Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Time 2 hours

Paper reference **4PM1/02**

Further Pure Mathematics
PAPER 2

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P71819A

©2023 Pearson Education Ltd.

J:1/1/1/1/

P 7 1 8 1 9 A 0 1 3 6

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\quad$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\quad$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

4PM1 | 2023 | January | Paper 2 | GradeMax

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Find the set of values of x for which

(a) $2(3x - 1) < 4 - 3x$

(2)

(b) $3x^2 - 8x - 3 < 0$

(4)

(c) **both** $2(3x - 1) < 4 - 3x$ **and** $3x^2 - 8x - 3 < 0$

(1)

(Total for Question 1 is 7 marks)

2

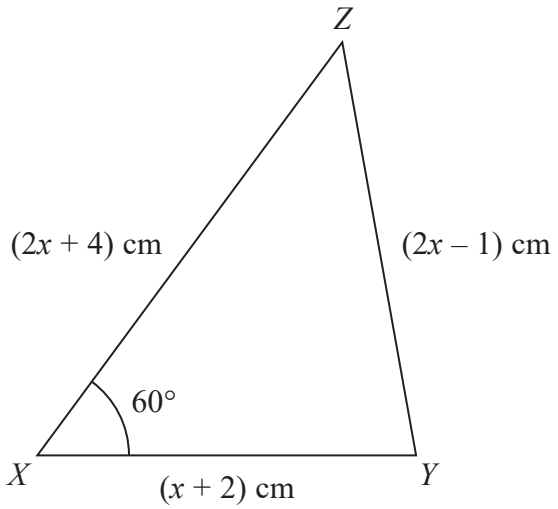

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows triangle XYZ in which

$XY = (x + 2)$ cm $\quad XZ = (2x + 4)$ cm $\quad YZ = (2x - 1)$ cm $\quad$ and $\angle YXZ = 60°$

Find the value of x

Give your answer in the form $p + q\sqrt{3}$ where p and q are integers to be found.

(4)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 2 continued

(Total for Question 2 is 4 marks)

3

$$f(x) = 8x^2 + 10x - 3$$

Given that $f(x)$ can be written in the form $A(x + B)^2 + C$ where A, B and C are constants,

(a) find the value of A, the value of B and the value of C.

(3)

(b) Hence, or otherwise, find,

(i) the value of x for which $f(x)$ has a minimum,

(ii) the minimum value of $f(x)$.

(2)

The curve C has equation $y = f(x)$.

(c) Find the x coordinate of each of the points where C crosses the x-axis.

(2)

The straight line l has equation $y = 2x + 13$

(d) Use algebra to find the coordinates of the two points of intersection of C and l.

(4)

Using the same axes and the results of parts (b), (c) and (d),

(e) sketch the curve C and the straight line l.

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 3 continued

Question 3 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 3 continued

(Total for Question 3 is 13 marks)

4 The equation of a curve is $y = x^3 \sin x$

Find an equation of the tangent to the curve at the point on the curve where $x = \frac{1}{2}\pi$

Give your answer in the form $y = mx + c$

(7)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 7 marks)

5

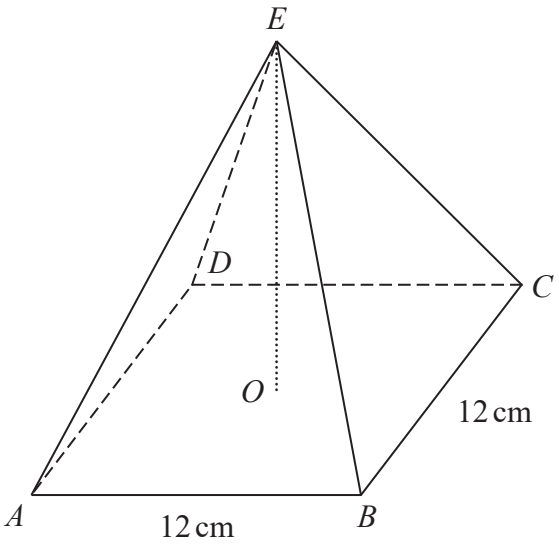

Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a right pyramid with a square base $ABCD$ and vertex E.

The base of the pyramid is horizontal with $AB = BC = 12$ cm.
The diagonals of the base intersect at the point O.

The vertex E of the pyramid is vertically above O and the angle between EA and the plane $ABCD$ is 30°

The height of the pyramid is h cm.

(a) Find the exact value of h

(3)

The point F lies on AD such that $AF : FD = 1 : 4$

(b) Calculate, to the nearest degree, the size of the angle EFO.

(4)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 5 continued

Question 5 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 5 continued

(Total for Question 5 is 7 marks)

6 The nth term of a geometric series G is U_n

The first three terms of G are given by

$$U_1 = q(4p + 1) \qquad U_2 = q(2p + 3) \qquad U_3 = q(2p - 3)$$

(a) Find the possible values of p

(5)

Given that G is convergent with sum to infinity 250

(b) find the value of q

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 6 continued

(Total for Question 6 is 8 marks)

7 $y = e^{2x} \cos 2x$

(a) Show that

$$\frac{dy}{dx} = 2y - 2e^{2x} \sin 2x$$

(4)

(b) Hence show that

$$\frac{d^2y}{dx^2} = 4\frac{dy}{dx} - 8y$$

(5)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 7 continued

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 7 continued

(Total for Question 7 is 9 marks)

8 The quadratic equation

$$x^2 - 4k\sqrt{2}x + 2k^4 - 1 = 0$$

where k is a positive constant, has roots α and β

Given that $\alpha^2 + \beta^2 = 66$ and that $\alpha^3 + \beta^3 = p\sqrt{2}$ where p is an integer,

find the value of p

(11)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 8 continued

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 8 continued

(Total for Question 8 is 11 marks)

9 A cube has edges of length x cm.

The total surface area, A cm^2, of the cube is increasing at a constant rate of 0.45 cm^2/s

Find the rate of increase, in cm^3/s, of the volume of the cube at the instant when the total surface area of the cube is 384 cm^2

(7)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 9 continued

(Total for Question 9 is 7 marks)

10 Using formulae given on page 2

(a) show that

(i) $\sin 2\theta = 2\sin\theta\cos\theta$

(ii) $\cos 2\theta = 2\cos^2\theta - 1$

(5)

Given that $\theta \neq (90° + 180° n)$ where $n \in \mathbb{Z}$

(b) use the results from part (a) to show that $\sin 2\theta - \tan\theta$ can be written as $\tan\theta\cos 2\theta$

(4)

(c) Solve for $0 < x < 360$

$$\sin 2x° - \tan x° = 0$$

(4)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 10 continued

Question 10 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 10 continued

(Total for Question 10 is 13 marks)

11

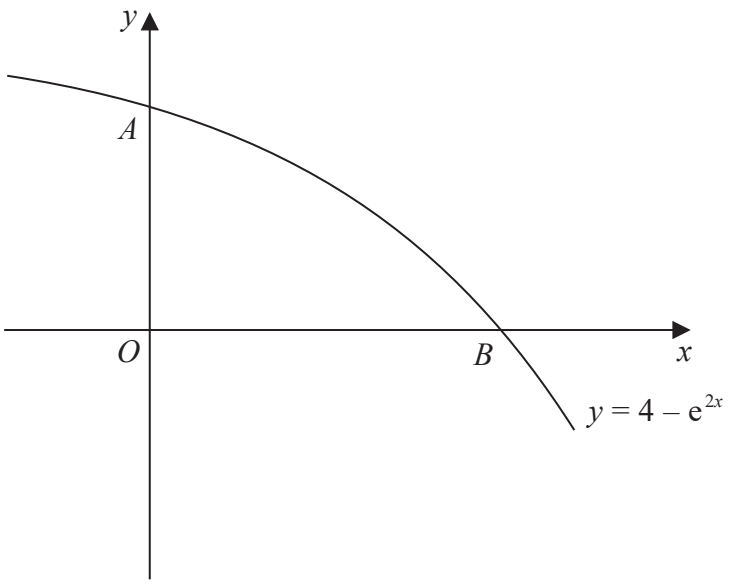

Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows part of the curve C with equation $y = 4 - e^{2x}$
The curve C crosses the y-axis at the point A and the x-axis at the point B.

(a) (i) Write down the y coordinate of point A.

(ii) Show that the x coordinate of B is $x = \ln 2$

(3)

The line l is the normal to C at the point B.

(b) Find an equation for l, giving your answer in the form $y = mx + c$

(4)

The finite region R is bounded by C, l and the y-axis.

(c) Using calculus, find the area of R.
Give your answer to one decimal place.

(7)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 11 continued

Question 11 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 11 continued

Question 11 continued

(Total for Question 11 is 14 marks)

TOTAL FOR PAPER IS 100 MARKS

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA