Follow through marks which involve more than one stage of calculation can only be awarded on sight of the relevant working, even if it appears obvious that there is only one way you could get the answer given.

- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially shows that the candidate did not understand the demand of the question.

- **Linear equations**

  Full marks can be gained if the solution alone is given, or otherwise unambiguously indicated in working (without contradiction elsewhere). Where the correct solution only is shown substituted, but not identified as the solution, the accuracy mark is lost but any method marks can be awarded.

- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$(x^2 + bx + c) = (x + p)(x + q) \text{ where } |pq| = |c|$$

$$(ax^2 + bx + c) = (mx + p)(nx + q) \text{ where } |pq| = |c| \text{ and } |mn| = |a|$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to

3. Completing the square:

Solving $x^2 + bx + c = \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$ where $q \neq 0$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1.

2. Integration:

Power of at least one term increased by 1.

Use of a formula:

Generally, the method mark is gained by

either quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication

from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

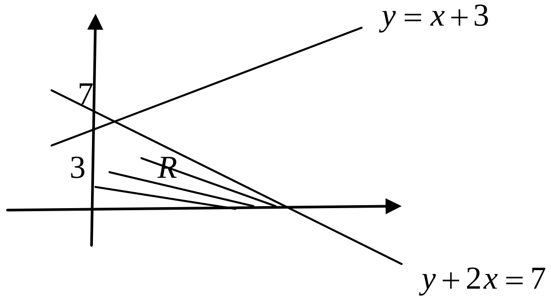
General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show....

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question Number	Answer	Marks
1.(a)		B1 B1 (2)
(b)	(Shade region in or out but R must be indicated.)	B1 (1)
(c)	$2 \leqslant 2+3, \quad 2+2\times 2 = 6 \leqslant 7$	M1
	$\therefore (2,2)$ lies in R	A1 (2) **[5]**

<u>Notes</u>

(a) B1 — for **EITHER** $y = x + 3$ drawn with a positive gradient, with a y intercept of (0, 3) or just 3 marked on the y axis, **OR** for $y + 2x = 7$ drawn with a negative gradient, with a y intercept of (0, 7) or just 7 marked on the y axis. Allow marks on y axis with no numbers provided they are complete for at least up to 7 graduations.

B1 — for BOTH lines drawn correctly

(b) B1 — for the correct region shaded in or out **(no ft)**

(c) M1 — for substitution of $x = 2$ and $y = 2$ into $y \le x + 3$ and $y + 2x \le 7$. (Verifying that $2 \ge 0$ for both x and y is NOT required)

A1 — for conclusion; therefore (2, 2) lies within the region R (cso) (Accept eg., so YES)

Special Case Allow $y = x + 3$ and $y + 2x = 7$ in part (c) with correct substitution for M1A0

Question Number	Answer	Marks
2. (a)	$\tan 2\theta = 1.5$	
	$2\theta = 56.30..., \ 236.30...$	M1
	$\theta = 28.2°, 118.2°$	A1,A1 (3)
(b)	$6\cos^2\theta + 11\cos\theta + 5 = 0$	M1
	$(6\cos\theta + 5)(\cos\theta + 1) = 0$	M1
	$\cos\theta = -\frac{5}{6} \quad \theta = 146.4°$	A1, A1 (4)
	$(\cos\theta = -1$ no solution$)$	
		[7]

Notes

(a) M1 for any valid value of 2θ
A1 for 28.2°
A1 for 118.2°
Use of Radians; allow M1 for $2\theta = 0.9827...$

Alternative

M1 using the correct double angle formula ($\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} \Rightarrow \frac{2\tan\theta}{1-\tan^2\theta} = \frac{3}{2}$) to form and solve a 3TQ $3\tan^2\theta + 4\tan\theta - 3 = 0 \Rightarrow \tan\theta = 0.53518...$, or $\tan\theta = -1.8685.....$

A1 for 28.2°

A1 for 118.2°

(b) M1 for attempting to form the 3TQ (3 terms in any order)
M1d for an attempt to solving their 3TQ as far as $\cos\theta$ (see general principles for definition of an attempt)
A1 for $\cos\theta = -\frac{5}{6}$
A1 for $\theta = 146.4°$ (Note: 180° is outside of the range, do not penalise)

(rounding subject to general principles)
For any extra values within the given range deduct one mark for each up to a maximum of 2 marks in (a) , and 1 mark in (b). Ignore extra values given outside of the range.

Question Number	Answer	Marks
3(a)	$y=\dfrac{e^{2x}}{2x-3}$	M1
	$\dfrac{dy}{dx}=\dfrac{2e^{2x}(2x-3)-e^{2x}\times 2}{(2x-3)^2}$	M1A1A1
	$\dfrac{dy}{dx}=\dfrac{4e^{2x}(x-2)}{(2x-3)^2}$ *	A1cso (5)
(b)	$\dfrac{dy}{dx}=\dfrac{-8}{9}$	B1 (1)
(c)	$x=0\Rightarrow y=-\dfrac{1}{3}$	B1
	$y+\dfrac{1}{3}=-\dfrac{8}{9}x$	M1
	$9y+8x+3=0$ oe	A1 (3) **[9]**

Notes

(a) **Method 1 (quotient rule)**

M1 for an attempt at factorising LHS **and** rearranging to make y the subject. y must be the subject of the equation with e^{2x} only in the numerator

M1d for an attempt at Quotient rule. The denominator must be squared. There must be two terms in the numerator irrespective of order and signs with a clear attempt at some differentiation.
OR, for an attempt at Product rule. There must be two terms irrespective of order and signs with a clear attempt at some differentiation, (see general guidance)

A1 for ONE term correct in the numerator, need not be simplified.

A1 for BOTH terms correct in the numerator, need not be simplified.

A1 for $\frac{dy}{dx}$ fully correct and simplified as shown. **cso** Note this is a show question; sufficient working must be seen to award marks.

(a) **Method 2 (implicit differentiation)**

M1 for the term $3y$ differentiated to give $3\frac{dy}{dx}$, an attempt at product rule on $2xy$, (there must be 2 terms added irrespective of order and signs) and e^{2x} differentiated to give $2e^{2x}$.

M1d for substituting a re-arranged expression for y, $\{ y = \frac{e^{2x}}{2x-3} \}$ into the above in an attempt to eliminate y

A1 for factorising the differentiated expression and making $\frac{dy}{dx}$ the subject to give $\frac{dy}{dx} = \frac{2e^{2x} - 2y}{(2x-3)}$.

A1 for simplifying sufficiently to achieve $(2x-3)^2$ in the denominator

A1 for a fully simplified $\frac{dy}{dx}$ as shown. **cso** Note this is a show question; sufficient working must be seen to award marks.

(b) B1 for $\frac{dy}{dx} = \frac{-8}{9}$ (no ft, cao)

(c) B1 for $y = -\frac{1}{3}$

M1 for using their x, their y, and their $\frac{dy}{dx}$ in $(y - y_1) = m(x - x_1)$, or use $y = mx + c$ to achieve a value for c.

A1 $9y + 8x + 3 = 0$ oe. (integer coefficients)

Question Number	Answer	Marks
4(a)	$a+2d=108 \quad a+11d=54$	M1A1
	$9d=-54 \quad d=-6$	A1 (3)
(b)	$a=108+12=120$	A1 (1)
(c)	$S_n=\frac{n}{2}\left(2\times120+(n-1)\times(-6)\right)$	M1A1
	$=n(120-3n+3)=3n(41-n)$ *	A1 cso (3)
(d)	$3n(41-n)=1200$	M1
	$n^2-41n+400=0$	A1
	$(n-16)(n-25)=0$	M1
	$n=16,\ 25$	A1 (4) **[11]**

Notes

(a) M1 for using $U_n = a + (n-1)d$, where U_3 = 108, U_{11} =54 and n must be 3 and 12 respectively.
A1 for both equation in *a* and *d* correct
A1 for solving the simultaneous equations to give $d = -6$

Alternative 1

M1 for $\frac{U_{12} - U_3}{n_{12} - n_3}$. Award even if the terms and *n*'s are the wrong way around, but numerator must be a difference of *U*'s and denominator must be difference of *n*'s.
A1 for $\frac{54-108}{12-3}$ or $\frac{108-54}{3-12}$
A1 for $d = -6$
.

(b) **B1** for *a* = 120. (A1 in epen)

Other Alternatives

Sight of correct values for *d* and *a* without working, from lists or any other working achieves full marks.

(c) M1 for attempting to use **their** *a* and *d* in $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$ (the formula must be seen first if there are errors in substitution for the award of this mark)
A1 for a fully correct substitution (**their** values) and expression for S_n, no simplification necessary for this mark
A1 for the fully correct simplified expression as shown cso
NB. This is a show question sufficient working must be seen for the award of these marks.

(d) M1 for equating the **given** S_n to 1200 to form an equation in *n*.
A1 for forming a correct 3TQ in *n* *eg* ($n^2 - 41n + 400 = 0$ or $3n^2 - 123n + 1200 = 0$)
M1 for an attempt at solving **their** 3TQ
A1 for *n* = 16, 25 cao

Question Number	Answer	Marks
5.	$\frac{dV}{dt}=72$	B1
	$V=\frac{1}{3}\pi r^2 h=\frac{4}{3}\pi r^3$	B1
	$\frac{dV}{dr}=4\pi r^2$	M1A1
	$\frac{dr}{dt}=\frac{dV}{dt}\times\frac{dr}{dV}$	M1
	$\frac{dr}{dt}=72\div 4\pi r^2=\frac{72}{4\pi\times 9}=0.637$	A1 **[6]**

Notes

Alternative 1 (*r* is the variable)

B1 for $\frac{dV}{dt}=72$

B1 for substituting $h = 4r$ into the formula for the vol of a cone in terms of the single variable r

M1 for an attempt at differentiating their V, (in terms of r only)

A1 for $\frac{dV}{dr}=4\pi r^2$

M1 for correct expression of chain rule (eg., $\frac{dr}{dV}=\frac{dr}{dt}\times\frac{dt}{dV}$) which can lead to $\frac{dr}{dt}$

A1 for using $r=3$ (from $h=4r$) to achieve $\frac{dr}{dt}=0.637$, correct to 3sf

Alternative 2 (*h* is the variable)

B1 for $\frac{dV}{dt}=72$

B1 for substituting $h = 4r$ into the formula for the vol of a cone in terms of the single variable h, ($r=\frac{h}{4}$)

M1 for an attempt to differentiate their V

A1 for $\frac{dV}{dh}=\frac{\pi h^2}{16}$

M1 for a correct expression of chain rule eg., $\frac{dr}{dt}=\frac{dr}{dh}\times\frac{dh}{dV}\times\frac{dV}{dt}$ which can lead to $\frac{dr}{dt}$

A1 for using $h = 12$ to achieve $\frac{dr}{dt}=\frac{1}{4}\times\frac{16}{\pi h^2}\times 72=\frac{2}{\pi}=0.637$

Question Number	Answer	Marks
6(a)	$\left(1+4x^2\right)^{-\frac{1}{5}}=1-\frac{1}{5}\times 4x^2+\frac{\left(-\frac{1}{5}\right)\left(-\frac{6}{5}\right)}{2!}\times\left(4x^2\right)^2+\frac{\left(-\frac{1}{5}\right)\left(-\frac{6}{5}\right)\left(-\frac{11}{5}\right)}{3!}\times\left(4x^2\right)^3$	M1
	$=1-\frac{4}{5}x^2+\frac{48}{25}x^4-\frac{704}{125}x^6$	A1A1A1 (4)
(b)	$\left(-1 \text{ or } 0<\right)4x^2<1, \quad -\frac{1}{2}<x<\frac{1}{2} \text{ or } \lvert x\rvert<\frac{1}{2}$	M1,A1 (2)
(c)	$\mathrm{f}(x)=\left(1+kx\right)\left(1-\frac{4}{5}x^2+\frac{48}{25}x^4-\frac{704}{125}x^6\right)$	M1
	$=1+kx-\frac{4}{5}x^2-\frac{4k}{5}x^3+\frac{48}{25}x^4+\frac{48k}{25}x^5$	M1A1 (3)
(d)	$\frac{48k}{25}=-\frac{4}{5}$	M1
	$k=-\frac{5}{12}$	A1 (2)
		[11]

Notes

(a) M1 for substituting the power and term in x into the correct formula. If there are errors in substitution and the correct formula is not seen do not award this mark.
A1 for a fully correct substitution
A1 for two correct algebraic terms in lowest terms
A1 for a fully correct expansion in lowest terms

(b) M1 allow (−1 or 0 <) $4x^2 < 1$ or $4x^2 \le 1$ for the method mark
A1 for $-\frac{1}{2}<x<\frac{1}{2}$ or $\lvert x\rvert < \frac{1}{2}$ accept $\lvert x\rvert \le \frac{1}{2}$ or $-\frac{1}{2}\le x\le\frac{1}{2}$

(c) M1 for setting $(1 + kx) \times$ their expansion in (a)
M1 for multiplying out $(1 + kx) \times$ their expansion in (a)
A1 for the correct expansion only as far as x^5 (ignore terms over x^5)

(d) M1 For setting their coefficients of x^2 and x^5 equal $\frac{48k}{25}=-\frac{4}{5}$ **Note:** $\frac{48kx^5}{25}=-\frac{4x^2}{5}$ scores M0
A1 for $k=-\frac{5}{12}$ cao

Question Number	Answer	Marks
7(a)	$v_p = -10 + t^2$	M1A1 (2)
(b)	$a_P = 2t$	M1A1 (2)
(c)	$x_Q = \frac{1}{3}t^3 - \frac{3}{2}t^2 + 4t \quad (+c)$	M1A1
	$t = 0,\ x_Q = 0 \Rightarrow c = 0$	B1 (3)
(d)	$8 - 10t + \frac{1}{3}t^3 = \frac{1}{3}t^3 - \frac{3}{2}t^2 + 4t$	M1
	$\frac{3}{2}t^2 - 14t + 8 = 0$ (oe)	A1
	$t = \frac{14 \pm \sqrt{14^2 - 48}}{3}$ $t = 0.6114...,\ 8.721...$	M1
	$\therefore T = 0.61....$ (allow t instead of T and awrt 0.61)	A1 (4)
		[11]

<u>Notes</u>

(a) M1 for an attempt at differentiation of x_p

A1 for the correct expression for v only ($v_p = -10 + t^2$)

(b) M1 for an attempt at differentiating their v_p.

A1 for the correct expression for a_p only ($a_P = 2t$)

(c) M1 for an attempt at integrating v_Q

A1 for the correct expression only $x_Q = \frac{1}{3}t^3 - \frac{3}{2}t^2 + 4t \quad (+c)$

B1 for substitution of $t = 0$ to give $c = 0$

(d) M1 for setting $x_P = x_Q$

A1 for forming the 3TQ (no ft)

M1 for an attempt to solve their 3TQ (usual rules) as far as t = a value

A1 $\Rightarrow t = 0.6114......, (8.721...)$, so $T = 0.61$ or better. If both are given as values for T, withhold this mark. (Accept $\frac{14 - 2\sqrt{37}}{3}$ or $\frac{14 - \sqrt{138}}{3}$). Minimally acceptable accuracy of 2dp.

Question Number	Answer	Marks
8		
(a)	$\alpha+\beta=-\frac{p}{3}\quad \alpha\beta=-\frac{7}{3}$	B1
	(i) $\alpha^2\beta^2=\frac{49}{9}$	B1ft
	(ii) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\frac{p^2}{9}+\frac{14}{3}$	M1A1ft (4)
(b)	$\alpha\beta=\frac{1}{3}(8+\beta)\beta=-\frac{7}{3}$	M1
	$\beta^2+8\beta+7=0$	A1
	$(\beta+7)(\beta+1)=0$	
	$\beta=-1,\ -7$	M1
	$\alpha=\frac{7}{3},\ \alpha=\frac{1}{3}$	A1
	$p=-3(\alpha+\beta)=-4,\ \ 20$	A1 (5)
	Alt: Find α first	
(c)	$x^2-\left(\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}\right)x+\frac{1}{\alpha^2\beta^2}=0$	M1 (= 0 not needed)
	$\dfrac{\frac{16}{9}+\frac{14}{3}}{\frac{49}{9}}$	M1ft(for finding a numerical value of the sum)
	$x^2-\frac{58}{49}x+\frac{9}{49}=0$	A1 (must have = 0) (3) **[12]**

Notes

(a) B1 for BOTH $\alpha+\beta=-\frac{p}{3}$ AND $\alpha\beta=-\frac{7}{3}$

(i) B1ft for $\alpha^2\beta^2=\frac{49}{9}$

(ii) M1 for the correct algebra on $(\alpha+\beta)^2$ to give $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ oe

A1ft for $\alpha^2+\beta^2=\frac{p^2}{9}+\frac{14}{3}$ oe (but need not be simplified)

Accept $\left(\frac{-p}{3}\right)^2+\frac{14}{3}$ and even $\left(\frac{p}{3}\right)^2+\frac{14}{3}$

(b) M1 for rearranging $3\alpha-\beta=8$ to give $\alpha=\frac{\beta+8}{3}$ oe, sub into their $\alpha\beta$, and equate to their $-\frac{7}{3}$

A1 for forming a 3TQ in β AND attempting to solve it

M1d for using their values of β ($\beta=-1,-7$) to find both values of α ft their values

A1 for both values of $\alpha=\frac{7}{3},\frac{1}{3}$ cao

A1 for finding values of p (p = 20, −4)

Alternative 1 – finding *α* first

(b) M1 for rearranging $3\alpha-\beta=8$ to give $\beta=3\alpha-8$ oe, substitute into their $\alpha\beta$, and equate to $-\frac{7}{3}$

A1 for forming a 3TQ { $9\alpha^2-24\alpha+7=0$ } AND attempting to solve it { $(3\alpha-7)(3\alpha-1)=0$ }

M1d for using their values of α ($\alpha=\frac{7}{3},\frac{1}{3}$) to find both values of β ft their values

A1 for both values of $\beta=-1,-7$ cao

A1 for finding p, (p = 20, −4)

Alternative 2 (finding β first)

(b) M1 for attempting to solve $3x^2 + px - 7 = 0$ to give $x = \dfrac{-p \pm \sqrt{p^2+84}}{6}$ and substituting this into $3\alpha - \beta = 8$, to give; $3\left(\dfrac{-p+\sqrt{p^2+84}}{6}\right) - \left(\dfrac{-p-\sqrt{p^2+84}}{6}\right) = 8 \Rightarrow \dfrac{-3p+3\sqrt{p^2+84}+p+\sqrt{p^2+84}}{6} = 8$

A1 for a fully correct expression

M1 for rearranging the above expression to give a 3TQ in p $p^2 - 16p - 80 = 0$

($p^2 - 16p - 80 = 0$)

$$\frac{-3p+3\sqrt{p^2+84}+p+\sqrt{p^2+84}}{6} = \frac{-2p+4\sqrt{p^2+84}}{6} = 8$$

$$-p+2\sqrt{p^2+84} = 24 \Rightarrow 2\sqrt{p^2+84} = 24+p \Rightarrow 4p^2+336 = 576+48p+p^2$$

$$\Rightarrow p^2 - 16p - 80 = 0$$

M1 for attempting to solve their 3TQ (this is an A mark, but treat as M mark)

A1 for p = 20, and $p = -4$

Alternative 3

Same as above but with roots reversed

M1 $3\left(\dfrac{-p-\sqrt{D}}{6}\right) - \left(\dfrac{-p+\sqrt{D}}{6}\right) = 8 \Rightarrow -3p - 3\sqrt{D} + p - \sqrt{D} = 48$ (roots reversed , D = discriminant)

A1 fully correct expression

M1 for forming the 3TQ $-2p - 4\sqrt{D} = 48 \Rightarrow -2\sqrt{D} = 24 + p \Rightarrow p^2 - 16p - 80 = 0$

M1 for attempting to solve their 3TQ (this is an A mark, but treat as M mark)

A1 for p = 20, and $p = -4$

(c) M1 for sum of roots $-\left(\dfrac{\alpha^2+\beta^2}{\alpha^2\beta^2}\right)$ and product of roots $\dfrac{1}{\alpha^2\beta^2}$ substituted into an equation in x, (=0 not required for the award of this mark)

M1 for substituting their values from part (a) to find a numerical SUM ft their values from (a)

A1 for fully correct equation $x^2 - \dfrac{58}{49}x + \dfrac{9}{49} = 0$. cao Need not be simplified, but do not accept un-evaluated double fractions in final answer. (= 0 required for this mark).

Question Number	Answer	Marks
9		
(a)	$x_D = \dfrac{2\times q + 16\times p}{p+q} (=8)$	M1
	$y_D = \dfrac{q\times 5 + p\times 12}{p+q} (=8)$	A1
	Solve to get $p=3,\ q=4$ (or equivalent numbers to give the same ratio)	A1 (3)
	ALT:	
	Use differences in x coords (or y coords) to get $\dfrac{p}{q} = \dfrac{6}{8}$ or $\dfrac{3}{4}$	M1A1
	So $p=3,\ q=4$ (or equivalent numbers to give the same ratio)	A1
(b)	Grad. $AB = \dfrac{12-5}{16-2} = \dfrac{1}{2}$	M1
	Grad perp. $=-2$	A1ft
	Equation perp: $y-8=-2(x-8)$ $(y+2x=24)$	M1A1 (4)
(c)	$y=6 \Rightarrow e=9$	B1 (1)
(d)	F is $(7,10)$ (or method shown and **both** answers incorrect B1B0)	B1B1 (2)
(e)	$EF = \sqrt{2^2+4^2} = 2\sqrt{5}$	M1
	Area kite $= \dfrac{1}{2}AB\times EF = \dfrac{1}{2}\times 7\sqrt{5}\times 2\sqrt{5} = 35$	M1A1 (3)
	ALT: $\dfrac{1}{2}\begin{vmatrix} 2 & 9 & 16 & 7 & 2 \\ 5 & 6 & 12 & 10 & 5 \end{vmatrix}$ oe	M1
	$=\dfrac{1}{2}\left[12+108+160+35-(20+84+96+45)\right]$	M1
	$=35$	A1 (3)
	Alt: Find DF or DE	M1
	Area = $2\times\dfrac{1}{2}AB\times DF$, = 35 accept awrt 35 (2sf)	M1,A1
		[13]

Notes

(a) M1 for using the correct formula with substitution the correct way for x OR y coordinates

A1 For using the correct formula with correct substitution for BOTH x AND y coordinates

A1 for the ratio $p : q = 3 : 4$ oe Accept $p = 3$, $q = 4$

Alternative 1

M1 for finding differences in similar triangles in x and/or y

A1 for finding the ratio $p : q = 3 : 4$

A1 so, $p : q = 3 : 4$ or accept $p = 3$, $q = 4$

Alternative 2

M1 for finding the length of either AD or DB ($3\sqrt{5}$ and $4\sqrt{5}$ respectively)

A1 for both lengths AD and DB

A1 for the ratio AD:DB = $3\sqrt{5} : 4\sqrt{5}$ (or 3 :4)

Special case: accept ratios of 3 :7 or 4 : 7 for M1A1A0 **provided** a correct method is shown.

(b) M1 for finding the gradient of the line AB using $m = \frac{y_1 - y_2}{x_1 - x_2}$ to give $m = \frac{1}{2}$

A1ft for using $m_1 \times m_2 = -1$ to give gradient of the perpendicular $= -2$

M1 for attempting to find the equation of the normal through D. The formula must be seen first if there are errors in substitution, using a 'changed' gradient from the tangent and which must be a numerical value.

Or, using a complete method using $y = mx + c$ as far as achieving a value for c.

A1 for $y - 8 = -2(x - 8)$ $(y + 2x = 24)$ oe

(c) B1 for $e = 9$

(d) B1 for correct coordinate of x (7) or y (10)

B1 for both x and y correct . If both are incorrect and the correct method has been used (eg, they have followed through from an incorrect value for e in part (c), Award B1B0

(e) M1 for finding the length (ft their coordinates) EF or DF ($2\sqrt{5}$) or the length AB $7\sqrt{5}$

M1d for using the formula for the area of a triangle $\frac{1}{2} \times \text{base} \times \text{height}$ or any other FULL method for the area of a kite

A1 for 35, accept awrt 35

Alternatives to find the area in the ms

Question Number	Answer	Marks
10		
(a)	Vol cuboid $=3\times DE\times BE$ Vol pyramid $=\frac{1}{3}\times\text{height}\times DE\times BE$	M1
	Same so height = 9 (cm) *	A1 (2)
(b)	$EC^2=5^2+4^2$	M1
	$AE^2=\text{height}^2+\frac{1}{4}\times 41$, $AE=9.552...=9.55$ (cm)	M1,A1 (3)
(c)	$EH^2=EC^2+CH^2=41+9$	M1
	$EH=\sqrt{50}=7.07$ (cm)	A1 (2)
(d)	$\tan\theta=\frac{\text{height}}{\frac{1}{2}BD}=\frac{9}{\frac{1}{2}\times\sqrt{41}}$ (or other appropriate trig ratio)	M1A1ft
	$\theta=70.41...=70.4°$	A1 (3)
(e)	Angle between plane ABE and plane $BCDE$:	
	$\tan\phi=\frac{9}{\frac{1}{2}\times 4}=\frac{9}{2}$ ($\phi=77.47...$)	M1A1
	Angle between plane $BCDE$ and plane $BEIH$	
	$\tan\psi=\frac{3}{4}$	M1A1
	Reqd angle $=\tan^{-1}\left(\frac{9}{2}\right)+\tan^{-1}\left(\frac{3}{4}\right)=114.34...=114.3°$	A1 (5)
		[15]

Notes

(a) M1 for using Vol cuboid $= 3 \times DE \times BE$ AND Vol pyramid $= \frac{1}{3} \times \text{height} \times DE \times BE$

A1 so $3 \times DE \times BE = \frac{1}{3} \times \text{height} \times DE \times BE$ and height = 9 (cm) *Show question*

(b) M1 for finding the length of EC^2 or EC, 41or $\sqrt{41}$ respectively

M1 for attempting to find the length AE^2 or AE

A1 for 9.552… correctly rounded to 9.55 (cm) (3sf)

(c) M1 for using $EH^2 = EC^2 + CH^2 = 41 + 9$

A1 for $EH = \sqrt{50} = 7.07$ (cm) rounded to 3 sf. (unless already penalized in (b))

(d) M1 for attempting to use any appropriate trigonometry to find the angle between AE and their $\sqrt{41} \div 2$. (ft their EC).

A1ft for using their correct lengths throughout.

A1 for $\theta = 70.41... = 70.4°$ rounded correctly to 1 dp

(e) M1 for using any appropriate trigonometry to find the angle between the planes ABE and $BCDE$

A1 for finding the angle $\tan^{-1}\left(\frac{9}{2}\right)$ or $\phi = 77.47.......$

M1 for using any appropriate trigonometry to find the angle between the planes $BCDE$ and $BEIH$

A1 for finding the angle $\tan^{-1}\left(\frac{3}{4}\right)$ or $\psi = 36.869...$

A1 for the required angle $(\phi + \psi)$ correct to 1dp = 114.3º

Alternative

M1 for attempting to find the length l of A to the midpoint of IH using Pythagoras theorem, (height of the triangle = 9 + 3cm, the base of the triangle = 2cm. $l=\sqrt{12^2+2^2}$), the length m of A to the midpoint of EB (height = 9 cm, base = 2cm, $m=\sqrt{2^2+9^2}=\sqrt{85}$) AND the length of $EF(=\sqrt{3^2+4^2}$)

A1 for $l=2\sqrt{37}$ or 12.16552…..and $m=\sqrt{85}$ or 9.219544…EF = 5 cm

M1 for attempting to use cosine rule with their l, m and EF. (If there are errors in substitution the correct formula must be seen first)

A1 for fully correct cosine rule with their values

$$\theta=\cos^{-1}\left(\frac{85+5^2-148}{2\times\sqrt{85}\times5}\right)=114.34.....\ \theta=\cos^{-1}\left(\frac{85+5^2-148}{2\times\sqrt{85}\times5}\right)=114.34.....$$

A1 for the required angle (correct to the nearest 0.1°) of 114.3° cao

Note A: there are two types of rounding in this question so;

- Deduct 1 mark for failing to round to 3 significant figures
- Deduct 1 mark separately for failing to round to 1 dp,

both subject to a maximum of one each for repeated rounding errors.

Note B: Please note General Guidance for rounding. For answers with rounding errors that arise as a result of accumulated rounding errors, General Guidance does not apply. The given answers must round to the required values. (ie., awrt etc.,)

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