Candidate surname

Other names

Pearson Edexcel International GCSE

Centre Number

Candidate Number

Time 2 hours

Paper reference **4PM1/02**

Further Pure Mathematics
PAPER 2

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question*.

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over ▶

P 6 6 0 2 7 R R A 0 1 3 6

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r \times$ slant height

Volume of sphere $= \dfrac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$

Geometric series

Sum to n terms, $S_n = \dfrac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \dfrac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule
In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B \qquad \sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B \qquad \cos(A-B) = \cos A \cos B + \sin A \sin B$

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

Logarithms

$\log_a x = \dfrac{\log_b x}{\log_b a}$

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DO NOT WRITE ON THIS PAGE

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (a) Use the formula for $\cos(A + B)$ to show that $\cos 2A = 1 - 2\sin^2 A$

(2)

Given that $\sin A = \dfrac{x + 1}{2}$ and $\cos 2A = \dfrac{4 - y}{3}$

(b) show that $y = \dfrac{1}{2}(3x^2 + 6x + 5)$

(3)

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Question 1 continued

(Total for Question 1 is 5 marks)

5

Turn over ▶

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2 The finite region enclosed by the curve with equation $y = 4 - x^2$ and the line with equation $y = x + 2$ is rotated through $360°$ about the x-axis.

Use algebraic integration to find the exact volume of the solid formed.

(6)

Question 2 continued

(Total for Question 2 is 6 marks)

3
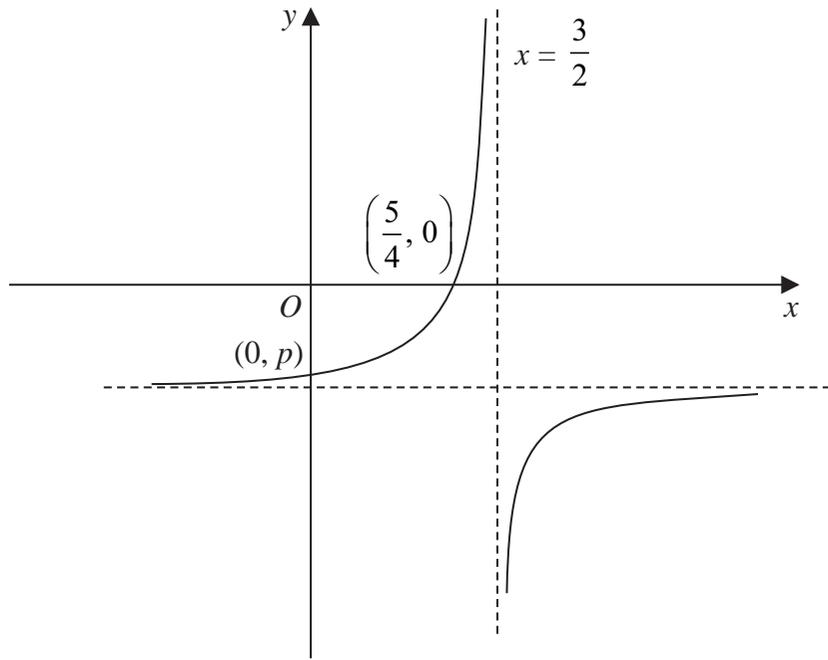
Figure 1

Figure 1 shows a sketch of the curve with equation

$$y = \frac{a - bx}{cx - d} \qquad x \neq \frac{d}{c}$$

where a, c and d are prime numbers and b is an integer.

The asymptote to the curve that is parallel to the y-axis has equation $x = \dfrac{3}{2}$

(a) Write down the value of c and the value of d

(2)

The curve crosses the x-axis at the point $\left(\dfrac{5}{4}, 0 \right)$

(b) Find the value of a and the value of b

(2)

The curve crosses the y-axis at the point $(0, p)$ where p is a rational number.

(c) Find the value of p

(2)

(d) Find an equation of the asymptote to the curve that is parallel to the x-axis.

(1)

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8

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 7 marks)

4 The curve C has equation $y = 2x^2 + px + q$ where p and q are integers.

The curve C has a stationary point at $(3, -5)$

(a) Show that $p = -12$ and find the value of q

(4)

(b) State, giving a reason, the nature of the stationary point.

(1)

(c) Find an equation of the normal to C at the point on C where $x = 1$

Give your answer in the form $ax + by + c = 0$

(6)

Question 4 continued

(Total for Question 4 is 11 marks)

Turn over ▶

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5 y and x vary so that $y = xe^{-2x}$

Given that the value of x increases by 3%, use calculus to find, in terms of x, an estimate for the percentage change in y

Give your answer in the form $a(b - cx)$ where a, b and c are integers.

(6)

Question 5 continued

(Total for Question 5 is 6 marks)

6 A particle P is moving along the x-axis. At time t seconds ($t \geqslant 0$) the displacement, s metres, of P from the origin O, is given by $s = t^3 - 4t^2 - 16t - 8$

(a) Find the distance of P from O when $t = 0$

(1)

(b) Find the value of t for which P is instantaneously at rest.

(4)

(c) Find the value of t for which P is accelerating at $10\,\text{m/s}^2$ in the positive x direction.

(3)

Question 6 continued

(Total for Question 6 is 8 marks)

Turn over ▶

7 A geometric series G has first term a and common ratio r

The sum of the first three terms of G is $\dfrac{61}{6}$

The sum to infinity of G is $\dfrac{125}{6}$

(a) (i) Show that $r = \dfrac{4}{5}$

   (ii) Find the value of a

(6)

The sum of the first n terms of G is S_n

Given that $S_n > 19.8$

(b) show that $n \lg\left(\dfrac{4}{5}\right) < \lg\left(\dfrac{31}{625}\right)$

(2)

(c) Hence find the least value of n

(2)

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Question 7 continued

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 10 marks)

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8 (a) Complete the table of values for $y = 2x + \dfrac{3}{x^2} - 3$ giving your answers to 2 decimal places where appropriate.

x	0.5	0.75	1	1.5	2	3	4	5
y	10		2				5.19	7.12

(2)

(b) On the grid opposite, draw the graph of $y = 2x + \dfrac{3}{x^2} - 3$ for $0.5 \leqslant x \leqslant 5$

(2)

(c) By drawing a suitable straight line on the grid, obtain estimates, to one decimal place, of the roots of the equation $4x^3 - 10x^2 + 3 = 0$ for $0.5 \leqslant x \leqslant 5$

(5)

Question 8 continued
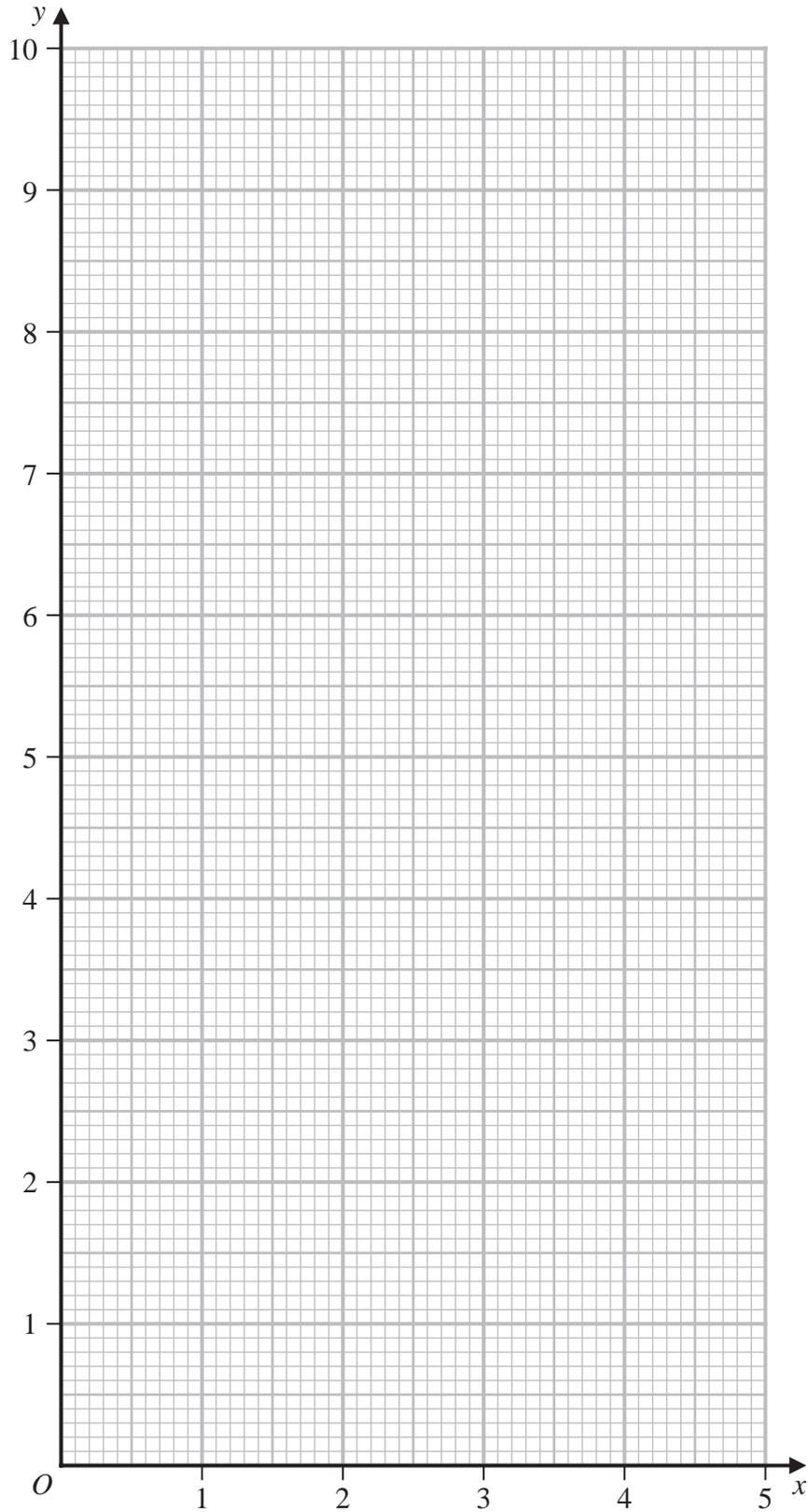
Turn over for a spare grid if you need to redraw your graph.

Question 8 continued

P 6 6 0 2 7 R R A 0 2 4 3 6

Question 8 continued

Only use this grid if you need to redraw your graph.
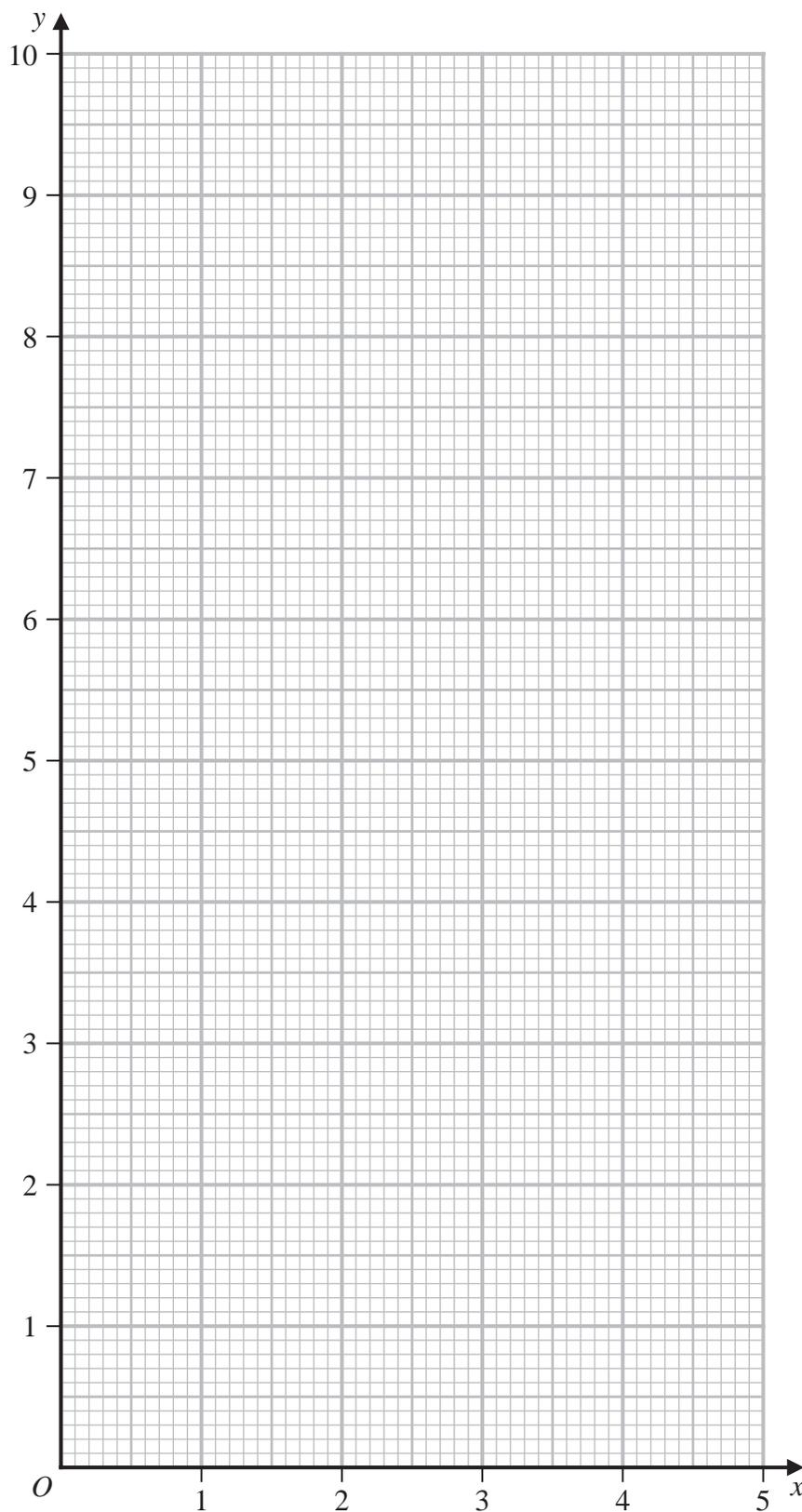
(Total for Question 8 is 9 marks)

9
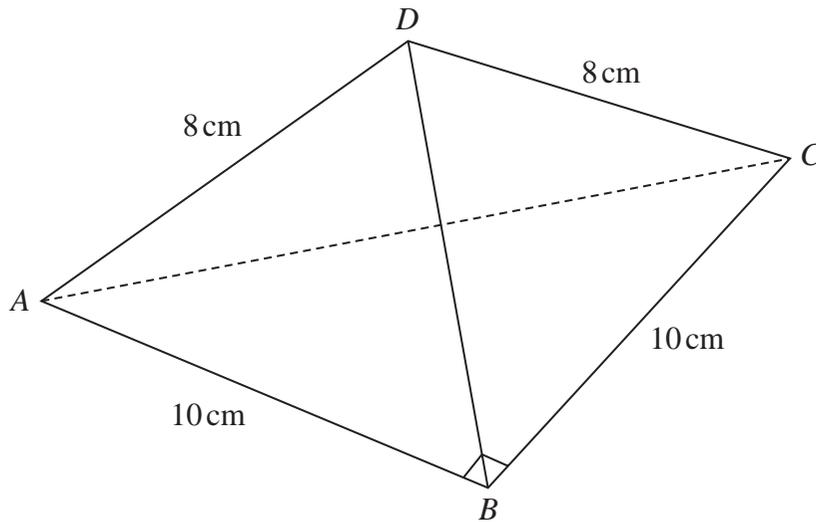
Figure 2

Figure 2 shows a triangular pyramid *ABCD* with base *ABC*

$$AB = BC = 10 \text{ cm} \qquad AD = CD = 8 \text{ cm} \qquad \angle ABC = 90°$$

(a) Find the exact length of *AC*

Give your answer in the form $p\sqrt{q}$ cm where *p* is an integer and *q* is a prime number.

(2)

The point *M* is the midpoint of *AC*

(b) Find the exact length of *BM*

Give your answer in the form $m\sqrt{n}$ cm where both *m* and *n* are prime numbers.

(2)

Given that *BD* = 6 cm,

(c) find, in degrees to one decimal place, the size of the acute angle between the plane *ACD* and the plane *ABC*

(4)

The base *ABC* of the pyramid is placed on a horizontal plane.

(d) Find, in cm to 3 significant figures, the vertical height of *D* above the base.

(2)

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Question 9 continued

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Question 9 continued

P 6 6 0 2 7 R R A 0 2 8 3 6

Question 9 continued

(Total for Question 9 is 10 marks)

10 (a) Show that $\dfrac{3}{\sqrt{9-3x}} = \left(1 - \dfrac{x}{3}\right)^{-\frac{1}{2}}$

(2)

(b) Hence expand $\dfrac{3}{\sqrt{9-3x}}$ in ascending powers of x up to and including the term in x^3 expressing each coefficient as an exact fraction in its lowest terms.

(3)

$$f(x) = \frac{1+2x}{\sqrt{9-3x}}$$

(c) Find the expansion of $3f(x)$ in ascending powers of x up to and including the term in x^3 expressing each coefficient as an exact fraction in its lowest terms.

(4)

(d) Hence, using algebraic integration, obtain an approximation to 6 significant figures for

$$\int_{0.1}^{0.2} \frac{1+2x}{\sqrt{9-3x}}\,\mathrm{d}x$$

(4)

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P 6 6 0 2 7 R R A 0 3 0 3 6

Question 10 continued

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Question 10 continued

P 6 6 0 2 7 R R A 0 3 2 3 6

Question 10 continued

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(Total for Question 10 is 13 marks)

11 The points A and B have coordinates $(-3, -5)$ and $(7, 5)$ respectively.

(a) Find an equation for the line AB

(2)

The point C has coordinates $(p, 1)$ where $p < 0$

Given that AC and BC are perpendicular,

(b) prove that $p = -5$

(7)

The point D, where BCD is a straight line, is such that C divides BD in the ratio $4:3$

(c) Find the coordinates of D

(2)

(d) (i) Find the exact length of AC

(ii) Hence, or otherwise, find the area of triangle ABD

(4)

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Question 11 continued

Question 11 continued

(Total for Question 11 is 15 marks)

TOTAL FOR PAPER IS 100 MARKS