Mark Scheme (Results)

Summer 2022

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 2

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Summer 2022
Question Paper Log Number P71666A
Publications Code 4PM1_02_2206_MS

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme - not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed out work should be marked **unless** the candidate has replaced it with an alternative response.

- **Types of mark**
    - M marks: method marks
    - A marks: accuracy marks
    - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
    - cao – correct answer only
    - ft – follow through
    - isw – ignore subsequent working
    - SC - special case
    - oe – or equivalent (and appropriate)
    - dep – dependent
    - indep – independent
    - awrt – answer which rounds to
    - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to $x=....$

$(ax^2+bx+c)=(mx+p)(nx+q)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=....$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=....$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either** quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication

from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."**Exact answers:**

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question number	Scheme	Marks
1	$b^2 - 4ac \geq 0 \Rightarrow (5k)^2 - 4(2k)(5k-3)[\geq 0]$	M1
	$[-15k^2 + 24k] \Rightarrow 3k(-5k+8)$	M1
	$'0' < k \leqslant '\frac{8}{5}'$	M1
	$0 < k \leqslant \frac{8}{5}$	A1 (4)
		Total 4 marks

Mark	Notes
M1	For correct substitution of a, *b* and *c* into $b^2 - 4ac$ Allow with any inequality, equals or even no sign at all.
M1	For solving their quadratic equation using any valid method, (provided the QE is either a 2TQ or a 3TQ). They must reach **TWO** critical values for the award of this mark. See General Guidance for the definition of a valid attempt to solve a QE If they use a calculator, then the Quadratic equation and the two critical values must be correct for the award of this mark.
M1	For forming an 'inside' region with **their** critical values. Allow use of either $<$ or $\leqslant$ here $'0' < k \leqslant '\frac{8}{5}'$ Allow for example $'0' \leqslant k \leqslant '\frac{8}{5}'$
A1	For $0 < k \leqslant \frac{8}{5}$ allow $0 < k < \frac{8}{5}$

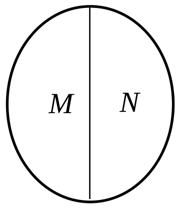
Question number	Scheme	Marks
2 a	$v=\frac{dx}{dt}=4t^3-13.5$	M1
	When $t=3$ $v=4(3)^3-13.5=94.5$ ms^{-1}	A1 (2)
b	$4t^3-13.5=0 \Rightarrow t^3=\frac{27}{8} \Rightarrow t=1.5$	M1 A1ft (2)
c	$a=\frac{dv}{dt}=12t^2$	M1
	When $t=2$ $a=12\times 2^2=48$ ms^{-2}	A1 (2)
		Total 6 marks

Part	Mark	Notes
Ignore incorrect/spurious notation through this question. e.g. ignore $\frac{dy}{dx}=...$ or the LHS		
(a)	**M1**	For an attempt to differentiate the given expression [with no terms integrated] See General Guidance for the definition of an attempt to differentiate.
	A1	For substituting the value of t = 3 into their differentiated expression and obtains 94.5 (units not required)
(b)	**M1**	For setting their $\frac{dx}{dt}=0$ which must be at a minimum of the form $\pm kt^3 \pm l$ and attempting to find a value for t
	A1ft	For $t=1.5$ Ft their expression for v which must have come from an acceptable attempt to differentiate the given x
(c)	**M1**	For differentiating their $\frac{dx}{dt}$ which must be of the form $\frac{dx}{dt}$ (or v) $=\pm kt^3 \pm l$ where k and l are constants, [with no term integrated]
	A1	For substituting t = 2 into their differentiated expression and obtains 48 (units not required)

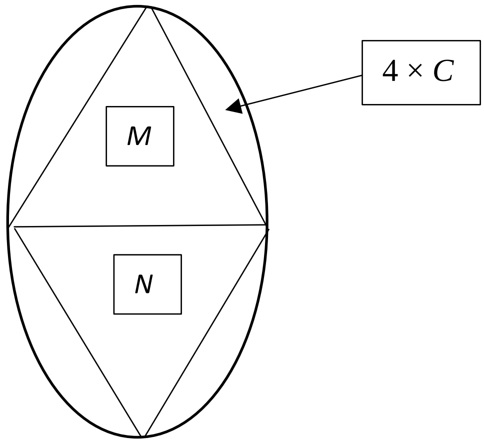
Question number	Scheme	Marks
3 a	$\left\|\overrightarrow{OA}\right\| = \sqrt{p^2+16}$ and $\left\|\overrightarrow{OB}\right\| = \sqrt{4p^2+4p+2}$	
	$\sqrt{2}\left\|\overrightarrow{OA}\right\| = \left\|\overrightarrow{OB}\right\| \Rightarrow 2p^2+32 = 4p^2+4p+2$	M1
	$2p^2+4p-30=0 \Rightarrow p^2+2p-15=0$	M1
	$(p+5)(p-3)=0$	M1
	$p=3$	A1 (4)
b	$\overrightarrow{AB} = -'3'\mathbf{i}+4\mathbf{j}+\mathbf{i}+(2\times'3'+1)\mathbf{j} = -2\mathbf{i}+11\mathbf{j}$	M1 A1
	$\left\|\overrightarrow{AB}\right\| = \sqrt{'4'+'121'}$ $\left[=5\sqrt{5}\right]$	M1
	$'\left[\dfrac{1}{5\sqrt{5}}\right]''(-2\mathbf{i}+11\mathbf{j})'$	dM1
	$(\pm)\ \dfrac{1}{5\sqrt{5}}(-2\mathbf{i}+11\mathbf{j})$	A1 (5)
		Total 9 marks

Part	Mark	Notes
(a)	M1	For use of $\sqrt{2}\left\|\overrightarrow{OA}\right\| = \left\|\overrightarrow{OB}\right\|$ i.e., $\sqrt{2}\times\sqrt{p^2+(-4)^2} = \sqrt{1+(2p+1)^2} \Rightarrow \left(\sqrt{2}\times\sqrt{p^2+16} = \sqrt{4p^2+4p+2}\right)$ They may find $\left\|\overrightarrow{OA}\right\|$ and $\left\|\overrightarrow{OB}\right\|$ separately. Award when combined with $\sqrt{2}$ and condone arithmetical slips.
	M1	For forming a 3TQ in any order. [The correct 3TQ is $2p^2+4p-30=0$ or $p^2+2p-15=0$]
	M1	For a correct attempt to solve **their** 3TQ by any valid method. They must reach a value of *p* for this mark.
	A1	For $p=3$ If they also give $p = -5$ without evidence of rejecting this solution, withhold the A mark
(b)	M1	For the vector statement $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$ o.e. This can be implied by sight of $\overrightarrow{AB} = '-2'\mathbf{i} + '11'\mathbf{j}$ or $\overrightarrow{AB} = \begin{pmatrix} '-2' \\ '11' \end{pmatrix}$ If there is no vector statement you must check their vector for substitution of their *p*.
	A1	For the correct $\overrightarrow{AB}$ (allow unsimplified) $\left(\overrightarrow{AB} = -2\mathbf{i}+11\mathbf{j}\right)$ and also allow $\overrightarrow{AB} = \begin{pmatrix} -2 \\ 11 \end{pmatrix}$ Award for sight of $-2\mathbf{i}+11\mathbf{j}$ only.
	M1	For using Pythagoras theorem on their $\overrightarrow{AB}$ i.e., $\sqrt{('-2')^2 + '11'^2}$
	dM1	For correct method to find a unit vector using their values and their $\overrightarrow{AB}$ NB: this mark is dependent on the previous M mark
	A1	For $\frac{1}{5\sqrt{5}}(-2\mathbf{i}+11\mathbf{j})$ oe $\left[\text{Allow } -\frac{1}{5\sqrt{5}}(-2\mathbf{i}+11\mathbf{j})\right]$ **OR** $\overrightarrow{AB} = \pm\frac{1}{5\sqrt{5}}\begin{pmatrix} -2 \\ 11 \end{pmatrix}$

Question number	Scheme	Marks
4	$O_1A = O_2A = O_1O_2 = 6$ so triangle AO_1O_2 is equilateral	B1
	So angle $AO_1O_2 = \frac{\pi}{3}$	M1
	Area of sector $AO_1BO_2 = \frac{1}{2} \times '\frac{2\pi}{3}' \times 6^2 = (12\pi)$	M1
	Area of triangle $AO_1B = \frac{1}{2} \times 6^2 \times \sin\left('\frac{2\pi}{3}'\right) = 9\sqrt{3}$	M1
	Area of segment $AO_1B = 12\pi - 9\sqrt{3}$	M1
	So $P = 2\times('12\pi - 9\sqrt{3}') = 24\pi - 18\sqrt{3}$ (cm^2) Decimal value for reference is 44.221… (cm^2)	M1 A1 (7)
		Total 7 marks

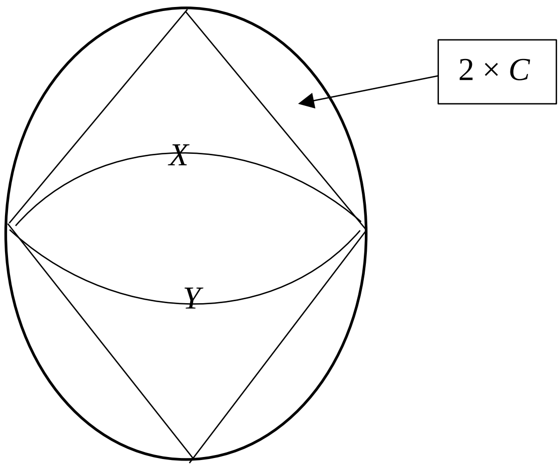
Mark	Notes
B1	For recognising that triangle AO_1O_2 is equilateral. This can be implied by further work. Look at the diagram. The angles are frequently marked on there. **This mark is common to all three methods of finding the area**. **Note**: Correct calculations for either the area of the sector or area of triangle implies the first two marks.
Finds area of segment AO_1B and doubles. (Area M + Area N)  	
M1	For angle $AO_1O_2 = \frac{\pi}{3}$ or 60^o **NB** This is often marked on the diagram. Please check that carefully.
M1	Correct method to find area of sector $AO_1BO_2 = \frac{1}{2}\times'\frac{2\pi}{3}'\times 6^2 = (12\pi)$ or $\frac{1}{3}\times\pi\times 6^2 = (12\pi)$
M1	Correct method to find area of triangle $AO_1B = \frac{1}{2}\times 6^2 \times \sin\left('\frac{2\pi}{3}'\right) = 9\sqrt{3}$
M1	So area of segment $AO_1B = 12\pi - 9\sqrt{3}$
dM1	For area of $R = 2\times\left('12\pi - 9\sqrt{3}'\right)$ **Dependent on all 4 previous M marks**
A1	For $24\pi - 18\sqrt{3}$ $\left(\text{cm}^2\right)$ (Units not required)
ALT	
M1	For angle $AO_1O_2 = \frac{\pi}{3}$ or 60^o **NB** This is often marked on the diagram. Please check that carefully.
M1	Correct method to find area of sector $AO_1O_2 = \frac{1}{2}\times'\frac{\pi}{3}'\times 6^2 = (6\pi)$ or $\frac{1}{6}\times\pi\times 6^2$
M1	Correct method to find area of triangle $AO_1O_2 = \frac{1}{2}\times 6^2 \times \sin\left('\frac{\pi}{3}'\right) = 9\sqrt{3}$ OR height of triangle $AO_1O_2 = 3\sqrt{3} \Rightarrow$ Area $AO_1O_2 = \frac{1}{2}\times 3\sqrt{3}\times 6 = 9\sqrt{3}$
M1	So area of segment C (on diagram) $='6\pi' - '9\sqrt{3}'$

Method 1 - 2 triangles + 4 segments = $M+N+4\times C$



dM1	Area of $R=2\times\Delta AO_1O_2+4\times\text{segment }AO_1O_2=2\times\left('9\sqrt{3}'\right)+4\left('6\pi'-'9\sqrt{3}'\right)$ **Dependent on all 4 previous M marks**
A1	For $24\pi-18\sqrt{3}\ \left(\text{cm}^2\right)$ (Units not required)

Method 2 – 2 sectors + 2 segments = $X+Y+2C$



dM1	Area of $R=2\times$ Sector $AO_1O_2+2\times\text{segment }AO_1O_2=2\times'6\pi'+2\left('6\pi'-'9\sqrt{3}'\right)$ **Dependent on all 4 previous M marks**
A1	For $24\pi-18\sqrt{3}\ \left(\text{cm}^2\right)$ (Units not required)

Question number	Scheme	Marks
5 a	$\alpha+\beta=-\frac{b}{a}=\frac{-(6+2p)}{2}=-3-p$	B1
	$\alpha\beta=\frac{c}{a}=\frac{2p}{2}=p$	B1 (2)
b	$(\alpha-\beta)^2=\alpha^2+\beta^2-2\alpha\beta$	M1
	$=(\alpha+\beta)^2-4\alpha\beta$	M1
	$=(-3-p)^2-4p$	M1
	$=9+2p+p^2$ *	A1 cso (4)
c	$(\alpha-\beta)=3$ implies $(\alpha-\beta)^2=9$	B1
	So $9=9+2p+p^2\Rightarrow 0=2p+p^2\Rightarrow 0=p(2+p)$	M1
	$p=0$ or $p=-2$	A1 (3)
		Total 9 marks

Part	Mark	Notes
(a)	**B1**	For the sum $\alpha+\beta=-3-p$ Need not be simplified. E.g., accept $\frac{-(6+2p)}{2}$
	B1	For the product $\alpha\beta=p$ $\left(\text{Accept } \frac{2p}{2}\right)$
(b)	**M1**	For the correct algebra on $(\alpha-\beta)^2$ $(\alpha-\beta)^2=\alpha^2+\beta^2-2\alpha\beta$ Accept un-simplified and terms in any order
	M1	For the **correct** algebra on $(\alpha-\beta)^2$ using $(\alpha+\beta)^2$ $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$ Note: $(\alpha+\beta)^2=9+6p+p^2$ which you may see substituted in without seeing the algebra
	M1	For substituting in their sum and product from part (a) $(\alpha-\beta)^2=(-3-p)^2-4p$
	A1 cso	For obtaining the given result $(\alpha-\beta)^2=9+2p+p^2$ Note: condone absence of seeing the LHS. **Note**: **This is a given result.** There must be **no errors or omissions** (including in algebra) in their work for this mark
(c)	**B1**	For $(\alpha-\beta)^2=9$
	M1	For setting the given answer = 9 $9+2p+p^2=9\Rightarrow 2p+p^2=0\Rightarrow p=...$ Two values.
	A1	For $p=0$ or $p=-2$

Question number	Scheme	Marks
6 a	$\cos 2A = \cos^2 A - \sin^2 A$ $= (1 - \sin^2 A) - \sin^2 A = 1 - 2\sin^2 A$ *	M1 A1 cso
		(2)
b	$\pi\int_0^{\frac{\pi}{4}} (3 + 2\sin x)^2 \mathrm{d}x$	M1
	$\pi\int_0^{\frac{\pi}{4}} (9 + 12\sin x + 4\sin^2 x)\ \mathrm{d}x$	M1
	$\pi\int_0^{\frac{\pi}{4}} (11 + 12\sin x - 2\cos 2x)\ \mathrm{d}x$	M1
	$\pi[11x - 12\cos x - \sin 2x]_0^{\frac{\pi}{4}}$	M1
	$\pi\left[\left(\frac{11\pi}{4} - 6\sqrt{2} - 1\right) - (0 - 12 - 0)\right] =$ awrt 35	M1 A1 (6)
		Total 8 marks

Part	Mark	Notes
(a)	**M1**	For the use of both $\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A = \left(1 - \sin^2 A\right)$ to attempt to form the required identity.
	A1	For $\cos 2A = 1 - 2\sin^2 A$ with no errors seen.
(b)	**M1**	For the correct use of use of $\pi\int_a^b y^2 \mathrm{d}x$ to obtain: $\pi\int_0^{\frac{\pi}{4}}(3+2\sin x)^2 \mathrm{d}x$ with the correct limits and π You may not see π and the limits at this stage. They may add these in at the end of their work. Check and if that is the case, award this mark.
	M1	For expanding the given quadratic correctly to achieve: This must be correct. $\pi\int_0^{\frac{\pi}{4}}\left(9+12\sin x+4\sin^2 x\right)\mathrm{d}x$ or accept $\pi\int_0^{\frac{\pi}{4}}\left(9+6\sin x+6\sin x+4\sin^2 x\right)\mathrm{d}x$ Ignore limits and π for this mark. **This is an A mark in Epen**
	M1	For using $\sin^2 A = \frac{1-\cos 2A}{2}$ and substituting this identity into their expansion to attempt to obtain: $\pi\int_0^{\frac{\pi}{4}}(11+12\sin x - 2\cos 2x)\,\mathrm{d}x$ As a minimum we need to see: $A + B\sin x \pm C\cos 2x$ Ignore limits and π for this mark.
	M1	For an attempt to integrate their expression which must be of the form: $A + B\sin x \pm C\cos 2x$ **or** $A + B\sin x \pm C\sin^2 x$ where there must be an acceptable attempt to integrate as a minimum at least **both** A **and** $B\sin x$ to obtain $A \to Ax$ and $B\sin x \to -B\cos x$ Ignore limits and π for this mark.
	M1	For the correct substitution of the correct given limits the correct way around into an integrated/changed expression. $\pi\left[\left(\frac{11\pi}{4} - 6\sqrt{2} - 1\right) - (0 - 12 - 0)\right] = 35.041....$ This mark can be implied by the correct final answer **ONLY** if the correct integration is seen explicitly in the response. If their integrated expression is incorrect, and no substitution is seen award M0 here.
	A1	For awrt 35

Question number	Scheme	Marks
7(i) (a)	$\tan 2\theta = \dfrac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta}$	M1
	$= \dfrac{2\tan\theta}{1-\tan^2\theta}$ *	A1 cso (2)
(b)	$1 = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$	
	$\tan^2\alpha + 2\tan\alpha - 1 = 0$	M1
	$\tan\alpha = -1 \pm \sqrt{2}$	M1 A1 (3)
(ii) (a)	$\cos(x-30) = \sin(x+30\ )$	
	$\Rightarrow \cos x\cos 30^\circ + \sin x \sin 30^\circ = \sin x\cos 30^\circ + \cos x \sin 30^\circ$	M1 M1
	$\left(\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}\right)\cos x = \left(\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}\right)\sin x$	
	$\dfrac{\sin x}{\cos x} = \tan x = \dfrac{\left(\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}\right)}{\left(\dfrac{\sqrt{3}}{2} - \dfrac{1}{2}\right)}$	M1
	$\tan x = 1$ *	A1 cso (4)
(b)	$\tan 2y = 1 \Rightarrow \tan y = -1 \pm \sqrt{2}$	
	$y = -67.5^\circ, 22.5^\circ$	M1 A1 (2)
		Total 11 marks

Part	Mark	Notes
(i)(a)	**M1**	For use of $\tan(A+B) \Rightarrow \tan 2\theta = \dfrac{\tan\theta + \tan\theta}{1-\tan\theta\tan\theta}$ and simplifying to obtain the given result.
	A1	For the given result with no errors seen
(i)(b)	**M1**	For setting $1 = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$ and for rearranging to obtain a 3TQ with terms in any order
	M1	For a correct attempt using a valid method to solve the 3TQ with two solutions for $\tan\alpha$ Accept use of θ for example in place of α
	A1	For $\tan\alpha = -1 \pm \sqrt{2}$
(ii)(a)	**General principle**. (The second and third M marks can be carried out in either order) First M1 – For expanding the two addition formulae **correctly and** equating. Second M1 – For collecting up like terms to obtain $\sin x(......) = \cos x(......)$ allow sign errors. Third M1 – For use of the $\tan = \dfrac{\sin}{\cos}$ identity Final A1 – For the given answer with no errors	

	They can work in terms of sin30º, cos30º or tan30º or alternatively $\frac{1}{2}, \frac{\sqrt{3}}{2}, \frac{1}{\sqrt{3}}$ respectively	
	M1	For the correct use of $\cos(A-B)$ and $\sin(A+B)$ to obtain $\cos x\cos 30^\circ + \sin x\sin 30^\circ = \sin x\cos 30^\circ + \cos x\sin 30^\circ$
	M1	For collecting up like terms to obtain: $\sin x(\sin 30^\circ - \cos 30^\circ) = \cos x(\sin 30^\circ - \cos 30^\circ)$ **NB**: This is an A mark in Epen
	M1	For use of $\tan = \frac{\sin}{\cos}$ at any stage in their working.
	A1	For $\tan x = 1$
	ALT	
	M1	For the correct use of $\cos(A-B)$ and $\sin(A+B)$ to obtain $\cos x\cos 30^\circ + \sin x\sin 30^\circ = \sin x\cos 30^\circ + \cos x\sin 30^\circ$
	M1	For dividing through by $\cos x$ and collecting up like terms to obtain: $\frac{\cos x\cos 30^\circ + \sin x\sin 30^\circ}{\cos x} = \frac{\sin x\cos 30^\circ + \cos x\sin 30^\circ}{\cos x}$ $\Rightarrow \frac{\sin x}{\cos x}(\sin 30^\circ - \cos 30^\circ) = \sin 30^\circ - \cos 30^\circ$ **OR** For dividing through by $\cos x$ **and** $\cos 30^\circ$ and also collecting up like terms to obtain: $\frac{\cos x\cos 30^\circ + \sin x\sin 30^\circ}{\cos x\cos 30^\circ} = \frac{\sin x\cos 30^\circ + \cos x\sin 30^\circ}{\cos x\cos 30^\circ}$ $\Rightarrow 1 + \frac{\sin x\sin 30^\circ}{\cos x\cos 30^\circ} = \frac{\sin x}{\cos x} + \frac{\sin 30^\circ}{\cos 30^\circ} \Rightarrow \frac{\sin x}{\cos x}\left(\frac{\sin 30^\circ}{\cos 30^\circ} - 1\right) = \frac{\sin 30^\circ}{\cos 30^\circ} - 1$
	M1	For use of $\tan = \frac{\sin}{\cos}$ at any stage in their working.
	A1	For $\tan x = 1$
(ii)(b)	**M1**	For solving the equation $\tan 2y = 1 \Rightarrow \tan y = -1 \pm \sqrt{2} \Rightarrow y = ..., ...$ (leading to two values for y) **NB**: This is an B mark in Epen
	A1	$y = -67.5^\circ, 22.5^\circ$ Both values required. **NB**: This is an B mark in Epen
	ALT	
	M1	For solving the equation $\tan 2y = 1$ to achieve at least one correct value for $2y$ $2y = \tan^{-1} 1 = 45^\circ, -135^\circ$ [Accept even for example 225° for this mark]
	A1	$y = -67.5^\circ, 22.5^\circ$ Both values required. For extra angles in range withhold this mark. Extra angles outside of the range – ignore. **NB**: This is an B mark in Epen

Question number	Scheme	Marks
8 a	Height of the waste paper basket = $\sqrt{(5x)^2-(3x)^2}=4x$ (cm)	M1
	$V=\frac{1}{2}(2x+8x)\times 4x\times h=20x^2h=2250\Rightarrow h=\frac{2250}{20x^2}$ oe	M1
	$S=2\times 20x^2+2xh+2(5xh)$	M1
	$S=40x^2+12x\left(\frac{2250}{20x^2}\right)$ oe	M1
	$S=40x^2+\frac{1350}{x}$ *	A1 cso (5)
b	$\frac{dS}{dx}=80x-\frac{1350}{x^2}$ oe	M1
	$\frac{dS}{dx}=80x-\frac{1350}{x^2}=0$ so $x^3=\frac{135}{8}\Rightarrow x=...$	M1
	$x=2.56$ awrt	A1
	$\frac{d^2S}{dx^2}=80+\frac{2700}{x^3}>0$ for all positive values of $x\therefore$ minimum	M1 A1ft (5)
c	When x = 2.56 $S=40(2.56)^2+\frac{1350}{2.56}=789$ awrt	M1 A1 (2)
		Total 12 marks

Part	Mark	Notes
(a)	**M1**	For finding the height of the waste-paper basket using Pythagoras theorem. $\sqrt{(5x)^2-(3x)^2}=4x$ (cm)
	M1	For finding the volume which must come from; V = correct area of trapezium (using their height) × h $V=\frac{1}{2}(2x+8x)\times'4x'\times h=2250\Rightarrow('20'x^2h=2250)$ **and** for obtaining an expression for the height h: $h=\frac{2250}{'20'x^2}$ or $xh=\frac{2250}{'20'x}$ Please check their algebra carefully, as some may even substitute $hx^2=...$ **NB: This is an A mark in Epen.**
	M1	For writing the surface area in terms of 2 unknowns [x and h] which need not be simplified. This must be **correct** using their expression in terms of x for the height of the prism. e.g. $S=2\times\frac{(8x+2x)\times'4x'}{2}+2xh+2(5xh)=\left[40x^2+12xh\right]$
	M1	For eliminating h from their expression for S S must be in the form Ax^2+Bxh and h must be of the form $\frac{C}{x^2}$ where A, B and C are constants.
	A1 cso	For the given result exactly as written. $S=40x^2+\frac{1350}{x}$
(b)	**M1**	For an acceptable attempt to differentiate the **given** expression for S. See General Guidance.
	M1	For setting their $\frac{dS}{dx}=0$ and attempting to find a value for x The minimum acceptable expression is $\frac{dS}{dx}=Px\pm\frac{Q}{x^2}$ where P and Q are constants.
	A1	For awrt $x=2.56$
	M1	For differentiating their $\frac{dS}{dx}$, which must be as a minimum $\frac{dS}{dx}=Px\pm\frac{Q}{x^2}$ to find the second derivative to achieve as a minimum $\frac{d^2S}{dx^2}=\pm M\pm\frac{N}{x^3}$
	A1ft	Concludes that as $\frac{d^2S}{dx^2}$ will always be positive, [either by substitution, or by inference] so the value of x obtained will be a minimum. $\left[\frac{d^2S}{dx^2}=80+\frac{2700}{2.56^3}=240.9...\right]$ with a conclusion. **NOTE:** The ft only applies to their value of x. Do not ft an incorrect $\frac{d^2S}{dx^2}$
(c)	**M1**	Substitutes their value of x, obtained using a correct method) into the **given** expression for S [provided it is a positive value, do **NOT** allow negative values of x]
	A1	For awrt 789

Question number	Scheme	Marks
9 a	$L_1 : y-7 = m(x-4)$	B1
	Gradient of $L_2 = -\frac{1}{m}$	M1
	$L_2 : y-k = -\frac{1}{m}(x-4)$	M1
	$x = \frac{y-7}{m} + 4$	M1
	$y-k = -\frac{1}{m}\left(\frac{y-7}{m} + 4 - 4\right)$	M1
	$m^2 y - m^2 k = 7 - y$	
	$y(m^2+1) = 7 + m^2 k$	dM1
	$Y = \frac{7+m^2k}{m^2+1}$ *	A1 cso (7)
b	Midpoint of $AB = \left([4], \frac{k+7}{2}\right)$	B1
	So $\frac{k+7}{2} = \frac{7+m^2k}{m^2+1}$	M1
	$m^2k + 7m^2 + k + 7 = 2m^2k + 14$	
	$m^2(k-7) = k-7$	M1
	As $k \neq 7$ $m = \pm 1$	A1
	As $m < 0$ $m = -1$	A1 (5)
		Total 12 marks

Part	Mark	Notes
(a)	**NOTE** ; The algebra must be correct for the first three marks.	
	B1	For writing an equation for L_1 e.g., $y-7=m(x-4)$ **OR** Uses $y=mx+c \Rightarrow y=mx+(7-4m)$
	M1	For the gradient of the perpendicular $\left(-\frac{1}{m}\right)$
	M1	For writing an equation for L_2 (ft a changed gradient) e.g., $y-k=-\frac{1}{m}(x-4)$ **OR** Uses $y=mx+c \Rightarrow y=-\frac{1}{m}x+\left(k+\frac{4}{m}\right) \Rightarrow \left[y=\frac{-x+km+4}{m}\right]$
	M1	For rearranging their equation of L_2 to make *x* the subject e.g., $x=\frac{y-7}{m}+4$ or $x=km-ym+4$ **ALT 1** Rearranges either equation to make (*x* – 4) the subject $x-4=\frac{y-7}{m}$ and/or $x-4=ym+km$ **ALT 2** Eliminates *y* from their equations to obtain $'x-4m+7'='\frac{-x+4+km}{m}' \Rightarrow x='\frac{4+km+4m^2-7m}{m^2+1}'$ They must reach *x* = ……. for the award of this mark. **Allow sign errors/slips at this stage.**
	M1	Forming a linear equation in *y* No simplification is required at this stage. $y-k=-\frac{1}{m}\left(\frac{y-7}{m}+4-4\right)$ **ALT 1** $\frac{y-7}{m}=ym+km$ **ALT 2** Forms a linear equation in *y* using their expression for *x* by substituting back into either L_1 or L_2 **Allow sign errors/slips at this stage.**
	dM1	For attempting to rearrange to obtain the required result. **Allow sign errors but there must be no missing terms. You must check carefully.**
	A1	For the required result with no errors seen. $Y=\frac{7+m^2k}{m^2+1}$ allow $y=\frac{7+m^2k}{m^2+1}$

(b)	**B1**	For identifying *y* coordinate of the midpoint of *AB* $\left([4], \frac{k+7}{2}\right)$ the *x* coordinate is not required for this mark.
	M1	Equating the *y* coordinate of *C* and *y* coordinate of the midpoint to give $\frac{k+7}{2} = \frac{7+m^2k}{m^2+1}$
	M1	For rearranging and factorising to attempt achieve $m^2(k-7) = k-7$ This mark can implied by the correct value for either m^2 or *m*
	A1	Concludes that $m = \pm 1$ $\left[m^2 = \frac{k-7}{k-7} = 1 \Rightarrow m = \pm 1\right]$ Again, this can be implied by the correct *m* = – 1
	A1	For $m = -1$
	ALT 1	
	B1	Let coordinates of *C* be (*X*, *Y*) Sets: length *AC* = length *BC* $(X-4)^2 + (Y-7)^2 = (X-4)^2 + (Y-k)^2 \Rightarrow \left[(Y-7)^2 = (Y-k)^2\right]$
	M1	Using the coordinates for *C* of (*X*, *Y*) or otherwise, performs the following algebraic manipulation. $(Y-7)^2 = (Y-k)^2 \Rightarrow Y^2 - 14Y + 49 = Y^2 - 2kY + k^2 \Rightarrow -14Y + 49 = -2kY + k^2$ $\Rightarrow 2Y(k-7) = (k+7)(k-7)$ $\Rightarrow 2Y = k+7$ **NB: Allow sign slips, but not missing terms**
	M1	Substitutes in the given expression for *Y* and attempts to obtain $m^2(k-7) = k-7$ $\frac{7+m^2k}{m^2+1} = \frac{k+7}{2}$ $\Rightarrow 14 + 2m^2k = km^2 + 7m^2 + k + 7 \Rightarrow 7 + km^2 = 7m^2 + k$ $\Rightarrow m^2(k-7) = k-7$ **NB: Allow sign slips, but not missing terms**
	A1	Concludes that *m* = ± 1 $m^2 = \frac{k-7}{k-7} = 1 \Rightarrow m = \pm 1$
	A1	For *m* = – 1
	ALT 2	
	If candidates deduces that *BAC* = 45º and concludes that the gradient must be *m* = – 1 award B1M1M1A1A1 If they leave their answer as *m* = ± 1 or just *m* = 1, withhold the final A mark	

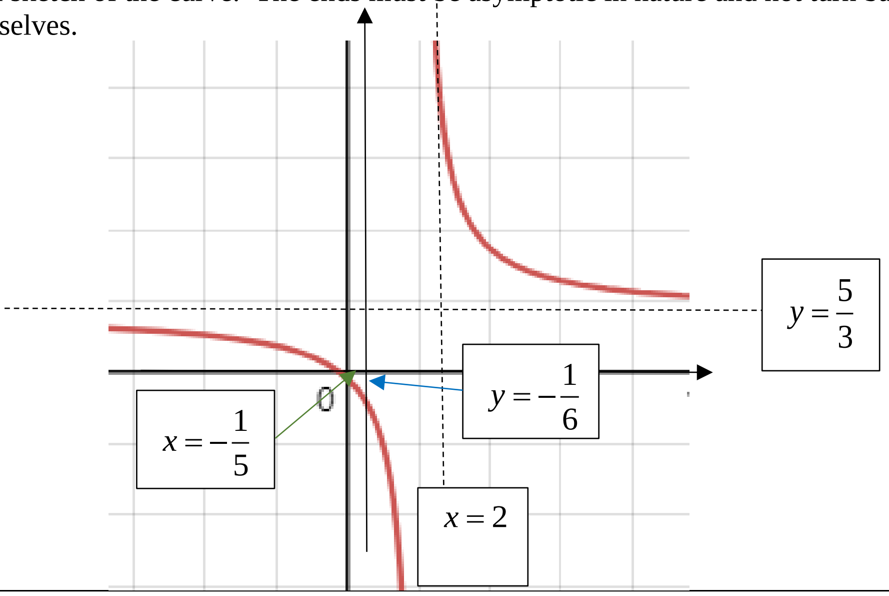
Question number	Scheme	Marks
10	$\frac{\log_2 x}{\log_2 4}+\frac{\log_2 x}{\log_2 16}+\log_2 x=10.5$	M1
	$\frac{\log_2 x}{2}+\frac{\log_2 x}{4}+\log_2 x=10.5$	M1
	$\frac{7}{4}\log_2 x=10.5$	M1
	$x=2^{'6'}$	M1
	$x=64$	A1 (5)
		Total 5 marks

Mark	Notes
The first two marks are common to both methods	
M1	Changes the base of any log correctly seen anywhere. Accept change to $\log_x$ or even $\log_{10}$ **Base 2** $\frac{\log_2 x}{\log_2 4}, \frac{\log_2 x}{\log_2 16}$ **Base 4** $\frac{\log_4 x}{\log_4 16}, \frac{\log_4 x}{\log_4 2}$ **Base 16** $\frac{\log_{16} x}{\log_{16} 4}, \frac{\log_{16} x}{\log_{16} 2}$
M1	For forming an equation (in any form) in a single base in **any** base <u>For example;</u> **Base 2** $\frac{\log_2 x}{2}+\frac{\log_2 x}{4}+\log_2 x=10.5$ or e.g., $\frac{\log_2 x}{\log_2 4}+\frac{\log_2 x}{4}+\log_2 x=10.5$ **Base 4** e.g $\log_4 x+\frac{\log_4 x}{2}+\frac{\log_4 x}{\frac{1}{2}}=10.5$ **Base 16** $\frac{\log_{16} x}{\frac{1}{2}}+\log_{16} x+\frac{\log_{16} x}{\frac{1}{4}}=10.5$ **NB - This is an A mark in Epen**

Method 1	
M1	For simplifying to the form $A\log_{(\text{any base})} = B$ where A and B are constants For example; **Base 2** $1.75\log_2 x = 10.5$ **Base 41** $3.5\log_4 x = 10.5$ **Base 16** $7\log_{16} x = 10.5$
M1	For undoing the log For example: **Base 2** $x = 2^6$ **Base 4** $x = 4^3$ **Base 16** $x = 16^{1.5}$
Method 2	
M1	Uses the addition law to simplify to the form: $\log_n\left(x^a \times x^b \times x^c\right) = 10.5 \Rightarrow (a+b+c)\log_n x = 10.5$ For example: B **Base 2** $\log_2\left(x^{\frac{1}{2}} \times x^{\frac{1}{4}} \times x\right) = 10.5 \Rightarrow \frac{7}{4}\log_2 x = 10.5$ **Base 4** $\log_4\left(x \times x^{\frac{1}{2}} \times 2x\right) = 10.5 \Rightarrow \frac{7}{2}\log_4 x = 10.5$ **Base 16** $\log_{16}(2x \times x \times 5x) = 10.5 \Rightarrow 7\log_{16} x = 10.5$
M1	For undoing the log: **Base 2** $2^{10.5} = x^{\frac{7}{4}}$ **Base4** $4^{10.5} = x^{\frac{7}{2}}$ **Base 16** $16^{10.5} = x^7$
A1	For $x = 64$

Question number	Scheme	Marks
11 a	$\dfrac{dy}{dx}=\dfrac{(2a-1)(ax-6)-a((2a-1)x+1)}{(ax-6)^2}$	M1A1A1 (3)
b (i)	At $x = 0$ $\dfrac{dy}{dx}=-\dfrac{11}{12}$	B1
	So $\dfrac{dy}{dx}=\dfrac{-6(2a-1)-a}{36}=-\dfrac{11}{12}$	M1
	$-13a+6=-33 \Rightarrow a=3*$	dM1 A1 cso (4)
b (ii)	$y=\dfrac{5x+1}{3x-6}*$	B1 cso (1)
c	Asymptote with equation drawn at $y=2$	B1
	Asymptote with equation drawn at $x=\dfrac{5}{3}$	B1
	Correct curve drawn with two branches	B1
	$-\dfrac{1}{5}$ and $-\dfrac{1}{6}$ labelled on x and y axes	B1 B1 (5)
d	$\dfrac{12}{11}x-\dfrac{1}{6}=\dfrac{5x+1}{3x-6}$	M1
	$(3x-6)(72x-11)=66(5x+1)$	M1
	$x(72x-265)=0$ oe	M1
	$x=\dfrac{265}{72}$ oe	A1 (4)
		Total 17 marks

Part	Mark	Notes
(a)	**M1**	For attempting to use a correct quotient rule: The definition of an attempt is as follows: • The denominator must be correct • The terms in the numerator must be subtracted either way around • The attempt to differentiate both terms must be as follows: $\left(2a-1\right)x+1 \to \pm\left(2a-1\right)$ and $ax-6 \to \pm a$
	A1	For one term in the numerator correct
	A1	For a fully correct differentiated expression. $\dfrac{dy}{dx}=\dfrac{(2a-1)(ax-6)-a((2a-1)x+1)}{(ax-6)^2}$ No simplification is required.

	Uses Product Rule	
	M1	For attempting to use a correct product rule: The definition of an attempt is as follows: • The terms in the numerator must be added. • The attempt to differentiate both terms must be as follows: $(2a-1)x+1 \to \pm(2a-1)$ and $(ax-6)^{-1} \to \pm a(ax-6)^{-2}$
	A1	For one term correct
	A1	For a fully correct differentiation $\frac{dy}{dx}=[(2a-1)x+1](a)(ax-6)^{-2}+(ax-6)^{-1}(2a-1)$
(b)(i)	**B1**	For finding the gradient of the tangent $=-\frac{11}{12}$
	M1	For using that at the point A $x = 0$ and setting their $\frac{dy}{dx}=\left(-\frac{11}{12}\right)$ which must be the gradient of their tangent to obtain $\frac{-6(2a-1)-a}{36}='-\frac{11}{12}' \Rightarrow \left(\frac{-13a+6}{36}=-\frac{11}{12}\right)$
	dM1	For simplifying to a linear equation and attempting to solve.
	A1	For $a=3$ with no errors seen
(b)(ii)	**B1**	For $y=\frac{5x+1}{3x-6}$ exactly in that form. You must see evidence of substitution of a = 3 for this mark. There must be $y=$ …. [Condone missing $x \neq 2$]
(c)		This is a sketch of the curve. The ends must be asymptotic in nature and not turn back on themselves. 
	B1	For asymptote **with equation drawn** at $y=\frac{5}{3}$ OR $x=2$
	B1	For both asymptotes **with equations drawn** at $y=\frac{5}{3}$ AND $x=2$
	B1	Correct curve drawn with two branches with one branch in 1st Quadrant and another branch in Quadrants 2, 3 and 4 with the graph drawn the correct way around. The graph must be asymptotic in nature.

	B1	$-\frac{1}{5}$ labelled on x-axis **or** $-\frac{1}{6}$ labelled on y- axis
	B1	$-\frac{1}{5}$ labelled on x axis **and** $-\frac{1}{6}$ labelled on y- axis
(d)	**M1**	For setting $\frac{12}{11}x-\frac{1}{6}=\frac{5x+1}{3x-6}$ [eliminating y from both equations]
	M1	For forming a quadratic equation, 2TQ or 3TQ in any order. $(3x-6)(72x-11)=66(5x+1) \Rightarrow 72x^2-265x=0$
	M1	For solving their QE provided it is either a 2TQ or a 3TQ
	A1	For $x=\frac{265}{72}$ oe

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