

Please check the examination details below before entering your candidate information

Candidates surname					Other names				
Centre Number					Candidate Number				
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**Pearson Edexcel International GCSE**

Time 2 hours

Paper reference **4PM1/01R**

**Further Pure Mathematics**  
**PAPER 1R**



**Calculators may be used.**

Total Marks

### Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.  
Anything you write on the formulae page will gain NO credit.

### Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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Pearson

## International GCSE in Further Pure Mathematics Formulae sheet

**Mensuration**Surface area of sphere =  $4\pi r^2$ Curved surface area of cone =  $\pi r \times$  slant heightVolume of sphere =  $\frac{4}{3}\pi r^3$ **Series****Arithmetic series**Sum to  $n$  terms,  $S_n = \frac{n}{2}[2a + (n-1)d]$ **Geometric series**Sum to  $n$  terms,  $S_n = \frac{a(1-r^n)}{(1-r)}$ Sum to infinity,  $S_\infty = \frac{a}{1-r}$   $|r| < 1$ **Binomial series** $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots$  for  $|x| < 1, n \in \mathbb{Q}$ **Calculus****Quotient rule (differentiation)**

$$\frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

**Trigonometry****Cosine rule**In triangle  $ABC$ :  $a^2 = b^2 + c^2 - 2bc \cos A$ 

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

**Logarithms**

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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**Answer all ELEVEN questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

1 (a) On the grid below, draw the graph of the line with equation

(i)  $4x + 5y = 20$       (ii)  $3y - 4x = -12$

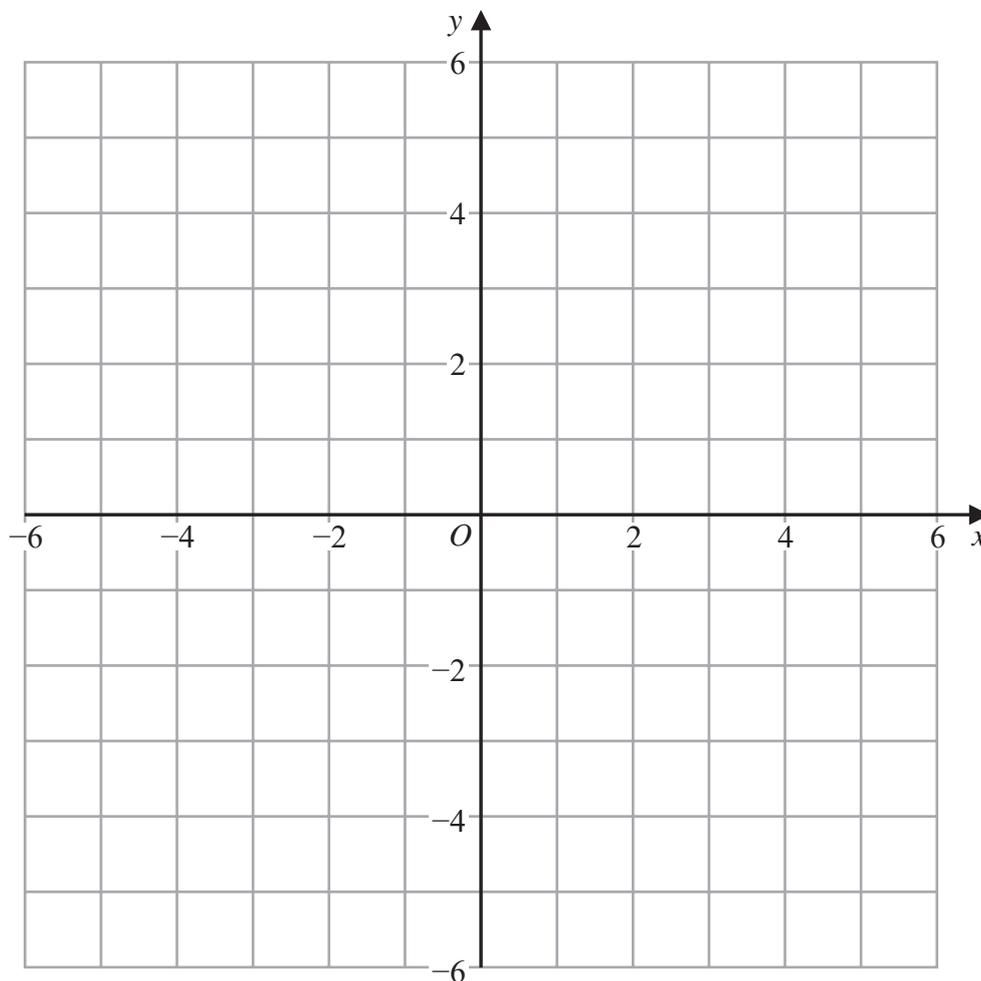
(2)

(b) Show, by shading on the grid, the region  $R$  defined by the inequalities

$4x + 5y \leq 20$        $3y - 4x \geq -12$        $y \leq 3$        $x \geq 1$

Label the region  $R$ .

(2)



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**Turn over for a spare grid if you need to redraw your graph.**









**Question 2 continued**

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**(Total for Question 2 is 8 marks)**





**Question 3 continued**

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**(Total for Question 3 is 10 marks)**





**Question 4 continued**

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**(Total for Question 4 is 8 marks)**





**Question 5 continued**

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**(Total for Question 5 is 8 marks)**





**Question 6 continued**

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**Question 6 continued**

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**(Total for Question 6 is 10 marks)**



7

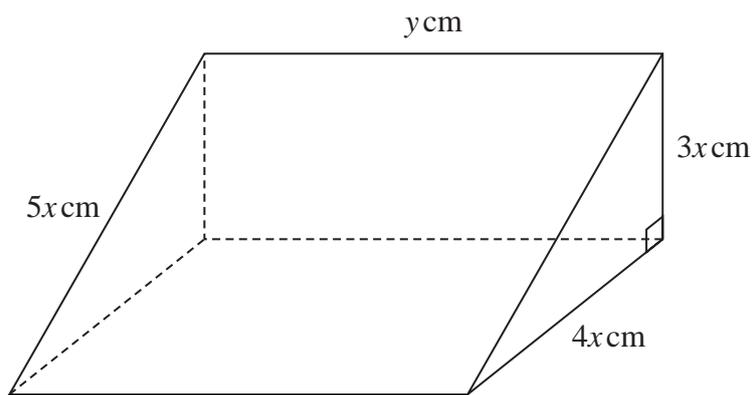


Diagram **NOT** accurately drawn

**Figure 2**

Figure 2 shows a block of wood in the shape of a right triangular prism.

The cross section of the prism is a right-angled triangle with sides of length  $3x\text{ cm}$ ,  $4x\text{ cm}$  and  $5x\text{ cm}$ .

The length of the prism is  $y\text{ cm}$ .

The total surface area of the five faces of the prism is  $144\text{ cm}^2$

The volume of the prism is  $V\text{ cm}^3$

(a) Show that

$$V = 72x - 6x^3 \tag{5}$$

Given that  $x$  can vary,

(b) use calculus to find the value of  $x$  for which  $V$  is a maximum, justifying that this value gives a maximum value of  $V$  (4)

(c) Find the maximum value of  $V$  (2)

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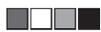
**Question 7 continued**

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**Question 7 continued**

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**(Total for Question 7 is 11 marks)**



8 The curve  $C$  has equation  $y = 2x^2 - \sin x$

The point  $A$  on  $C$  has  $x$  coordinate  $\pi$

Show that an equation of the normal to  $C$  at the point  $A$  is

$$x + (4\pi + 1)y - \pi(8\pi^2 + 2\pi + 1) = 0 \tag{8}$$

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**Question 8 continued**

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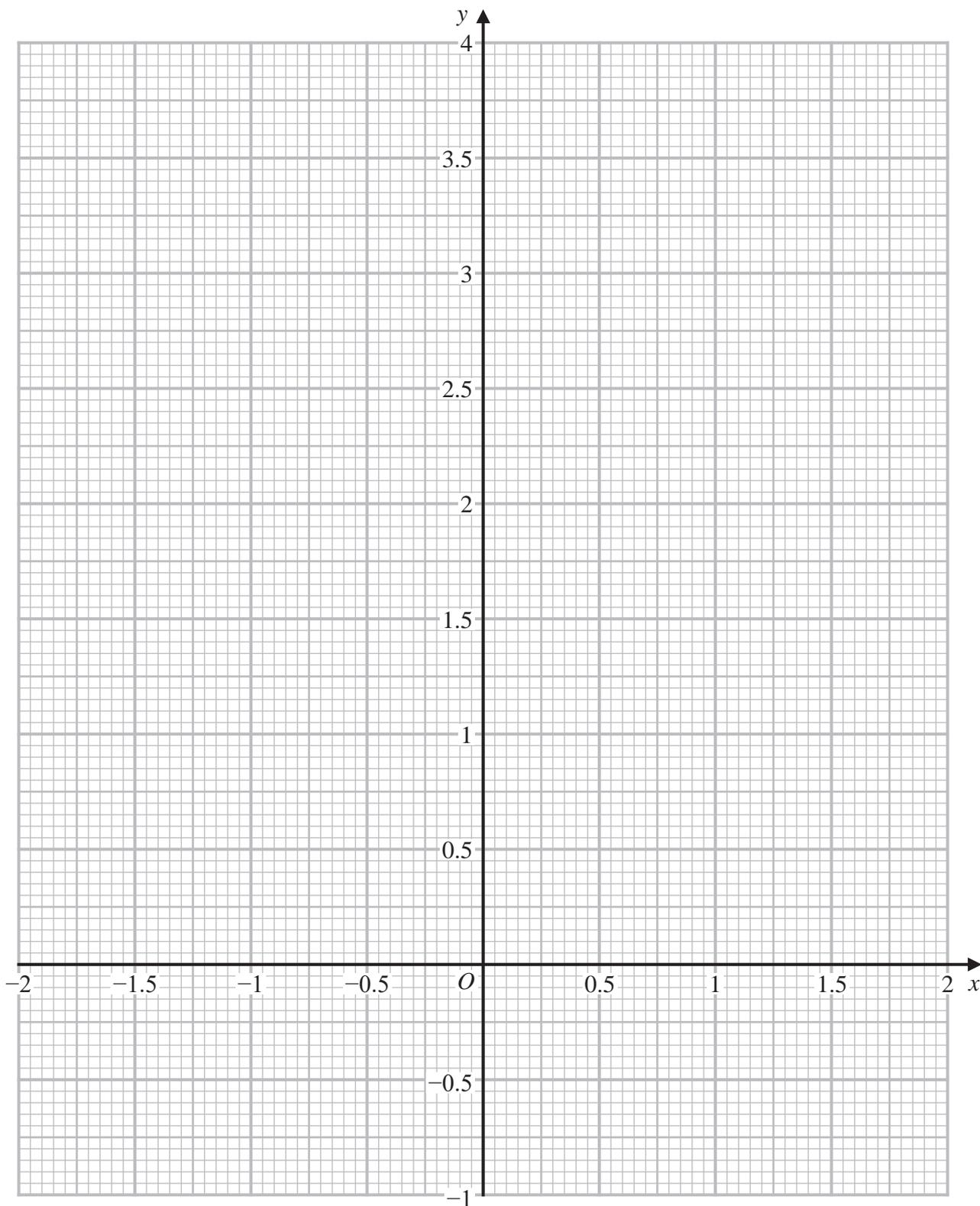
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**(Total for Question 8 is 8 marks)**





**Question 9 continued**



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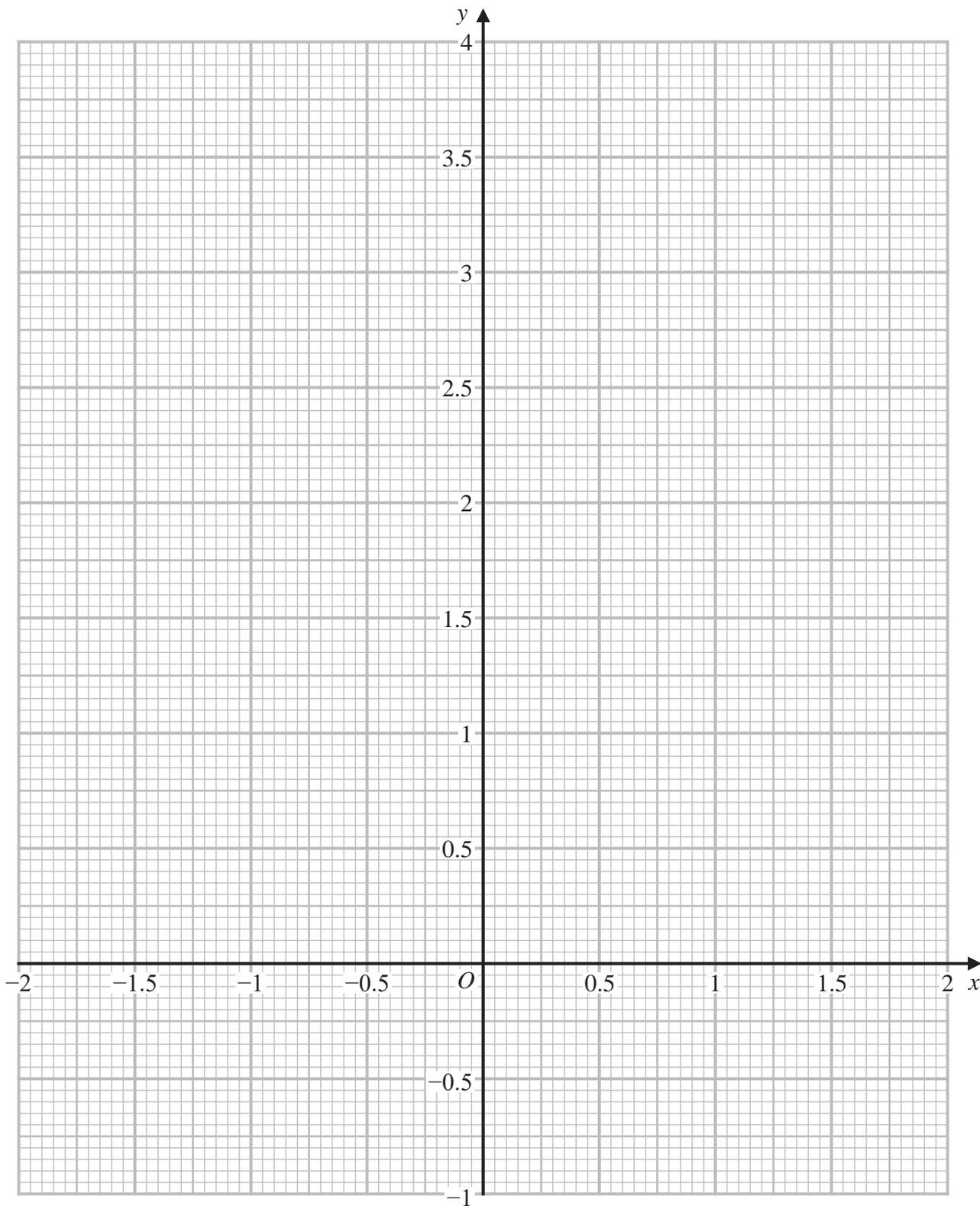
**Turn over for a spare grid if you need to redraw your graph.**





**Question 9 continued**

**Only use this grid if you need to redraw your graph.**



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**(Total for Question 9 is 9 marks)**





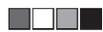
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**Question 10 continued**

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**(Total for Question 10 is 15 marks)**



11

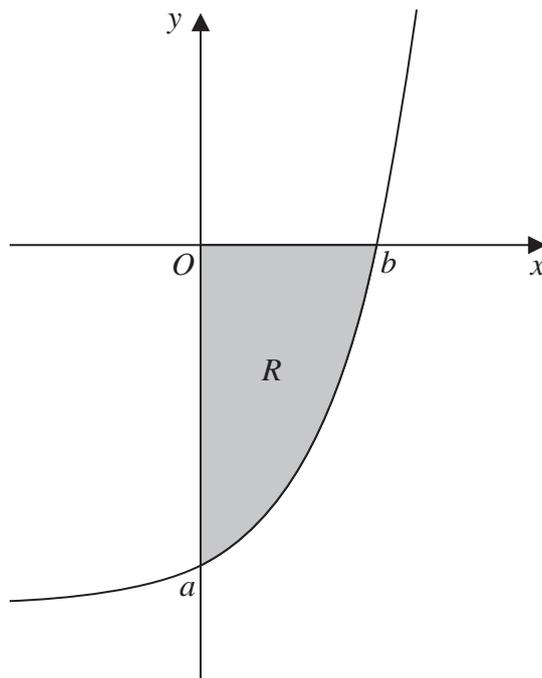


Diagram NOT accurately drawn

Figure 3

The finite region  $R$ , shown shaded in Figure 3, is bounded by the curve with equation  $y = e^{2x} - 9$  and the coordinate axes.

The curve crosses the coordinate axes at the points with coordinates  $(0, a)$  and  $(b, 0)$

- (a) (i) Find the value of  $a$
- (ii) Show that  $b = \ln 3$

(3)

The region  $R$  is rotated through  $360^\circ$  about the  $x$ -axis.

- (b) Use calculus to find the volume of the solid generated.

Give your answer in the form  $\pi(p \ln 3 + q)$ , where  $p$  and  $q$  are integers.

(6)

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**Question 11 continued**

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**Question 11 continued**

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