

Please check the examination details below before entering your candidate information

Candidate surname	Other names
Pearson Edexcel	Centre Number
International GCSE	Candidate Number
Monday 21 January 2019	
Morning (Time: 2 hours)	Paper Reference 4PM0/02
Further Pure Mathematics	
Paper 2	
Calculators may be used.	Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
– *there may be more space than you need.*

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Solve the equation $3 \log_3 x - 8 \log_x 3 = 10$

(6)

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Question 1 continued

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(Total for Question 1 is 6 marks)



2 (a) Using the axes below, sketch the line with equation

$$(i) y + 2x = -5 \quad (ii) y = x + 4$$

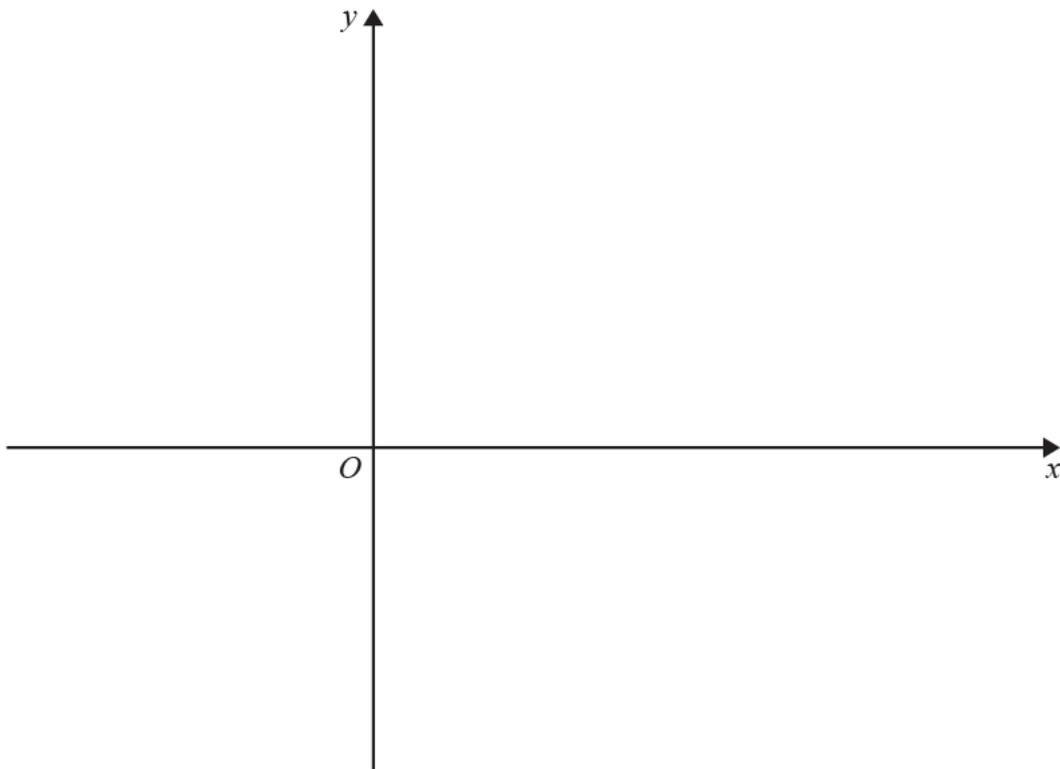
Show the coordinates of the points where each line crosses the coordinate axes.

(2)

(b) Show, by shading, the region R defined by the inequalities

$$y + 2x > -5 \quad y < x + 4 \quad x < 1$$

(1)



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Question 2 continued

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(Total for Question 2 is 3 marks)



3 Referred to a fixed origin O , the position vectors of the points P and Q are $(5\mathbf{i} + 6\mathbf{j})$ and $(3\mathbf{i} - 4\mathbf{j})$ respectively.

(a) Find, as a simplified expression in terms of \mathbf{i} and \mathbf{j} , \overrightarrow{PQ} . (2)

(b) Find a unit vector parallel to \overrightarrow{PQ} . (2)

The position vector of the fixed point R is $(13\mathbf{i} + a\mathbf{j})$, where a is a constant.

Given that $\overrightarrow{QR} = 5\overrightarrow{QP}$

(c) find the value of a . (2)

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Question 3 continued

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(Total for Question 3 is 6 marks)



4 A particle P is moving along the x -axis. At time t seconds ($t \geq 0$) the velocity, v m/s, of P is given by $v = 4 \sin 2t$

(a) Find the least value of t for which the velocity of P is 2 m/s. (2)

(b) Find the magnitude of the acceleration of P when its velocity is 2 m/s. (3)

The particle P is at the point with coordinates $(3, 0)$ when $t = \frac{\pi}{4}$

(c) Find the distance of P from the origin when $t = 0$ (4)

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Question 4 continued

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(Total for Question 4 is 9 marks)



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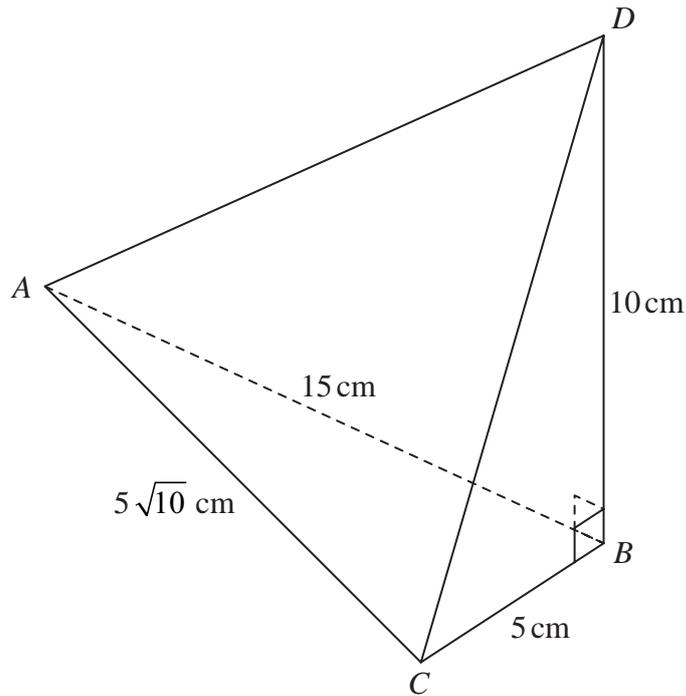
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Figure 1

Figure 1 shows a triangular pyramid $ABCD$ where triangle ABC is the base and BD is perpendicular to the base.

$$AB = 15 \text{ cm} \quad AC = 5\sqrt{10} \text{ cm} \quad BC = 5 \text{ cm} \quad BD = 10 \text{ cm}$$

- (a) Show that $\angle ABC = 90^\circ$ (2)
- (b) Find, in degrees to 1 decimal place, the size of $\angle DAC$. (4)

The point X on AC is such that BX is perpendicular to AC .

- (c) Find, in degrees to 1 decimal place, the size of $\angle DXB$. (4)

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Question 5 continued

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Question 5 continued

(Total for Question 5 is 10 marks)



6

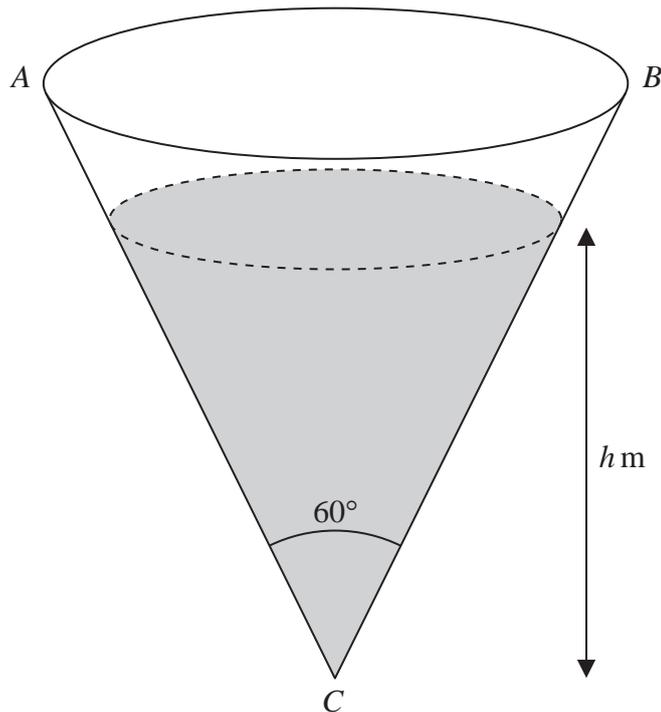
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Figure 2

Figure 2 shows a water tank in the shape of a hollow right circular cone fixed with its axis of symmetry vertical. A diameter of the circular rim of the cone is AB . The vertex, C , of the cone is below AB such that $\angle ACB = 60^\circ$

Initially, the tank is empty and water flows into the tank at a constant rate of $0.03 \text{ m}^3/\text{s}$. At time t seconds after the water starts to flow into the tank, the height of the surface of the water in the tank above C is h metres.

Find, in m/s to 3 significant figures, the rate of change of the height of the surface of the water above C at the instant when $h = 1.5$

(6)

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Question 6 continued

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(Total for Question 6 is 6 marks)



- 7 (a) Complete the table of values for $y = \ln(3x + 1) + 2$, giving your answers to 2 decimal places.

x	0	1	2	3	4	5	6
y	2		3.95	4.30			4.94

(2)

- (b) On the grid opposite, draw the graph of $y = \ln(3x + 1) + 2$ for $0 \leq x \leq 6$

(2)

- (c) Use your graph to obtain an estimate, to 1 decimal place, for the value of $\ln 10.6$. You **must** show clearly how you have used your graph.

(3)

- (d) By drawing a straight line on the grid, obtain estimates, to 1 decimal place, for the roots of the equation $(3x + 1)^2 = e^{(x+1)}$ in the interval $0 \leq x \leq 6$

(5)

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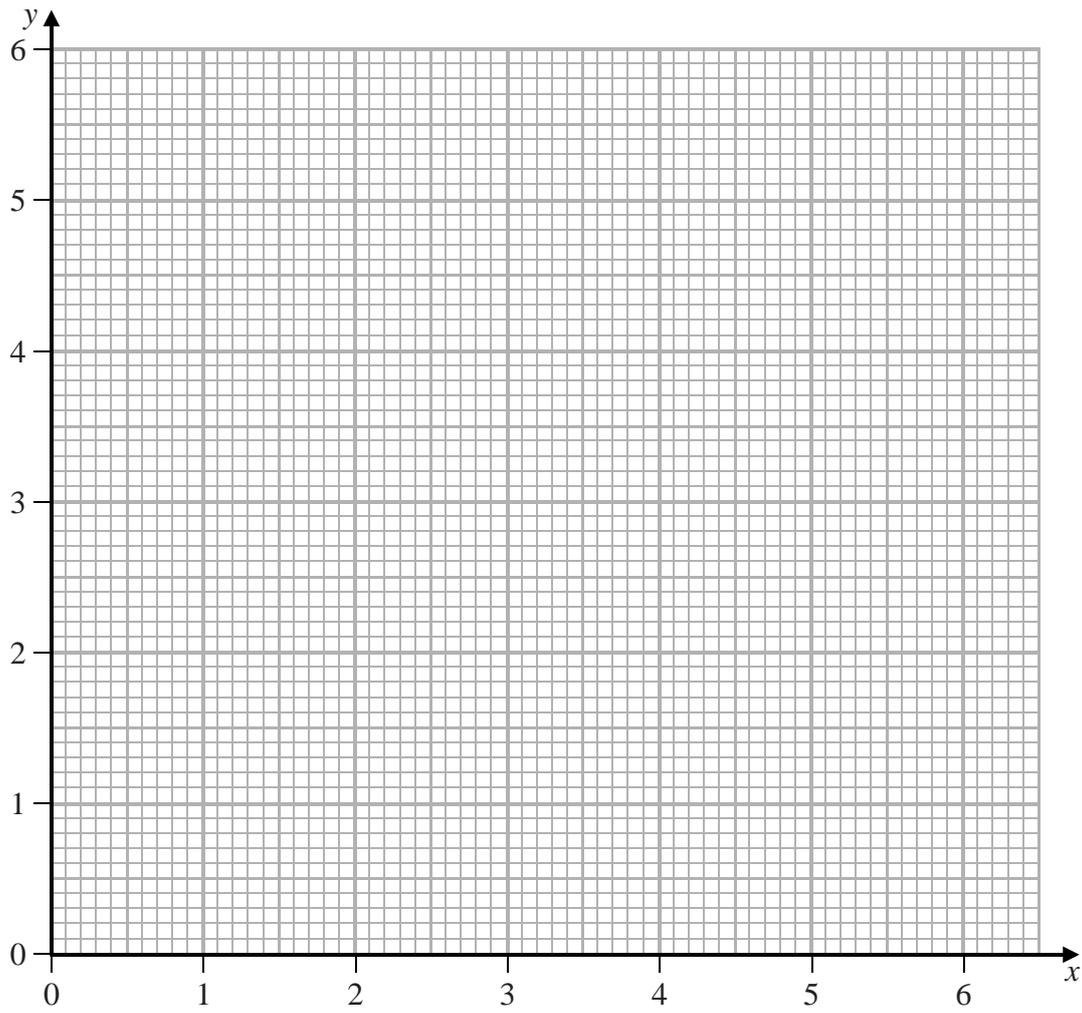


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Question 7 continued



Turn over for a spare grid if you need to redraw your graph.



Question 7 continued

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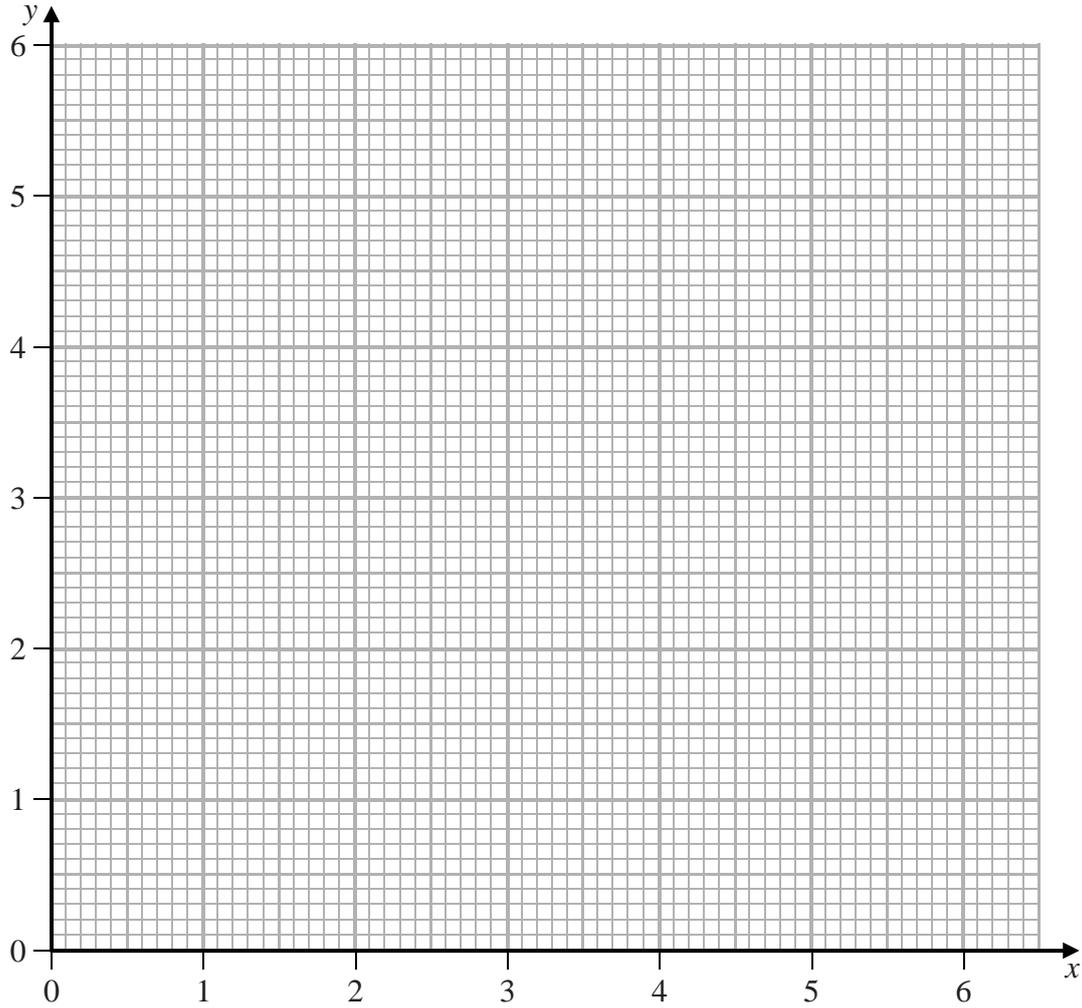
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Question 7 continued

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(Total for Question 7 is 12 marks)



8 The roots of the equation $3x^2 - 2x - 1 = 0$ are α and β , where $\alpha > \beta$

Without solving the equation,

(a) find the value of $\alpha^2 + \beta^2$ (3)

(b) show that $\alpha - \beta = \frac{4}{3}$ (2)

(c) form a quadratic equation, with integer coefficients, that has roots $\frac{\alpha + \beta}{\alpha}$ and $\frac{\alpha - \beta}{\beta}$ (6)

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Question 8 continued

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(Total for Question 8 is 11 marks)



9

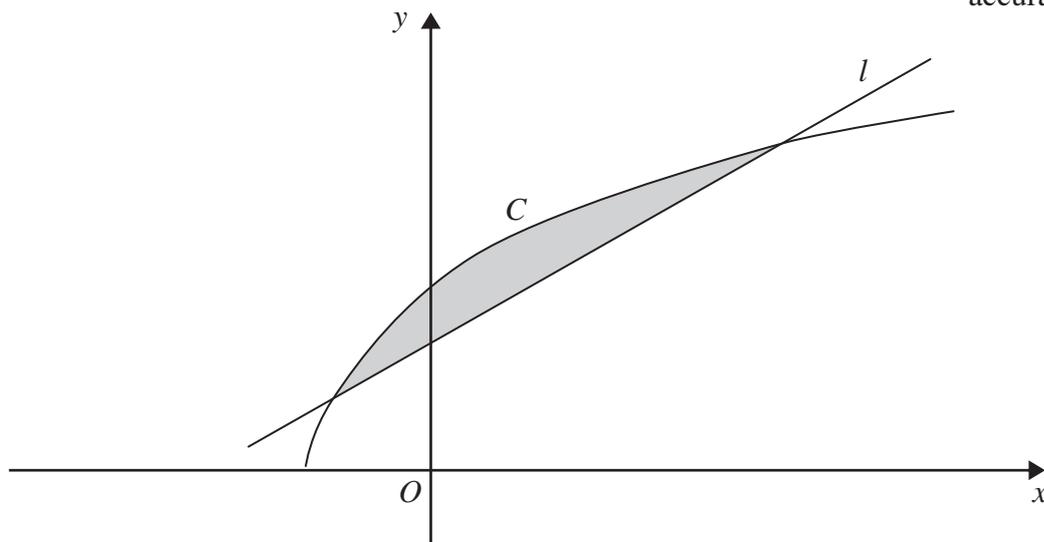
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Figure 3

Figure 3 shows part of the curve C with equation $y = (2x + 3)^{\frac{1}{2}}$ and the line l with equation $2y = x + 3$.
The line l crosses C at two points.

(a) Find the coordinates of each of these points.

(5)

The finite region bounded by C and l , shown shaded in Figure 3, is rotated through 360° about the x -axis.

(b) Use algebraic integration to find, in terms of π , the volume of the solid generated.

(5)

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Question 9 continued

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(Total for Question 9 is 10 marks)



10 A geometric series has first term a and common ratio r ($r > 0$)

The n th term of the series is U_n

Given that $U_1 + 3U_2 = 8$ and that $U_2 \times U_3 = 4U_5$

(a) find

(i) the value of r

(ii) the value of a

(5)

(b) Hence show that $U_n = \frac{2^{n+2}}{3^n}$

(2)

(c) Find the least value of n such that $U_n < 0.05$

(3)

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Question 10 continued

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(Total for Question 10 is 10 marks)



11

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

(a) (i) Using the above identity, show that

$$\cos 2x = 1 - 2 \sin^2 x$$

(ii) Hence show that

$$\frac{13 \sin x - 2 \cos 2x - 10}{4 \sin x - 3} = 4 + \sin x \quad (7)$$

(b) Hence solve, in radians to 3 significant figures, the equation

$$10 + 2 \cos\left(2\theta + \frac{\pi}{3}\right) - 13 \sin\left(\theta + \frac{\pi}{6}\right) = 2 \sin\left(\theta + \frac{\pi}{6}\right) + 8$$

$$\text{for } \pi \leq \theta \leq 2\pi$$

(5)

(c) Find the exact value of

$$\int_0^{\frac{\pi}{2}} \left(\frac{13 \sin x - 2 \cos 2x - 10 + 4x \sin x - 3x}{4 \sin x - 3} \right) dx \quad (5)$$

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TOTAL FOR PAPER IS 100 MARKS

