Please check the examination details below before entering your candidate information

Candidate surname | Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Friday 26 May 2023

Afternoon (Time: 2 hours) | **Paper reference** **4PM1/01**

Further Pure Mathematics
PAPER 1

Calculators may be used. | Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r \times$ slant height

Volume of sphere $= \dfrac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \dfrac{n}{2}\big[2a + (n-1)d\big]$

Geometric series

Sum to n terms, $S_n = \dfrac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \dfrac{a}{1-r}\quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Trigonometry

Cosine rule
In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$ $\qquad$ $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$ $\qquad$ $\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$ $\qquad$ $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

Logarithms

$\log_a x = \dfrac{\log_b x}{\log_b a}$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (a) Show that $\displaystyle\sum_{r=1}^{n}\left(3r+2\right)=\frac{n}{2}\left(3n+7\right)$

(3)

(b) Hence, or otherwise, evaluate $\displaystyle\sum_{r=10}^{40}\left(3r+2\right)$

(2)

(Total for Question 1 is 5 marks)

2
$$y = (\sin 2x)\sqrt{3 + 2x}$$

Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\sin 2x + (A + Bx)\cos 2x}{\sqrt{3 + 2x}}$ where A and B are integers to be found.

(5)

Question 2 continued

(Total for Question 2 is 5 marks)

3

Figure 1

Figure 1 shows part of the curve with equation $y = \dfrac{x}{2} + \dfrac{4}{x^2}$ in the interval $0.8 < x < 7$

By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the roots of the equation $3x^3 - 12x^2 + 8 = 0$ in the interval $0.8 < x < 7$

(5)

Question 3 continued

(Total for Question 3 is 5 marks)

4 A particle P is moving along the x-axis. At time t seconds, $t \geqslant 0$, the velocity, v m/s, of P is given by $v = 2t^2 - 16t + 30$

(a) Find the acceleration, in m/s^2, of P when $t = 5$

(2)

P comes to instantaneous rest at the points M and N at times t_1 seconds and t_2 seconds where $t_2 > t_1$

(b) Find the exact distance MN

(8)

Question 4 continued

(Total for Question 4 is 10 marks)

5 A solid cuboid has width x cm, length $4x$ cm and height h cm.
The volume of the cuboid is 75 cm^3 and the surface area of the cuboid is S cm^2

(a) Show that $S = 8x^2 + \dfrac{375}{2x}$

(4)

Given that x can vary, using calculus,

(b) (i) find to 3 significant figures, the value of x for which S is a minimum,

(ii) justify that this value of x gives a minimum value of S

(5)

(c) Find, to 3 significant figures, the minimum value of S

(2)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 11 marks)

6 Solve the equation

$$\log_2 x^3 + \log_4 x^2 - 3\log_x 2 = 0$$

giving your answers to 3 significant figures.

(8)

Question 6 continued

(Total for Question 6 is 8 marks)

7   The equation of a curve is $y = \sqrt{\dfrac{e^{4x}}{2x-3}}$

When x is increased to $(x + \delta x)$, y increases to $(y + \delta y)$ where δx and δy are small.

(a) Show that $\delta y \approx \dfrac{e^{2x}(4x-7)}{(2x-3)^{\frac{3}{2}}}\, \delta x$

(7)

Given that $x = 2.5$

(b) find an estimate, to 2 significant figures, of the value of δy when the value of x increases by 0.2%

(3)

Question 7 continued

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Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)

8
$$f'(x) = 18x^2 - 2x + 13$$

Given that $(2x - 1)$ is a factor of $f(x)$

show that the curve with equation $y = f(x)$ has only one intersection with the x-axis.

(9)

Question 8 continued

(Total for Question 8 is 9 marks)

Turn over ►

9 (a) Using the formulae on page 2, show that

(i) $\cos^2 A = \dfrac{\cos 2A + 1}{2}$

(ii) $\sin^2 A = \dfrac{1 - \cos 2A}{2}$

(4)

(b) Show that

$$(2\sin x - \cos x)(\sin x - 3\cos x) = \frac{1}{2}(\cos 2x - 7\sin 2x + 5)$$

(5)

$$y = (2\sin x - \cos x)(\sin x - 3\cos x)$$

(c) Solve, for $0° \leqslant x \leqslant 180°$ the equation, $\dfrac{dy}{dx} = 0$

Give your answers to the nearest whole number.

(4)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 13 marks)

10 O, A and B are fixed points such that

$\overrightarrow{OA} = (b + 1)\mathbf{i} + b\mathbf{j}$

$\overrightarrow{AB} = 3\mathbf{i}$

The unit vector parallel to $\overrightarrow{OB}$ is $\dfrac{\sqrt{17}}{34}[(3a + 2)\mathbf{i} + b\mathbf{j}]$

Given that a and b are constants where $a > 0$ and $b > 0$

find the exact value of

(i) a

(ii) b

(10)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 10 marks)

11
$$f(x) = 10 + 6x - x^2$$

Given that $f(x)$ can be written in the form $A(x + B)^2 + C$ where A, B and C are constants,

(a) find the value of A, the value of B and the value of C

(4)

(b) Hence, or otherwise, find

   (i) the value of x for which $f(x)$ has its greatest value

   (ii) the greatest value of $f(x)$

(2)

The curve C has equation $y = f(x)$

The curve S with equation $y = x^2 - x + 13$ intersects curve C at two points.

(c) Find the x coordinate of each of these two points.

(3)

(d) Use algebraic integration to find the exact area of the finite region bounded by the curve C and the curve S.

(5)

Question 11 continued

Question 11 continued

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(Total for Question 11 is 14 marks)

TOTAL FOR PAPER IS 100 MARKS