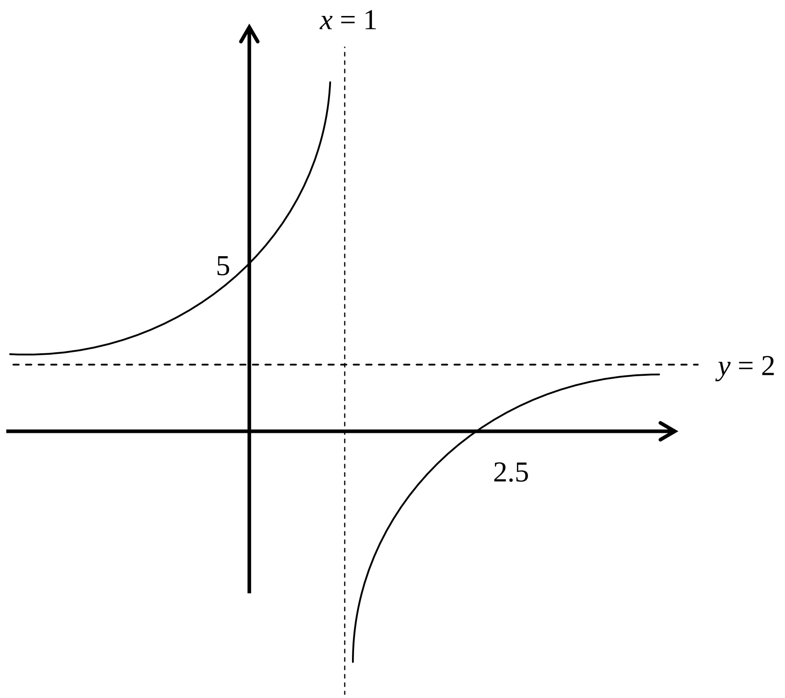
International GCSE Futher Pure Mathematics – Paper 2 mark scheme

Paper 2					
Question	**Working**	**Answer**	**Mark**	**AO**	**Notes**
1	First term = $3e^{-1}$, Common Ratio = e^{-2}		B1B1	1	
	$S = \frac{3e^{-1}}{1-e^{-2}} \times \left(\frac{e^2}{e^2}\right) = \frac{3e}{e^2-1}$	$S_\infty = \frac{3e}{e^2-1}$	M1M1 A1	2	
	(So, $a = 3$ and $b = 2$)		**(5)**		
2		$x > 4$	B1		
(a)	$3 + x < 2x - 1 \Rightarrow x > 4$			1	
	$x(x-1) > 6 \Rightarrow x^2 - x - 6 > 0 \Rightarrow (x-3)(x+2) > 0$		M1		
(b)	critical values are $x = 3, -2$ so $x > 3$ OR $x < -2$	$x > 3$ OR $x < -2$	M1A1	1	(Outside region)
(c)	$x > 3$ OR $x < -2$, $x > 4$ so $x > 4$	$x > 4$	B1 **(5)**	1	

Question	Working	Answer	Mark	AO	Notes
3 (a)	$\overrightarrow{AB} = -(4\mathbf{i}+3\mathbf{j})+(8\mathbf{i}+p\mathbf{j}) = 4\mathbf{i}+(p-3)\mathbf{j}$		B1	1	
	$\lvert\overrightarrow{AB}\rvert = \sqrt{52} = \sqrt{4^2+(p-3)^2} \Rightarrow p^2-6p-27=(p-9)(p+3)=0$		M1A1		
	$\Rightarrow p=9, p=-3$				
(b)	$(p>0 \quad \text{so } p=9)$				
	$\overrightarrow{AB} = 4\mathbf{i}+(p-3)\mathbf{j} = 4\mathbf{i}+6\mathbf{j} \quad \Rightarrow \frac{1}{2\sqrt{13}}(4\mathbf{i}+6\mathbf{j}) = \frac{1}{\sqrt{13}}(2\mathbf{i}+3\mathbf{j})$ (oe)	$\frac{1}{2\sqrt{13}}(4\mathbf{i}+6\mathbf{j})$	B1B1	1	
			(5)		
4 (a)	$f(-3)=2\times(-3)^2+p\times(-3)^2+q\times(-3)+12=0$				Mark parts (i) and (ii) together
	$\Rightarrow 42=9p-3q \Rightarrow 14=3p-q$		M1	3	
	$f'(x)=6x^2+2px+q$		M1		
	$f'(-3)=6\times(-3)^2+2p\times(-3)+q=37$		M1A1		
	$\Rightarrow -6p+q=-17 \Rightarrow q=17-6p$				
	$14=3p-1(6p-17) \Rightarrow p=1, q=-11$	$q=-11$	M1A1		Solving simultaneous equations (by any method)
(b)	$\frac{(2x^3+x^2-11x+12)}{(x+3)} = 2x^2-5x+4$		M1A1	2	
	$\Rightarrow (x+3)(2x^2-5x+4)$				
(c)	$b^2-4ac=(-5)^2-4\times2\times4=-7$ (7 hence no real roots for quadratic factor, so x = -3 only real root.)		M1A1	1	
			(10)		

Question	Working	Answer	Mark	AO	Notes
5 (a)	$\cos(A-B)-\cos(A+B)=\cos A\cos B+\sin A\sin B-\cos A\cos B+\sin A\sin B$ $\Rightarrow 2\sin A\cos B$		M1A1	1	
(b)	$2\sin 5x\sin 3x=\cos 2x-\cos 8x$	$\cos 2x-\cos 8x$	B1	2	
(c)(i)	$\int 4\sin 5\theta\sin 3\theta \mathrm{d}\theta=2\int \cos 2\theta-\cos 8\theta \mathrm{d}\theta=2\left[\frac{\sin 2\theta}{2}-\frac{\sin 8\theta}{8}\right](+c)$		M1A1	3	
(ii)	$\int_0^{\frac{\pi}{6}} 4\sin 5\theta\sin 3\theta \ \mathrm{d}x=2\int_0^{\frac{\pi}{6}}(\cos 2\theta-\cos 8\theta)\ \mathrm{d}\theta=2\left[\frac{\sin 2\theta}{2}-\frac{\sin 8\theta}{8}\right]_0^{\frac{\pi}{6}}=\frac{5\sqrt{3}}{8}$	$\frac{5\sqrt{3}}{8}$	M1A1 **(7)**		
6	$\log_x 2=\frac{\log_2 2}{\log_2 x}=\frac{1}{\log_2 x}$ $\log_2 x+\frac{6}{\log_2 x}=7\Rightarrow(\log_2 x)^2-7\log_2 x+6=0$ $\Rightarrow(\log_2 x-6)(\log_2 x-1)=0\Rightarrow\log_2 x=6,\ \log_2 x=1$ $\Rightarrow x=64,\ \ x=2$	64, 2	B1 M1M1 A1 M1M1 A1 **(7)**	2, 3	

Question	Working	Answer	Mark	AO	Notes
7 **(a)**	(i) $0=\dfrac{ax-5}{x-b}\Rightarrow ax-5=0\Rightarrow a\dfrac{5}{2}-5=0\Rightarrow a=2$		M1A1	2	
	(ii) $b=1$	$b=1$	B1	2	
(b)	When $x=0$, $y=\dfrac{-5}{-b}\Rightarrow y=\dfrac{5}{1}\Rightarrow y=5$	(0, 5)	B1		Accept $y=5$
(c)	$y=2$	$y=2$	B1	3	
(d)			B1 B1 B1 **(8)**	3	Curve in correct quadrants Correct asymptotes drawn Correct intersections with axes

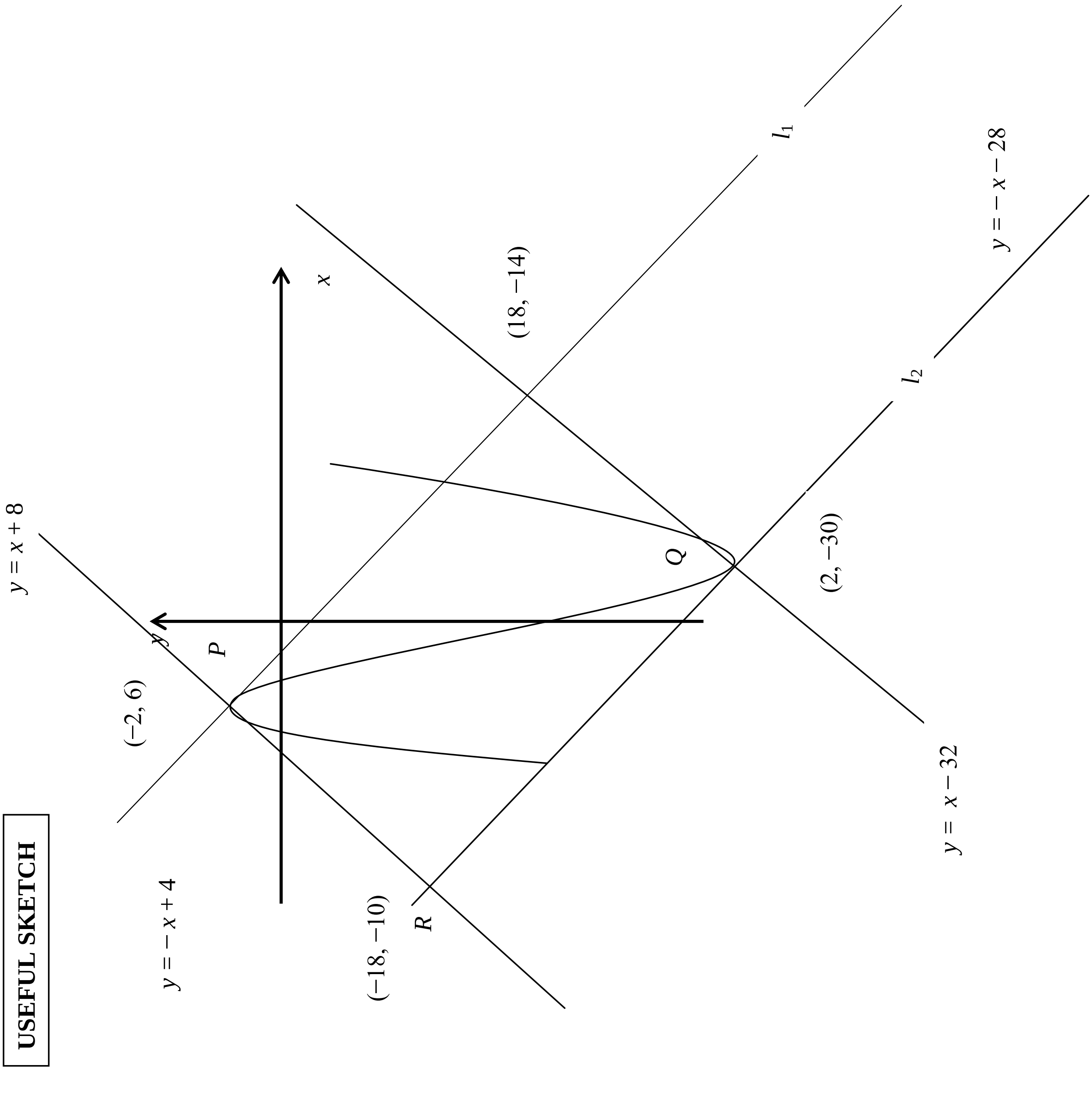
Question	Working	Answer	Mark	AO	Notes
8 (a)	$\frac{3}{\sqrt{1-2x}}=3(1-2x)^{-\frac{1}{2}}\Rightarrow$		B1	2	
	$=3\left\{1+\left(-\frac{1}{2}\right)(-2x)+\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\frac{(-2x)^2}{2!}+\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\frac{(-2x)^3}{3!}...\right\}$		M1A1		
	$\frac{3}{\sqrt{1-2x}}=3+3x+\frac{9}{2}x^2+\frac{15}{2}x^3$		A1		
(b)	$-\frac{1}{2}<x<\frac{1}{2}$ or $\lvert x\rvert<\frac{1}{2}$	$-\frac{1}{2}<x<\frac{1}{2}$	B1	2	
(c)	$\frac{3}{\sqrt{0.9}}=\frac{3}{\sqrt{9/10}}=\frac{3}{3/\sqrt{10}}=\sqrt{10}$		B1	3	
(d)	$\frac{1}{\sqrt{10}-3}\times\frac{(\sqrt{10}+3)}{(\sqrt{10}+3)}=\frac{(\sqrt{10}+3)}{1}=(\sqrt{10}+3)$	$(\sqrt{10}+3)$	M1A1	3	
(e)	$1-2x=0.9\Rightarrow x=0.05$		B1	3	
	$\sqrt{10}+3=3+3\times(0.05)+\frac{9}{2}\times(0.05)^2+\frac{15}{2}\times(0.05)^3+...+3=3.1621875...+3$	6.16219	M1A1		
	≈ 6.16219		**(11)**		

Question	Working	Answer	Mark	AO	Notes
9 (a)	$7+4x-2x^2=-2\left[(x-1)^2-1\right]+7=-2(x-1)^2+9$ $P=-2, Q=-1, R=9$		M1A1 A1	1	
(b)	(i) 9 (ii) 1	(i) 9 (ii) 1	B1B1	2	
(c)	$7+4x-2x^2=4-x \Rightarrow 2x^2-5x-3=0$ $\Rightarrow (2x+1)(x-3)=0 \Rightarrow x=-\frac{1}{2},3$	$x=-\frac{1}{2}, x=3$	M1M1 A1	2	
(d)	$V=\pi\int_{-0.5}^{3}\left(7+4x-2x^2\right)^2 \mathrm{d}x=\pi\int_{-0.5}^{3}(4-x)^2 \mathrm{d}x$ $\Rightarrow \pi\int_{-0.5}^{3} 33+64x-13x^2-16x^3+4x^4 \mathrm{d}x$ $\Rightarrow \pi\left[33x+32x^2-\frac{13x^3}{3}-4x^4+\frac{4x^5}{5}\right]_{-0.5}^{3}=\pi\frac{4459}{30}=466.945\ldots\approx 467$	467	M1 M1A1 M1A1 **(13)**	3	

Question	Working	Answer	Mark	AO	Notes
10 (a)	$y = f(-2) = 6$		B1	1	
	$\frac{dy}{dx} = 3x^2 - 13 \Rightarrow f'(-2) = 13 - 13 = -1$		M1A1		
	Equation of tangent $y - 6 = -1(x - (-2)) \Rightarrow y + x - 4 = 0$	$y + x - 4 = 0$	M1A1		
(b)	$-1 = 3x^2 - 13 \Rightarrow 3x^2 - 12 = 0 \Rightarrow x^2 - 4 = 0 \Rightarrow (x-2)(x+2) = 0$ at $Q, x = 2$		M1		
	$f(2) = -30$		B1	1, 2	
	$y - -30 = -(x - 2) \Rightarrow \{y = -x - 28\}$	$y = -x - 28$	M1A1		
(c)	Gradient of normal = 1		B1	2	
	Equation of Normal $y - 6 = x - (-2) \Rightarrow \{y = x + 8\}$ At R, $x + 8 = -x - 28 \Rightarrow x = -18$		M1		
	When $x = -18$, $y = -10$	$(-18, -10)$	A1A1		

Question	Working	Answer	Mark	AO	Notes
10 cont'd (d)	{Coordinates of R $(-18, -10)$}				
	$PR=\sqrt{(6--10)^2+(-2--18)^2}=16\sqrt{2}$	$16\sqrt{2}$	M1A1	3	
(e)	Area of rectangle				
	Length of $QR=\sqrt{(-18-2)^2+(-10--30)^2}=20\sqrt{2}$		M1	3	
	Area = $16\sqrt{2}\times 20\sqrt{2}=640$	640	M1A1		
(e)	**ALTERNATIVE** Equation of normal at Q $y--30=x-2\Rightarrow y=x-32$ Coordinates of 4th vertex of rectangle			3	
	$x-32=-x+4\Rightarrow x=18$ so $y=-14$		M1		
	Area = $\frac{1}{2}\begin{bmatrix}-2 & -18 & 2 & 18 & -2\\ 6 & -10 & -30 & -14 & 6\end{bmatrix}=640$	640	M1A1 **(18)**		

USEFUL SKETCH



Question	Working	Answer	Mark	AO	Notes
11	Total Surface Area of the cone				
	$l = \frac{x}{\sin 30^\circ} = 2x$		B1	2, 3	
	$A = \pi r l + \pi r^2 \Rightarrow \pi x \times 2x + \pi x^2 = 3\pi x^2$		M1A1		
	Volume of cone				
	$h = \frac{x}{\tan 30^\circ} = \sqrt{3}x$		B1		
	$\text{Vol} = \frac{1}{3}\pi r^2 h \Rightarrow V = \frac{\sqrt{3}}{3}\pi x^3$		M1A1		
	$\frac{dA}{dt} = 10\ (\text{cm}^2/s)$ $\quad \frac{dV}{dx} = \sqrt{3}\pi x^2, \frac{dA}{dx} = 6\pi x$		B1M1		M1 for differentiating **either** $\frac{dV}{dx}$ or $\frac{dA}{dx}$
	$\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dA}{dt} \times \frac{dx}{dA}$		M1		
	$\Rightarrow \frac{dV}{dt} = \sqrt{3}\pi x^2 \times 10 \times \frac{1}{6\pi x} = \frac{5\sqrt{3}}{3} x \Rightarrow 10\sqrt{3}\ (\text{cm}^3/s)$	$10\sqrt{3}\ (\text{cm}^3/s)$	M1A1		
			(11)		
		Total	**100**		