



Mark Scheme (Results)

Summer 2019

Pearson Edexcel International GCSE
In Mathematics B (4PM1)
Paper 01

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Summer 2019

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.
Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
 - M marks: method marks
 - A marks: accuracy marks
 - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
 - cao – correct answer only
 - ft – follow through

- isw – ignore subsequent working
- SC - special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- eoo – each error or omission

- **No working**

If no working is shown then correct answers may score full marks

If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

Always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.

If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.

Any case of suspected misread loses 2A (or B) marks on that part, but can gain the M marks.

If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

- If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used.

- **Ignoring subsequent work**

It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.

- **Parts of questions**

Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another.

- **Multiple attempts**

Where a candidate offers more than one attempt at part of a question with all or none crossed out NO marks can be given.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c| \text{ leading to } x = \dots$$

$$(ax^2 + bx + c) = (mx + p)(nx + q) \text{ where } |pq| = |c| \text{ and } |mn| = |a| \text{ leading to } x = \dots$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for a , b and c , leading to $x = \dots$

3. Completing the square:

$$x^2 + bx + c = 0: \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0 \quad \text{leading to } x = \dots$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \rightarrow x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n \rightarrow x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required.

(Mark schemes may override this eg in a case of "prove or show....")

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the rule may allow the mark to be awarded before the final answer is given.

June 2019
4PM1 Paper 1
Mark Scheme

| Question | Scheme | Marks |
|----------|--|-----------------------|
| 1 (a) | $x^2 - x - 2 = (x - 2)(x + 1)$ | B1 [1] |
| (b) | $f(2) = 2^3 + 2^3 - 10 - 6 = 0$ $f(-1) = -1^3 + 2 \times 1^2 + 5 - 6 = 0$ M1 for either of these, A1 for both correct As $(x - 2)$ and $(x + 1)$ are factors of $f(x)$ then so is $(x^2 - x - 2)$ | M1A1 A1 cso |
| | ALT 1 Divide $f(x)$ by $x^2 - x - 2$ Rem = 0 | M1A1 |
| | ALT 2 By inspection: $(x^2 - x - 2)(x + 3) = x^3 + 3x^2 - x^2 + 3x - 2x - 6$ | M1A1 |
| | Both Correct work and conclusion stated | A1 |
| | | Total 4 marks |

| Additional Guidance | | |
|---------------------|--------------|--|
| (a) | B1 | For the correct factorisation only $(x - 2)(x + 1)$ or $(x + 1)(x - 2)$ |
| (b) | M1 | For substituting either $x = 2$ or $x = -1$ into $f(x)$ AND equating = 0 |
| | A1 | For substituting both with no errors |
| | A1 | For the correct conclusion. Note this is a show question so there must be no errors in substitution and there must be a conclusion. |
| | ALT 1 | |
| | M1 | An attempt to divide $f(x)$ by $x^2 - x - 2$ A minimally acceptable attempt in the division is to reach a quotient of $(x \pm 3)$ |
| | A1 | For the correct division and NO remainder |
| | A1 | For the correct conclusion – there is no remainder hence $x^2 - x - 2$ is a factor of $f(x)$ |
| | ALT 2 | |
| | M1 | Uses inspection and must reach $(x^2 - x - 2)(x \pm 3)$ |
| | A1 | For the correct factorisation with no errors. |
| | A1 | For the correct conclusion – there is no remainder hence $x^2 - x - 2$ is a factor of $f(x)$ |

| Question | Scheme | Marks |
|----------|---|------------------------|
| 2 | $\frac{4+2\sqrt{3}}{5-2\sqrt{3}} \times \frac{5+2\sqrt{3}}{5+2\sqrt{3}}$ $= \frac{20+10\sqrt{3}+8\sqrt{3}+12}{25-4 \times 3} =$ (At least one intermediate step must be shown for M1 to be awarded) | M1 dM1A1 [3] |
| | | Total 3 marks |

| Additional Guidance | |
|---------------------|--|
| M1 | For the correct use of $5+2\sqrt{3}$ seen as intention to multiply both the numerator and denominator. This need not be on one line. . Do not accept $\frac{4+2\sqrt{3}}{5-2\sqrt{3}} \times (5+2\sqrt{3})$ for this mark unless multiplication of the numerator and denominator is later seen explicitly. This mark can be implied by later correct work, for example $\frac{20+10\sqrt{3}+8\sqrt{3}+12}{25-4 \times 3}$ Ignore poor/missing bracketing provided the work is correct. |
| M1 | Multiplies out both denominator and numerator. The multiplication must be correct for the award of this mark. |
| A1 | For the correct final answer $\frac{32+18\sqrt{3}}{13}$ There must be no errors seen for the award of this mark. |

| Question | Scheme | Marks |
|---------------|---|-------------------|
| 3 (a) | $\frac{\sin B}{7} = \frac{\sin 65^\circ}{10}$ $B = 39.37\dots = 39.4^\circ$ | M1A1 A1 [3] |
| (b) | $\text{Area} = \frac{1}{2} \times 10 \times 7 \sin(180 - 65 - "39.37")$ $= 33.90\dots = 33.9 \text{ (cm}^2\text{)}$ | M1A1 A1 [3] |
| Total 6 marks | | |

| Additional Guidance | | |
|---------------------|----|---|
| (a) | M1 | For the correct use of a correct Sine rule $\frac{\sin B}{7} = \frac{\sin 65^\circ}{10}$ seen only is sufficient for the award of this mark. |
| | A1 | For reaching $\sin B = \frac{7 \times \sin 65}{10}$ |
| | A1 | For $B = 39.37\dots^\circ \approx 39.4^\circ$ correctly rounded answer only. |
| (b) | M1 | For writing a correct expression for the area reaching a value for the area. $\text{Area} = \frac{1}{2} \times 10 \times 7 \sin(180 - 65 - "39.37")$ |
| | A1 | For using the correct value for angle B ($39.37\dots^\circ$ or 39.4°) |
| | A1 | For an area of $33.9 \text{ (cm}^2\text{)}$ rounded correctly. |
| Rounding | | In this question penalise rounding separately for part (a) and part (b) |

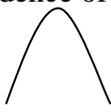
| Question | Scheme | Marks |
|---------------|---|-----------------------|
| 4 (a) | $\frac{1}{2} \times 15^2 \theta - \frac{1}{2} \times 10^2 \theta = 100$ $\theta = \frac{200}{125} = \frac{8}{5} \quad (\text{accept } 1.6)$ | M1M1 A1 [3] |
| (b) | Perimeter = $10 \times \frac{8}{5} + 15 \times \frac{8}{5} + 2 \times 5 = 50$ (cm) | M1M1A1 [3] |
| Total 6 marks | | |

| Additional Guidance | | |
|---------------------|----|--|
| (a) | M1 | For the correct length of <i>OC</i> (and <i>OD</i>) $OC = OD = 10$ (cm) |
| | M1 | For using the correct formula for the area of a sector and evaluating to reach a value for θ $\frac{1}{2} \times 15^2 \theta - \frac{1}{2} \times 10^2 \theta = 100 \Rightarrow \theta = \dots$ (using their value for <i>OC</i> or <i>OD</i>) |
| | A1 | For the correct value of $\theta = \frac{200}{125}$ o.e |
| (b) | M1 | For using the correct arc formula for the length of both arcs using their values for <i>OC</i> and θ $10 \times \frac{8}{5}$ and $15 \times \frac{8}{5}$ |
| | M1 | For a complete expression for the perimeter for the region <i>ABDC</i> and for evaluating their expression. Perimeter = $10 \times \frac{8}{5} + 15 \times \frac{8}{5} + 2 \times 5 = \dots$ (cm) |
| | A1 | For 50 (cm) |

| Question | Scheme | Marks |
|---------------|---|-------------------------|
| 5 (a) | $f(x) = 3(x^2 - 3x) + 5 = 3\left(x - \frac{3}{2}\right)^2 - \frac{27}{4} + 5 = 3\left(x - \frac{3}{2}\right)^2 - \frac{7}{4}$ $a = 3, b = \frac{3}{2}, c = -\frac{7}{4}$ <p>ALT Expand and equate coefficients</p> | M1M1A1 [3] |
| (b) | (i) $f(x)_{\min} = -\frac{7}{4}$ (ii) $x = \frac{3}{2}$ (Allow both if obtained by differentiation) | B1ft B1ft [2] |
| Total 5 marks | | |

| Additional Guidance | | |
|---|--------------|--|
| (a) | M1 | For an attempt to take out 3 as a common factor to achieve; $3\left(x^2 - 3x + \frac{5}{3}\right)$ or $3(x^2 - 3x) + 5$ This must be correct for this mark |
| | M1 | For an acceptable attempt to complete the square. Minimally acceptable attempts are; $3\left(x \pm \frac{3}{2}\right)^2 \pm l$ or $3\left[\left(x \pm \frac{3}{2}\right)^2 \pm m\right]$ where l and m are constants |
| | A1 | For either $3\left(x - \frac{3}{2}\right)^2 - \frac{7}{4}$ or $a = 3, b = \frac{3}{2}, c = -\frac{7}{4}$ The values of a, b and c can be embedded and need not be explicitly stated. |
| ALT | | |
| | M1 | For a correct expansion of $a(x-b)^2 + c$ and set = to $3x^2 - 9x + 5$ $a(x-b)^2 + c = ax^2 - 2abx + ab^2 + c \Rightarrow 3x^2 - 9x + 5 = ax^2 - 2abx + ab^2 + c$ |
| | dM1 | For an attempt to equate coefficients. Minimally acceptable attempt; $a = 3, 9 = 2 \times 3 \times b \Rightarrow b = \dots, 5 = 3 \times b^2 + c \Rightarrow c = \dots$ |
| | A1 | For either $3\left(x - \frac{3}{2}\right)^2 - \frac{7}{4}$ or $a = 3, b = \frac{3}{2}, c = -\frac{7}{4}$ The values of a, b and c can be embedded and need not be explicitly stated. |
| (b) | (i) B1ft | For $f(x)_{\min} = -\frac{7}{4}$, $f(x)_{\min} = \text{'their } c\text{'}$ Note this a follow through mark. |
| | (ii) B1ft | For $x = \frac{3}{2}$, $x = -\text{their } b$ Note this a follow through mark. |
| Allow B1B1 for correct values obtained using differentiation. Their answers must be clearly labelled and there must be no ambiguity for the minimum value of $f(x)$ and the value of x at which the minimum occurs. | | |

| Question | Scheme | Marks |
|----------|--|-----------------------------------|
| 6 (a) | $2y + 2\pi x = 90$ $S = 2yx + \pi x^2, = (90 - 2\pi x)x + \pi x^2 = 90x - \pi x^2 \quad k = 90$ | B1 M1, M1A1 [4] |
| (b) | $\frac{dS}{dx} = "90" - 2\pi x$ $\frac{dS}{dx} = 0 \Rightarrow x = \frac{"90"}{2\pi} = 14.32$ $\frac{d^2S}{dx^2} = -2\pi, < 0 \therefore \text{max}$ <p>ALT Use graph of $S = 90x - \pi x^2$ to justify max or values of ds/dx either side of their x</p> | M1 M1A1 M1A1 [5] |
| (c) | $S_{\max} = 90 \times \frac{90}{2\pi} - \pi \left(\frac{90}{2\pi} \right)^2 = 644.57\dots = 645 \text{ (Allow 644)}$ | M1, A1 [2] |

| Additional Guidance | | |
|----------------------------|---|---|
| (a) | B1 | For the correct expression for the perimeter $2y + 2\pi x = 90$ |
| | M1 | For the correct expression for the area. $S = 2yx + \pi x^2$ (That is all that is required for this mark) |
| | M1 | For substituting their rearranged expression ($2y = 90 - 2\pi x$) into their expression for the area ($S = 2yx + \pi x^2$) |
| | A1 | For simplifying their expression to achieve $S = 90x - \pi x^2$ AND the value of k must be stated; $k = 90$ |
| (b) | M1 | For an attempt to differentiate their expression for S . Minimally acceptable attempt $S = '90'x - \pi x^2 \Rightarrow \frac{dS}{dx} = '90' - p\pi x$ where p is a constant |
| | M1 | For setting their $\frac{dS}{dx} = 0$ and attempting to find a value for $x \Rightarrow x = \frac{'90'}{p\pi}$ and obtaining a numerical value for x . |
| | A1 | $x = 14.32$ cao |
| | Verifying the value of x is a maximum. Using 2nd derivative | |
| | M1 | For finding $\frac{d^2S}{dx^2}$ of their $\frac{dS}{dx}$. Minimally acceptable attempt $\frac{d^2S}{dx^2} = \pm q$ where q is a constant |
| | A1 | For the correct $\frac{d^2S}{dx^2} = -2\pi$ with a correct conclusion. |
| | ALT | |
| | M1 | (i) Uses a graph (must be evidence of this) to draw the conclusion. Eg General graph of S  so value is a maximum |
| | | (ii) Uses value of x around their value for the maximum. For example, tests $x = 14$ and $x = 15$ in their $\frac{dS}{dx}$ |
| | A1 | For the correct conclusion derived from correct working throughout. For example is $x = 14$ and $x = 15$ then $\frac{dS}{dx} = 2.035$ and $\frac{dS}{dx} = -4.25$ respectively so the gradient moves from positive to negative and therefore the turning point is a maximum. |
| (c) | M1 | For substituting their values of k and x into the given expression for S and obtaining a value. $S_{\max} = 90 \times 'x' - \pi 'x'^2 = \dots$ |
| | A1 | $S_{\max} = 645$ or 644 |

| Question | Scheme | Marks |
|----------------|---|---------------------------------------|
| 7 (a) | $\left(\cos \theta = -\frac{5}{3} \text{ not possible} \right)$ $\sin \theta = \frac{3}{5} \quad \theta = 36.9^\circ \text{ or } 143.1^\circ$ | M1A1 [2] |
| (b) | $8(\sin x \cos \alpha - \cos x \sin \alpha) = 3(\sin x \cos \alpha + \cos x \sin \alpha)$ $5 \sin x \cos \alpha = 11 \cos x \sin \alpha$ $5 \frac{\sin x}{\cos x} = 11 \frac{\sin \alpha}{\cos \alpha}$ $5 \tan x = 11 \tan \alpha *$ | M1 A1 M1 M1A1 [5] |
| | $5 \tan 2y = 11 \tan 30^\circ$ $\tan 2y = \frac{11}{5} \tan 30^\circ$ $2y = 51.78\dots, 231.78\dots$ $y = 25.89\dots 115.89\dots$ $y = 25.9^\circ \quad 115.9^\circ$ | M1 A1 M1 A1A1 [5] |
| Total 12 marks | | |

| Additional Guidance | | |
|--|----|--|
| (a) | M1 | For rejecting a value for $\cos \theta$ and finding a value for $\sin \theta$ $\sin \theta = \frac{3}{5} \Rightarrow \theta = 36.9^\circ$ or any other valid value (143.1°) Accept even for example -216.9° Working in radians is not acceptable unless a correct conversion to degrees is later applied. |
| | A1 | For both correct values of $\theta = 36.9^\circ, 143.1^\circ$ rounded correctly. |
| (b) | M1 | For the correct expansion of $\sin(x + \alpha)$ and $\sin(x - \alpha)$ Allow $8(\sin x \cos \alpha + \cos x \sin[-\alpha]) = 3(\sin x \cos \alpha + \cos x \sin \alpha)$ |
| | A1 | For rearranging and obtaining $5 \sin x \cos \alpha = 11 \cos x \sin \alpha$ |
| | M1 | For rearranging their expression to obtain $'5' \frac{\sin x}{\cos x} = '11' \frac{\sin \alpha}{\cos \alpha}$ |
| | M1 | For using the identity $\tan A = \frac{\sin A}{\cos A}$ on their $'5' \frac{\sin x}{\cos x} = '11' \frac{\sin \alpha}{\cos \alpha}$ to obtain $'5' \tan x = '11' \tan \alpha$ |
| | A1 | For a fully correct method to achieve $5 \tan x = 11 \tan \alpha$ Note: This is a show question. |
| (c) | M1 | $[x = 2y \text{ and } \alpha = 30^\circ]$ For using the given identity $5 \tan x = 11 \tan \alpha$ with the correct substitution for x and α to give $5 \tan 2y = 11 \tan 30^\circ$ |
| | A1 | For achieving $\tan 2y = \frac{11}{5} \tan 30^\circ$ or $\tan 2y = 2.2 \tan 30^\circ$ |
| | M1 | For achieving any valid value for $2y$ i.e. 51.78° or 231.78° Accept values out of range for this mark, even for example -128.22° |
| | A1 | For either 25.9° or 115.9° |
| | A1 | For both 25.9° and 115.9° |
| Penalise rounding only once in this question. However, the values given must round to 25.9° and 115.9° otherwise it is the incorrect value. Values given of 25.90° and/or 115.90° would score A0A1 | | |

| Question | Scheme | Marks |
|----------------|--|---|
| 8 (a) | $(5p-4)(p-1) = 0$ $p = \frac{4}{5}, p = 1$ | M1A1 [2] |
| (b) | $5^{2x+1} - 9(5^x) + 4 = 0 \Rightarrow 5 \times 5^{2x} - 9(5^x) + 4 = 0$ $5^x = 1 \quad x = 0$ $5^x = \frac{4}{5}, \quad x \ln 5 = \ln\left(\frac{4}{5}\right) \quad x = -0.1386... = -0.139$ | M1 A1 M1A1 [4] |
| (c) | $5^{2x+1} + 5^x = 2(5^{x+1}) - 4$ $5^{2x+1} - 9(5^x) + 4 = 0$ $x = 0 \quad y = 5 + 1 = 6 \quad (\text{or } y = 2 \times 5 - 4 = 6) \quad (0, 6)$ $x = -0.1386... \quad 5^x = \frac{4}{5} \quad y = 5 \times \left(\frac{4}{5}\right)^2 + \frac{4}{5} = 4 \quad (\text{or } y = 10 \times \frac{4}{5} - 4 = 4)$ $(-0.139, 4)$ | M1 A1 M1 A1 [4] |
| Total 10 marks | | |

| Additional Guidance | | |
|---------------------|----|--|
| (a) | M1 | For an attempt to solve the given quadratic. Note: they must arrive at two values of x that must follow from their factorisation. Minimally acceptable attempt are as follows; $(5p \pm 4)(p \pm 1) \Rightarrow p = \pm \frac{4}{5}, \pm 1$ $(5p \pm 1)(p \pm 4) \Rightarrow p = \pm \frac{1}{5}, \pm 4$ $(5p \pm 2)(p \pm 2) \Rightarrow p = \pm \frac{2}{5}, \pm 2$ |
| | A1 | For $p = \frac{4}{5}, p = 1$ or $p = 0.8, p = 1$ |
| (b) | M1 | For changing $5^{2x+1} = 5 \cdot 5^{2x}$ forming the 3TQ shown $5 \times 5^{2x} - 9(5^x) + 4 = 0$ and using the factorisation in (a) to find one value for x . |
| | A1 | For $x = 0$ |
| | M1 | For finding the second value of x arising from their factorisation. |
| | A1 | For $x = -0.139$ |
| (c) | M1 | For setting the equation of the curve = to the equation of the line, forming a 3TQ and finding one set of coordinates. Follow through their factorisation from part (a). |
| | A1 | For the coordinates of $(0, 6)$ |
| | M1 | For finding the second set of coordinates. |
| | A1 | For $(-0.139, 4)$ |

| Question | Scheme | Marks |
|----------------|---|---|
| 9 (a) | $2\log_p 9 + 3\log_3 p = 8$ $2\frac{\log_3 9}{\log_3 p} + 3\log_3 p = 8$ $2\log_3 9 + 3(\log_3 p)^2 = 8\log_3 p$ $3(\log_3 p)^2 - 8\log_3 p + 4 = 0$ $(3\log_3 p - 2)(\log_3 p - 2) = 0$ $\log_3 p = \frac{2}{3} \quad p = 3^{\frac{2}{3}} = \sqrt[3]{9} \quad (= 2.08)$ $\log_3 p = 2 \quad p = 3^2 = 9$ | <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1 [6]</p> |
| (b) | $\log_2 3 = \frac{\log_4 3}{\log_4 2} = \frac{\log_4 3}{\frac{1}{2}} = 2\log_4 3 = \log_4 3^2 \Rightarrow k = 2$ | <p>M1A1 [2]</p> |
| (c) | $6x\log_4 x - 3x\log_2 3 - 5\log_4 x + 10\log_2 3$ $= 6x\log_4 x - 5\log_4 x - 3x\log_4 3^2 + 10\log_4 3^2$ $= \log_4 x^{6x} - \log_4 x^5 - \log_4 3^{6x} + \log_4 3^{20}$ $= \log_4 \frac{x^{6x} \times 3^{20}}{x^5 \times 3^{6x}}$ $= \log_4 \frac{x^{6x-5}}{3^{6x-20}} *$ | <p>M1</p> <p>M1</p> <p>M1</p> <p>A1 [4]</p> |
| Total 12 marks | | |

| Additional Guidance | | |
|----------------------------|---|---|
| (a) | Method 1 – Changing to \log_3 | |
| | M1 | For changing the base of $\log_p 9$ to $\log_3 9$ |
| | M1 | For forming a 3TQ |
| | A1 | For the correct 3TQ $3(\log_3 p)^2 - 8\log_3 p + 4 = 0$ in any form. Accept for example, $2\log_3 9 + 3(\log_3 p)^2 = 8\log_3 p$ |
| | M1 | For an attempt to solve their 3TQ. They must obtain two values for $\log_3 p$ Minimally acceptable attempts to solve the 3TQ are defined in General Guidance |
| | A1 | For either $p = 9$ or $p = \sqrt[3]{9}$ [accept 2.08] |
| | A1 | For both $p = 9$ and $p = \sqrt[3]{9}$ [accept 2.08] |
| | Method 2 - Changing to \log_p | |
| | M1 | For changing the base of $\log_3 p$ to $\log_p 3 \Rightarrow 3 \frac{\log_p p}{\log_p 3}$ |
| | M1 | For forming a 3TQ $4(\log_p 3)^2 + 3 = 8\log_p 3$ |
| | A1 | For the correct 3TQ in any form. $4(\log_p 3)^2 - 8\log_p 3 + 3 = 0$ |
| | M1 | For an attempt to solve their 3TQ. They must obtain two values for $\log_p 3$ Minimally acceptable attempts to solve the 3TQ are defined in General Guidance |
| | A1 | For either $p = 9$ or $p = \sqrt[3]{9}$ [accept 2.08] |
| | A1 | For both $p = 9$ and $p = \sqrt[3]{9}$ [accept 2.08] |
| (b) | Method 1 – Changing the base to $\log_4 3$ | |
| | M1 | For changing $\log_2 3 \Rightarrow \log_4 3$ and finding the value of k |
| | A1 | For $k = 2$ |
| | Method 2 – Changing the base to $\log_2 3$ | |
| | M1 | For changing the base of the log and obtaining a value for k $\log_2 3 = \log_4 3^k \Rightarrow \log_2 3 = k \log_4 3 \Rightarrow k = \frac{\log_2 3}{\log_4 3}$ $\Rightarrow k = \frac{\log_2 3}{\frac{\log_2 3}{\log_2 4}} \Rightarrow k = \frac{\log_2 3}{\frac{\log_2 3}{2}} = 2$ |
| | A1 | For $k = 2$ |
| (c) | Mark this part in any order | |
| | M1 | For changing the base of both terms with \log_2 to \log_4 |
| | M1 | For dealing with all the powers |
| | M1 | For combining the logs together into one log term. |
| | A1 | For the final correct answer. This is a show question, there must be no errors for the award of the final A mark |

| Question | Scheme | Marks |
|----------------|--|---|
| 10 (a) | $(1+2x^2)^{-\frac{1}{3}} = 1 - \frac{1}{3}(2x^2) + \frac{-\frac{1}{3} \times \left(-\frac{4}{3}\right)}{2!} (2x^2)^2 + \frac{-\frac{1}{3} \times \left(-\frac{4}{3}\right) \left(-\frac{7}{3}\right)}{3!} (2x^2)^3$ $= 1 - \frac{2x^2}{3} + \frac{8x^4}{9} - \frac{112x^6}{81}$ | M1 A1A1 [3] |
| (b) | $ x < \frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ (But accept \leq for either or both) | B1 [1] |
| (c) | $f(x) = (2+kx^2) \left(1 - \frac{2x^2}{3} + \frac{8x^4}{9} - \frac{112x^6}{81} \right)$ $= 2 - \frac{4x^2}{3} + \frac{16x^4}{9} - \frac{224x^6}{81} + kx^2 - \frac{2kx^4}{3} + \frac{8kx^6}{9} + \dots$ $= 2 + \left(k - \frac{4}{3}\right)x^2 + \left(\frac{16}{9} - \frac{2k}{3}\right)x^4 + \left(\frac{8k}{9} - \frac{224}{81}\right)x^6$ | M1 dM1 A1 [3] |
| (d) | $\frac{16}{9} = \frac{2k}{3}, \quad k = \frac{8}{3}$ | M1,A1 [2] |
| (e) | <p>Coeffs of $x^2 = \frac{4}{3}, \quad x^6 = -\frac{32}{81}$</p> $\int_0^{0.5} f(x) dx \approx \int_0^{0.5} \left(2 + \frac{4}{3}x^2 - \frac{32}{81}x^6 \right) dx$ $= \left[2x + \frac{4x^3}{9} - \frac{32x^7}{567} \right]_0^{0.5}$ $= 2 \times 0.5 + \frac{4}{9}(0.5)^3 - \frac{32}{567}(0.5)^7, \approx 1.0551$ | B1 (for both) M1A1ft dM1,A1 [5] |
| Total 14 marks | | |

| Additional Guidance | | |
|---|-----|--|
| (a) | M1 | For an attempt to use the Binomial Expansion The minimally acceptable attempt is as follows; <ul style="list-style-type: none"> • The power of x must be correct in each term. $\left[(2x^2), (2x^2)^2 \text{ and } (2x^2)^3 \right]$ • The first term is 1 • The denominators in terms 2, 3 and 4 must be correct. |
| | A1 | For the first term and at least one term in x correct and simplified. |
| | A1 | All terms correct and simplified $1 - \frac{2x^2}{3} + \frac{8x^4}{9} - \frac{112x^6}{81}$ |
| (b) | B1 | For the correct inequality. $ x < \frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ Accept $ x < \frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$ |
| (c) | M1 | Shows that they intend to multiply their expansion from part(a) by $(2 + kx^2)$ |
| | dM1 | Multiplies out the two brackets to at least 4 terms up to and including the term in x^6 with a constant, term in x^2 , term in x^4 and a term in x^6 |
| | A1 | For the expansion as shown, simplified and the coefficient of each term in x in terms k $2 + \left(k - \frac{4}{3}\right)x^2 + \left(\frac{16}{9} - \frac{2k}{3}\right)x^4 + \left(\frac{8k}{9} - \frac{224}{81}\right)x^6$ |
| (d) | M1 | For $\frac{16}{9} = \frac{2k}{3}$ Allow the presence of x^4 provided it is on both sides of the equality. |
| | A1 | For $k = \frac{8}{3}$ |
| (e) | B1 | Finds the coefficients of x^2 and x^6 $x^2 = \frac{4}{3}, \quad x^6 = -\frac{32}{81}$ |
| In this part the question clearly states using algebraic integration No evidence of algebraic integration – no marks | | |
| | M1 | For an attempt to integrate their expansion in (c) with their value for k provided it has as a minimum; a constant term and at least two algebraic terms. Ignore the limits for this mark |
| | A1 | For a fully correct integrated expression as shown (ignore limits) |
| | M1 | Attempts to evaluate their integrated expression using the correct limits Substitution of 0 need not be seen |
| | A1 | For a value of 1.0551 rounded correctly. |

| Question | Scheme | Marks |
|----------|--|---|
| 11 (a) | $x = 4 \Rightarrow y = 6$ $\frac{dy}{dx} = \frac{2}{3}x$ Grad tgt = $\frac{8}{3}$ $y - 6 = \frac{8}{3}(x - 4)$ $3y = 8x - 14$ | B1 M1 A1 M1A1 A1 [6] |
| (b) | $y - 6 = -\frac{3}{8}(x - 4)$ $8y = -3x + 60$ | M1 A1 [2] |
| (c) | Base = $\left(20 - \frac{7}{4}\right)$ $\left(\frac{73}{4}\right)$ Area = $\frac{1}{2} \times \frac{73}{4} \times 6 = 54\frac{3}{4}$ or 54.75 ALT Vertices $\left(\frac{7}{4}, 0\right)$ and $(20, 0)$, Area = $\frac{1}{2} \begin{vmatrix} 4 & \frac{7}{4} & 20 & 4 \\ 6 & 0 & 0 & 6 \end{vmatrix} = 54\frac{3}{4}$ | B1 M1,A1 [3] B1,M1A1 [3] |
| (d) | $\text{Vol} = \pi \int_0^4 \left(\frac{1}{3}(x^2 + 2)\right)^2 dx + \text{cone, base 6 and ht 16}$ $(\pi) \int_0^4 \left(\frac{1}{3}(x^2 + 2)\right)^2 dx = (\pi) \frac{1}{9} \int_0^4 (x^4 + 4x^2 + 4) dx = (\pi) \frac{1}{9} \left[\frac{x^5}{5} + \frac{4x^3}{3} + 4x \right]_0^4$ Cone = $\frac{\pi}{3} \times 6^2 \times 16$ ($= 192\pi$) (or equation of normal squared and integrated correctly) $\text{Vol} = \frac{\pi}{9} \left(\frac{4^5}{5} + \frac{4^4}{3} + 4^2 \right) + 192\pi = 710$ | M1 M1A1 B1 M1A1 [6] |
| | | Total 17 marks |

| Additional Guidance | | |
|----------------------------|------------|--|
| (a) | B1 | Finds the value of y when $x = 6$. $x = 4 \Rightarrow y = 6$ |
| | M1 | Differentiates the given expression to give $\frac{dy}{dx} = \frac{2}{3}x$ This must be correct. |
| | A1 | For the gradient of the tangent = $\frac{8}{3}$ |
| | M1 | For the equation of the line k . $y - '6' = '\frac{8}{3}'(x - 4)$ Follow through their y and their gradient. |
| | A1 | For the correct equation in any form. |
| | A1 | For the correct equation in the required form. $3y = 8x - 14$ |
| (b) | M1 | For the equation of the normal to the curve. Allow their y and the negative inverse of their gradient of the tangent. $y - '6' = '-\frac{3}{8}'(x - 4)$ The gradient of the normal must be the negative inverse of their numerical value |
| | A1 | For the correct equation in the required form. $8y = -3x + 60$ |
| (c) | B1 | For the length of the base $\left(\frac{73}{4}\right)$ |
| | M1 | For the area of the triangle Area = $\frac{1}{2} \times '\frac{73}{4}' \times '6'$ |
| | A1 | For the correct area of the triangle $54\frac{3}{4}$ or 54.75 |
| | ALT | |
| | B1 | For the correct coordinates of the vertices Vertices $\left(\frac{7}{4}, 0\right)$ and $(20, 0)$ |
| | M1 | For the correct method to find the area using their vertices $\frac{1}{2} \begin{vmatrix} 4 & '\frac{7}{4}' & '20' & 4 \\ '6' & 0 & 0 & '6' \end{vmatrix} = \dots$ |
| | A1 | For the correct area of the triangle $54\frac{3}{4}$ or 54.75 |
| (d) | M1 | For a correct statement of the volume of the solid. $\pi \int_0^4 \left(\frac{1}{3}(x^2 + 2)\right)^2 dx$ + cone, with a radius of the base of their value of y (6) and the height derived from their equation of line l (normal) Alternatively, they can use Volume of cone = $\pi \int_4^{20} \left(-\frac{3}{8}x + '\frac{60}{8}'\right)^2 dx$ The limits must be correct for this mark. |

| | |
|----|--|
| M1 | For squaring the equation of the curve only and attempting to integrate. (See general guidance for the definition of an attempt) Ignore the limits and π for this mark. |
| A1 | For a fully correct integrated expression for the cone. Ignore the limits and π for this mark. |
| B1 | For finding the volume of the cone. Volume of cone = $= \frac{\pi}{3} \times 6^2 \times 16$ ($= 192\pi$) ALT Volume of cone = $\pi \int_4^{20} \left(-\frac{3}{8}x + \frac{15}{2}\right)^2 dx = \pi \int_4^{20} \left(\frac{9}{64}x^2 - \frac{45}{8}x + \frac{225}{4}\right) dx$ $= \pi \left[\frac{3}{64}x^3 - \frac{45}{16}x^2 + \frac{225}{4}x \right]_4^{20} = 192\pi$ If candidates chose to use the ALT method – it must be fully correct. |
| M1 | Evaluating their integrated expression which must have as a minimum a term in x^4 , a term in x^3 , and a term in x , AND adding their volume of the cone. |
| A1 | For 710 (cm^3) |

