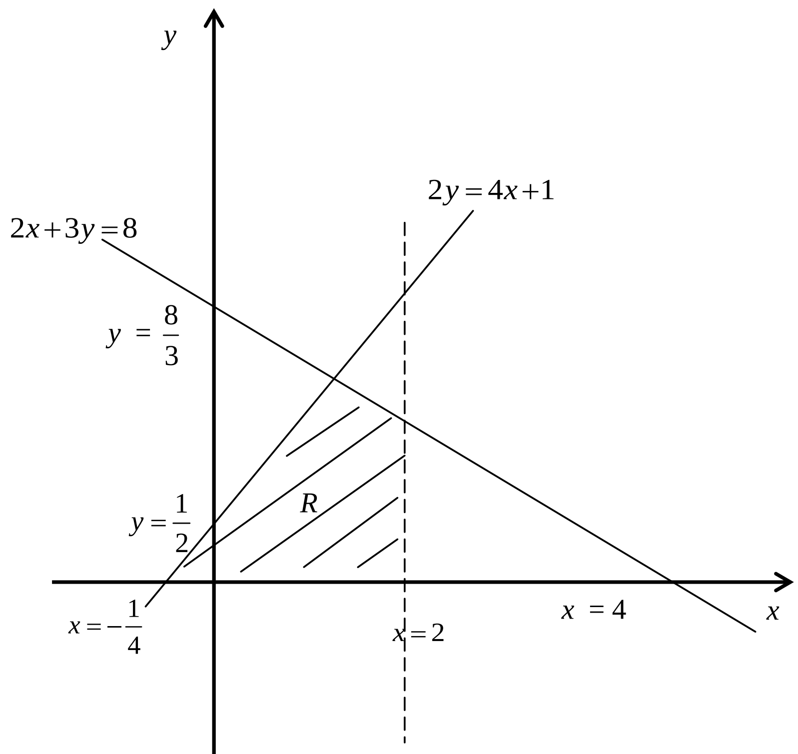
International GCSE Futher Pure Mathematics – Paper 1 mark scheme

Paper 1					
Question	Working	Answer	Mark	AO	Notes
1 (a)			B1 B1	1	One correct line Both correct lines
(b)			B1 B1 **(4)**	1	Correct line x = 2 drawn Correct region shaded in or out.

Question	Working	Answer	Mark	AO	Notes
2 (a)	$\pi = 6\theta \Rightarrow \theta = \frac{\pi}{6} \Rightarrow AOB = \frac{5\pi}{6} = (150^{\circ})$		B1	1	
	$AB = \sqrt{10^2 + 6^2 - 2\times10\times6\times\cos\left(\frac{5\pi}{6}\right)} = 15.4894... = 15.5$ cm	15.5 (cm)	M1A1	2	Accept working in degrees $AOB = 150^{\circ}$
(b)	$\text{Area} = \frac{1}{2}\times10\times6\times\sin\frac{5\pi}{6} + \frac{\pi}{6}\times\frac{6^2}{2} = 24.424...$	24.4 (cm^2)	M1M1A1	2	
	ALTERNATIVE $\text{Area} = \frac{1}{2}\times10\times6\times\sin\frac{5\pi}{6} + \frac{1}{2}\times\pi\times6 = 24.424...$		M1M1A1 **(6)**		
3	$2\cos(2\theta+30) + \frac{\sin(2\theta+30)}{\cos(2\theta+30)} = 0 \Rightarrow 2\cos^2(2\theta+30) + \sin(2\theta+30) = 0$		M1	2	
	$\Rightarrow 2 - 2\sin^2(2\theta+30) + \sin(2\theta+30) = 0$		M1A1		
	$\sin(2\theta+30) = \frac{1\pm\sqrt{1-4\times2\times(-2)}}{2\times2} = 1.2807..., -0.7807....$		M1	3	Solves 3 TQ
	$2\theta+30 = -51.33167..., 231.33167, 308.66833$		A1A1		Finds one angle from their 3TQ
	$\theta = 100.7,\ 139.3$	$\theta = 100.7,\ 139.3$	**(6)**		

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4 **(a)**	$v = 0$ so $4t^2 - 19t + 12 = 0 \Rightarrow (4t-3)(t-4) = 0 \Rightarrow t = \frac{3}{4}, 4$	$t = \frac{3}{4}, 4$	M1A1	1	
(b)	$s = \int 4t^2 - 19t + 2\,\mathrm{d}t = \frac{4t^3}{3} - \frac{19t^2}{2} + 12t + c$ when $t = 0$, $s = -4 \Rightarrow c = -4$		M1M1A1	2	
	When $t = 6$, $s = \frac{4\times 6^3}{3} - \frac{19\times 6^2}{2} + 12\times 6 - 4 = 14$	14	A1		
(c)	$a = \frac{\mathrm{d}v}{\mathrm{d}t} = 8t - 19 \Rightarrow 8t - 19 = 0 \Rightarrow t = \frac{19}{8}$	$t = \frac{19}{8}$	M1M1A1 **(9)**	3	
5	$2x + y = 13 \Rightarrow y = 13 - 2x$		B1	1	
(a)	$S = 4x^2 + (13-2x)^2 = 4x^2 + 169 - 52x + 4x^2 = 8x^2 - 52x + 169$	$8x^2 - 52x + 169$	M1A1		
(b)	$\frac{\mathrm{d}S}{\mathrm{d}x} = 16x - 52$, $\frac{\mathrm{d}S}{\mathrm{d}x} = 0 \Rightarrow 16x - 52 = 0 \Rightarrow x = \frac{13}{4}$	$x = \frac{13}{4}$	M1M1A1	2,3	
	$\frac{\mathrm{d}^2S}{\mathrm{d}x^2} = 16$ $\quad 16 > 0$,	Hence minimum	B1		
(c)	$S = 8\times\left(\frac{13}{4}\right)^2 - 52\times\frac{13}{4} + 169 = \frac{169}{2} = 84.5$	$\frac{169}{2} = 84.5$	M1A1 **(9)**	3	

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6	$\frac{dy}{dx}=e^x(x^2-3x)+e^x(2x-3)\left[\Rightarrow e^x(2x-3)=\frac{dy}{dx}-y\right]$		M1M1A1	4	
	$\frac{d^2y}{dx^2}=e^x(x^2-3x)+e^x(2x-3)+e^x(2x-3)+2e^x=y+2\left(\frac{dy}{dx}-y\right)+2e^x$		M1A1	4	
	$2e^x=\frac{d^2y}{dx^2}-2\left(\frac{dy}{dx}-y\right)-y\Rightarrow 2e^x=y-2\frac{dy}{dx}+\frac{d^2y}{dx^2}$ *		M1M1A1 **(8)**		
7					
(a)	$\begin{array}{c\|c\|c\|c\|c\|c\|c} x & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline y & 3 & \mathbf{3.83} & \mathbf{5} & \mathbf{6.66} & 9 & 12.31 \end{array}$		B1B1	1	
(b)	All points plotted within an accuracy of half of a square. A smooth curve drawn through their points		B1B1	1	
(c)	$\log_2(4x-6)^2-x=2\Rightarrow 2\log_2(4x-6)=x+2\Rightarrow\log_2(4x-6)=\frac{x}{2}+1$ $\Rightarrow 4x-6=2^{\left(\frac{x}{2}+1\right)}\Rightarrow 4x-5=2^{\left(\frac{x}{2}+1\right)}+1$		M1M1	2	
	Line $y=4x-5$ drawn on graph $\Rightarrow$ so x = 2.8(36)	x = 2.8	M1A1 **(8)**		

Question	Working	Answer	Mark	AO	Notes
8 **(a)**	$a = S_1 = 2\times1\times(1+3) = 8$	$a = 8$	B1	1	
(b)	$S_2 = 2\times2\times(2+3) = 20$ $S_2 = a + T_2 \Rightarrow T_2 = S_2 - a = 20 - 8 = 12$ $d = 12 - 8 = 4$	$d = 4$	M1A1	1	
(c)	Uses given formula for $S_n = 2n(n+3)$ and formula for *nth* term $T_n = a + (n-1)d$ $6\left[2(n-4)(n-4+3)\right] = 7\left[8+(n+3-1)4\right]$ $\Rightarrow 12n^2 - 88n - 64 = 0 \Rightarrow 3n^2 - 22n - 16 = 0$ $(n-8)(3n+2) = 0$ $\Rightarrow n = 8, \left(n = -\frac{2}{3}\right)$	$n = 8$	M1M1 M1A1 M1A1	2, 3	
	ALT Using formula $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$ $6\left[\frac{(n-4)}{2}\left(2\times8 + ((n-4)-1)4\right)\right] = 7\left[8+(n+3-1)4\right]$ $\Rightarrow 12n^2 - 88n - 64 = 0 \Rightarrow 3n^2 - 22n - 16 = 0$ $(n-8)(3n+2) = 0$ $\Rightarrow n = 8, \left(n = -\frac{2}{3}\right)$	$n = 8$	M1M1 M1A1 M1A1 **(9)**	2, 3	

Question	Working	Answer	Mark	AO	Notes
9 **(a)**	$\alpha+\beta=-\frac{7}{3}\quad \alpha\beta=-2=-\frac{6}{3}$ so $a=3$, $b=7$ and $c=-6$		B1B1	1	
	Hence quadratic equation $\Rightarrow 3x^2+7x-6=0$ oe with integer coefficients		M1A1		
(b)	$(\alpha-\beta)^2=\alpha^2+\beta^2-2\alpha\beta=(\alpha+\beta)^2-4\alpha\beta=\left(-\frac{7}{3}\right)^2-4\times-2=\frac{121}{9}$			3	
	$\alpha>\beta$ so $\alpha-\beta=\frac{11}{3}$ *		M1A1		
(c)	Sum				
	$\frac{\alpha+\beta}{\alpha}+\frac{\alpha-\beta}{\beta}=\frac{\beta(\alpha+\beta)+\alpha(\alpha-\beta)}{\alpha\beta}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{\left(-\frac{7}{3}\right)^2-2\times-2}{-2}=-\frac{85}{18}$		M1M1A1	3	
	Product $\frac{(\alpha+\beta)}{\alpha}\times\frac{(\alpha-\beta)}{\beta}=\frac{\left(-\frac{7}{3}\right)\times\left(\frac{11}{3}\right)}{-2}=\frac{77}{18}$		M1A1		
	Equation $18y^2+85y+77=0$ oe with integer coefficients		M1A1		
			(13)		

Question	Working	Answer	Mark	AO	Notes
10 **(a)**	Let M be the midpoint of diagonals AC or DB $AC = \sqrt{8^2+8^2} = 8\sqrt{2} \Rightarrow AM = 4\sqrt{2}$ $h = \sqrt{12^2 - \left(4\sqrt{2}\right)^2} = \sqrt{112} = \left(4\sqrt{7}\right)$	$4\sqrt{7}$	M1M1A1	1	
(b)	$\tan^{-1}\left(\frac{4\sqrt{7}}{4\sqrt{2}}\right) = 61.87449... \approx 61.9^\circ$	61.9°	M1A1	1	Or any equivalent trigonometry
(c)	Let N be the midpoint of AB Angle the plane AOB makes with horiz = $\tan^{-1}\left(\frac{4\sqrt{7}}{4}\right) = 69.295... \approx 69.3^\circ$	69.3°	M1A1	3	Or any equivalent trigonometry
(d)	**By using the symmetrical properties of the pyramid** Let S be the perpendicular from P to diagonal AC Let R be the perpendicular from S to side BC In triangle PSR → $PR = \sqrt{\left(2\sqrt{17}\right)^2 - 2^2} = 8$ In triangle PRQ → $PQ = \sqrt{8^2+4^2} = 4\sqrt{5}$	$4\sqrt{5}$	M1A1 M1A1	3	Or any equivalent system of right angle triangles
(e)	Length $AQ = \sqrt{8^2+6^2} = 10$ Angle of $PQA = \cos^{-1}\left(\frac{10^2+\left(4\sqrt{5}\right)^2-6^2}{2\times10\times4\sqrt{5}}\right) = 36.39124... \approx 36.4^\circ$	36.4°	M1 M1A1A1 **(15)**	3	

Question	Working	Answer	Mark	AO	Notes
(d)	**ALTERNATIVE without using the symmetrical properties of the pyramid** $\cos OAB = \frac{12^2+8^2-12^2}{2\times8\times12} = \frac{1}{3}$ $\Rightarrow PB = \sqrt{6^2+8^2-2\times6\times8\times\frac{1}{3}} = 2\sqrt{17}$ In triangle *PBC* $PC = \sqrt{6^2+\left(8\sqrt{2}\right)^2-2\times6\times\left(8\sqrt{2}\right)\times\cos\left(\tan^{-1}\left(\frac{\sqrt{7}}{\sqrt{2}}\right)\right)} = 10$ $\Rightarrow$ Angle $PBC = \cos^{-1}\left(\frac{8^2+68-10^2}{2\times8\times2\sqrt{17}}\right) = 75.9637.....^\circ$ In triangle *PBQ;* $PQ = \sqrt{6^2+68-2\times6\times2\sqrt{17}\times\cos 75.96375...} = 4\sqrt{5}$	$4\sqrt{5}$	M1 M1 M1A1	3	

Question	Working	Answer	Mark	AO	Notes
11	Mark parts (i) and (ii) together			1, 3	
(a)	$y=\int 6x^2-26x+12\mathrm{d}x=\left[\frac{6x^3}{3}-\frac{26x^2}{2}+12x+c\right]$		M1		
	At the point $(-1,0)$ $0=2(-1)^3-13(-1)^2+12(-1)+C\Rightarrow C=27$		M1A1		
	$(2x^3-13x^2+12x+27)\div(x+1)=2x^2-15x+27=(2x-9)(x-3)$		M1M1A1		
	$\Rightarrow a=\frac{9}{2}, b=3$	$a=\frac{9}{2}, b=3$	B1B1		
(b)	Area $=\int_0^3 2x^3-13x^2+12x+27\mathrm{d}x+\left\|\int_3^{\frac{9}{2}} 2x^3-13x^2+12x+27\mathrm{d}x\right\|=$		M1M1	3	
	$\left[\frac{2}{4}x^4-\frac{13}{3}x^3+\frac{12}{2}x^2+27x\right]_0^3+\left\|\left[\frac{2}{4}x^4-\frac{13}{3}x^3+\frac{12}{2}x^2+27x\right]_3^{\frac{9}{2}}\right\|=\frac{2043}{32}$		A1M1A1		
	So Area $=\frac{2043}{32}$	Area $=\frac{2043}{32}$	**(13)**		
		Total	**100**		