Mark Scheme (Results)

November 2023

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 1

November 2023
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme - not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed out work should be marked **unless** the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.
- **With working**
  If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right),\text{ where }\left|pq\right|=\left|c\right| \quad \text{leading to } x=....$$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right)\text{ where}\left|pq\right|=\left|c\right|\text{ and }\left|mn\right|=\left|a\right| \quad \text{leading to } x=....$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$$x^2+bx+c=0:\ \left(x\pm\tfrac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0 \qquad \text{leading to } x=....$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either** quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:
The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."**Exact answers:**
When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)
Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

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4PM1 Paper 1
Mark Scheme

Question	Scheme	Marks
1	$34+11\sqrt{5}=\frac{1}{2}\times\left(2+4\sqrt{5}\right)\left(a+b\sqrt{5}\right)$	M1
	$\Rightarrow a+b\sqrt{5}=\frac{2\times\left(34+11\sqrt{5}\right)}{2+4\sqrt{5}}$ or $a+b\sqrt{5}=\frac{\left(34+11\sqrt{5}\right)}{1+2\sqrt{5}}$ oe	M1
	$=\frac{\left(34+11\sqrt{5}\right)}{\left(1+2\sqrt{5}\right)}\times\frac{\left(1-2\sqrt{5}\right)}{\left(1-2\sqrt{5}\right)}=\frac{34-68\sqrt{5}+11\sqrt{5}-110}{-19}$ oe or $=\frac{\left(68+22\sqrt{5}\right)}{\left(2+4\sqrt{5}\right)}\times\frac{\left(2-4\sqrt{5}\right)}{\left(2-4\sqrt{5}\right)}=\frac{136-272\sqrt{5}+44\sqrt{5}-440}{-76}$ oe	M1
	$=4+3\sqrt{5}$ or $a=4,\quad b=3$	A1 [4]
	ALT	
	$34+11\sqrt{5}=\frac{1}{2}\times\left(2+4\sqrt{5}\right)\left(a+b\sqrt{5}\right)$ oe	M1
	$\left(34+11\sqrt{5}\right)=a+b\sqrt{5}+2a\sqrt{5}+10b$ oe eg $68+22\sqrt{5}=2a+2b\sqrt{5}+4a\sqrt{5}+20b$ $\left(34+11\sqrt{5}\ \right)=\left(a+10b\right)+\left(2a+b\right)\sqrt{5}$ oe eg $\left(68+22\sqrt{5}\right)=2a+20b+\left(4a+2b\right)\sqrt{5}$	M1
	$\Rightarrow 34=a+10b$ and $11=2a+b$ oe eg $\Rightarrow 68=2a+20b$ and $11=4a+2b$ Solves simultaneous equations by any method	M1
	$=4+3\sqrt{5}$ or $a=4,\quad b=3$	A1 [4]
		Total 4 marks

Mark	Notes
M1	For the correct statement for the area, this may be explicit or implied by working, but **must** be a fully correct statement. Allow $a+b\sqrt{5}$ to be written as *BC* or denoted by a letter.
M1	For **correctly rearranging** to the form $a+b\sqrt{5}=\frac{2\times\left(34+11\sqrt{5}\right)}{2+4\sqrt{5}}$ oe or $a+b\sqrt{5}=\frac{\left(34+11\sqrt{5}\right)}{1+2\sqrt{5}}$ oe This mark may also be awarded for rearranging an initial formula, missing the $\frac{1}{2}$ ie for correctly rearranging $34+11\sqrt{5}=\left(2+4\sqrt{5}\right)\left(a+b\sqrt{5}\right)$ to $a+b\sqrt{5}=\frac{\left(34+11\sqrt{5}\right)}{2+4\sqrt{5}}$ oe Allow $a+b\sqrt{5}$ to be written as *BC* or denoted by a letter.
M1	For multiplying eg $\frac{\left(34+11\sqrt{5}\right)}{\left(1+2\sqrt{5}\right)}\times\frac{\left(1-2\sqrt{5}\right)}{\left(1-2\sqrt{5}\right)}$ or $\frac{\left(68+22\sqrt{5}\right)}{\left(2+4\sqrt{5}\right)}\times\frac{\left(2-4\sqrt{5}\right)}{\left(2-4\sqrt{5}\right)}$ - allow the 2 to remain factorised. or, if missing the $\frac{1}{2}$ for the first method mark $\frac{\left(34+11\sqrt{5}\right)}{\left(2+4\sqrt{5}\right)}\times\frac{\left(2-4\sqrt{5}\right)}{\left(2-4\sqrt{5}\right)}=\frac{68-136\sqrt{5}+22\sqrt{5}-220}{-76}$ oe Allow one error in multiplying out. Look for equivalences. The question precludes the use of a calculator, so method must be shown.
A1	$4+3\sqrt{5}$ or $a=4,\quad b=3$
ALT	
M1	For the correct statement for the area, this may be explicit or implied by working, but **must** be a fully correct statement. Allow $a+b\sqrt{5}$ to be written as *BC* or denoted by a letter.
M1	For multiplying out correctly as shown in the scheme. But if the initial formula is missing the $\frac{1}{2}$, the mark may also be awarded for $\left(34+11\sqrt{5}\right)=2a+2b\sqrt{5}+4a\sqrt{5}+20b$
M1	For correctly equating coefficients and any valid complete method to solve a pair of simultaneous equations, each equation in *a* and *b*, allow one error. This work may also be done with the $\frac{1}{2}$ missing from the formula and that doesn't count as 'one error'. The question precludes the use of a calculator, so method must be shown.
A1	$4+3\sqrt{5}$ or $a=4,\quad b=3$

Question	Scheme	Marks
For part (a) of this question, mark using the scheme which gives the most marks.		
2a	$p=2,\quad q=\frac{1}{8},\quad r=-\frac{97}{32}$	B1B1B1
ALT	$\left(\mathrm{g}(x)=2x^2+\frac{1}{2}x-3=\right)2\left(x^2+\frac{1}{4}x-\frac{3}{2}\right)$ or $2\left(x^2+\frac{1}{4}x\right)-3$ $("2")\left(\left[x+\frac{1}{8}\right]^2-\frac{1}{64}-\frac{3}{2}\right)$ or $("2")\left(\left[x+\frac{1}{8}\right]^2-\frac{1}{64}\right)-3$ $2\left[x+\frac{1}{8}\right]^2-\frac{97}{32}$ or $p=2,\quad q=\frac{1}{8},$ oe $r=-\frac{97}{32}$ oe	M1 M1 A1 [3]
(b)	(i) $\left(\mathrm{g}(x)_{\min}=\right)"-\frac{97}{32}"$ (ii) $\left(x=\right)-"\frac{1}{8}"$	B1ft B1ft [2]
(c)	(i) $\mathrm{h}(x)_{\min}="-\frac{97}{32}"$ (ii) $x=\sqrt[3]{"-\frac{1}{8}"}=-\frac{1}{2}$	B1ft B1ft [2]
		Total 7 marks

Part	Mark	Notes
- **Mark using the B scheme first.** - If not full marks – use the MMA scheme also, if appropriate. - Same score – apply the B marks. - Higher score – apply the marks from the MMA scheme.		
(a) **Different to Epen marks**	B1	For one of *p, q,* or *r* correct.
	B1	For two of *p, q,* or *r* correct.
	B1	For all of *p, q,* or *r* correct – note *q* and *r* can be written as equivalent fractions.
ALT	M1	Correctly takes out 2 as a common factor to give either of the 2 expressions shown.
	M1	Completes the square correctly, **regardless of any factor on the outside – follow through their factorisation.** $\left(x^2+ax+b\right)$ or $\left(x^2+ax\right)+c\Rightarrow$ ie $\left[\left(x+\frac{a}{2}\right)^2-\left(\frac{a}{2}\right)^2+b\right]$ or $\left[\left(x+\frac{a}{2}\right)^2-\left(\frac{a}{2}\right)^2\right]+c\qquad a,b,c\neq 0$
	A1	Fully correct answer – note *q* and *r* can be written as equivalent fractions.
Note part (a) does not ask students to show working nor preclude the use of a calculator, so if values are simply listed, these can be given marks from the B scheme		
(b) (i)	B1ft	For the correct value **or** ft their *r*
(ii)	B1ft	For the correct value **or** ft their – *q*
If no labelling of (i) and (ii) is present for parts (b) and (c) – marks may be awarded for the values presented in the correct order. **B0 B0 if not labelled and work doesn't meet this condition.**		
(c) (i)	B1ft	For the correct value **or** ft their $-\frac{97}{32}$
(ii)	B1ft	For the correct value **or** ft the cube root of their $-\frac{1}{8}$ (evaluated).

Question	**Scheme**	**Marks**
3(a)	$\left(y=\int\left(mx^2-10x-37\right)\mathrm{d}x\ =\right)\frac{mx^3}{3}-\frac{10x^2}{2}-37x(+c)$	M1
	$\left[\text{At }(1, 20)\right]$	
	$20=\frac{m\times1^3}{3}-\frac{10\times1^2}{2}-37\times1+c\left(\Rightarrow 62=\frac{m}{3}+c\right)$ oe	dM1
	$\left[\text{Factor of }(x-5)\right]$	
	$0=\frac{m\times5^3}{3}-\frac{10\times5^2}{2}-37\times5+c\left(\Rightarrow 310=\frac{125m}{3}+c\right)\Rightarrow m=6 \qquad c=60$	M1 M1
	$\Rightarrow\left(\mathrm{g}(x)=\right)2x^3-5x^2-37x+60$ *	A1cso [5]
ALT	3rd and 4th method marks $(x-5)\left(\frac{m}{3}x^2+bx\mp\frac{c}{5}\right)=\frac{m}{3}x^3-5x^2-37x\pm c\Rightarrow m=6 \qquad c=60 \qquad (b=5)$	M1 M1
Special case up to SC3 – mark as M1dM1M1 M0A0 in Epen	Differentiates the given expression for g (x)correctly to give $2(3x^2)-5(2x)-37\left(=6x^2-10x-37\right)$ **and** concludes m = 6	SC1
	Correctly substitutes x = 5 into the given function and shows g (x) = 0 $\left(\mathrm{g}(5)=\right)2\times5^3-5\times5^2-37\times5+60=0$	SC1
	Correctly substitutes x = 1 into the given function and shows and g (x) = 20 $\left(\mathrm{g}(1)=\right)2\times1^3-5\times1^2-37\times1+60=20$	SC1
(b)	$x-5\overline{\smash{)}2x^3-5x^2-37x+60}$ with quotient $2x^2+5x-12$	M1
	$2x^2+5x-12=(2x-3)(x+4)$	M1
	$\left(\mathrm{g}(x)=(x-5)(2x-3)(x+4)=[0]\right)$	
	$\Rightarrow x=5, \quad \frac{3}{2}, \quad -4$	A1 [3]
		Total 8 marks

Part	Mark	Notes
(a)	M1	For a minimally acceptable attempt to integrate the given expression (see general guidance). No power of *x* to decrease. +*c* is not required for this mark
	dM1	Correctly substitutes *x* = 1 and *y* = 20 to form an equation – simplification is not required Dependent on 1st method mark. A fully correct equation with no incorrect working can imply this mark. We will be lenient on not seeing the substitution of *x* = 1 as this is trivial, so long as the resulting equation is correct from their g (1) = 20. +*c* is not required for this mark
	M1	Uses the information correctly that (*x* – 5) is a factor of g (*x*) by correct substitution of *x* = 5 and g(*x*) = 0 into their g (*x*). Their g (*x*) **must** be a 4 term cubic expression. A fully correct equation with no incorrect working can imply this mark – but if the equation is incorrect we must see substitution of *x* = 5 such that g(5) = 0.
	M1	Uses a valid and complete method to solve their resulting simultaneous equations which must be in two variables. Allow up to 2 errors.
	A1 cso	For the correct function as shown or the correct expression. This is a show question. There must be no errors for the award of this mark.
ALT	3rd M1	Uses the information correctly that (*x* – 5) is a factor of g (*x*) by writing a correct statement like the one shown for their g (*x*). Their g (*x*) **must** be a 4 term cubic expression.
	4th M1	Uses a valid and complete method to solve their resulting simultaneous equations in three variables. Allow up to 2 errors. It is not necessary to see the value for "*b*".
(b)	M1	For an attempt at polynomial division or equate coefficients to reach a 3 term quotient. The minimally acceptable quotient is $2x^2 \pm 5x \pm k \quad k \neq 0$ where *k* is a constant. In the absence of any other working, $2x^2 \pm 5x \pm k \quad k \neq 0$ may be accepted as evidence of working.
	M1	For attempting to factorise their quotient which must be of the form $2x^2 \pm 5x \pm k \quad k \neq 0$ - we must at least see a quadratic factor to factorise. See General Guidance for the definition of an attempt. It would also be possible to see a minimal attempt to use the quadratic formula or completing the square to solve (see general guidance) if their quadratic, of the form $2x^2 + 5x \pm k \quad k \neq 0$ does not factorise. If using the formula the substitution of their *a*, *b* and *c* must be completely correct.
	A1	For the correct values of *x*
Candidates have clearly been directed to use algebra in the question and therefore 0 marks can be scored without seeing M1 M1		

Question	Scheme	Marks
4(a)	$\left(3\times14-2\times12-p=0\Rightarrow\right)p=18$ $3y-2q-\text{"}18\text{"}=0\Rightarrow3\times2-2q-\text{"}18\text{"}=0\Rightarrow q=$ $q=-6$	B1 M1 A1 [3]
(b)	$\left(X\left(x,y\right)=\right)\left(\frac{1\times\text{"}-6\text{"}+2\times12}{3},\frac{1\times2+2\times14}{3}\right)$ oe $=\left(6,10\right)$ $\left(\text{Gradient of }AB=\right)=\frac{2}{3}$ oe $\left(\text{Gradient of }L=\right)-\frac{1}{\text{"}\frac{2}{3}\text{"}}\left(=-\frac{3}{2}\right)$ $y-\text{"}10\text{"}=-\frac{1}{\text{"}\frac{2}{3}\text{"}}\left(x-\text{"}6\text{"}\right)\left(\Rightarrow y=-\frac{3}{2}x+19\right)$ $\Rightarrow3x+2y-38=0$ oe Where *a*, *b* and *c* are integers.	B1B1 or M1A1 B1 B1ft M1 A1 [6]
		Total 9 marks

	Mark	Notes
(a)	B1	For p = 18
	M1	Uses their value of p and substitutes y = 2 and finds a value for q.
	A1	For the correct value of q.
First 2 marks of (b) – use the scheme which gives the most marks to candidates.		
(b)	B1 (M1 ePen)	For one correct value (6, 10).
	B1 (A1 ePen)	For both correct values (6, 10).
	or M1	Correct method for finding both coordinates, using their q. Look for any equivalent methods eg similar triangles.
	or A1	Both correct values.
	B1	For a fully correct unsimplified or simplified gradient of *AB* Note this mark is awarded for **finding or stating** the gradient and if simplified, it must be clear that $\frac{2}{3}$ is not a fraction from the given ratio. Accept $\frac{14-2}{12--6}$ This mark may be implied for a correct gradient of $-\frac{3}{2}$ used in further work.
	B1ft	For finding the negative reciprocal of their gradient – does not need to be simplified.
	M1	For a full and correct method to find the equation for the line L using **their** coordinates of X and **their** perpendicular gradient. This need not be simplified. If $y = mx + c$ is used, the correct value of c must be found for their coordinates and gradient. It must be clear the candidate is using their coordinates for X and not, for example simply the numbers from the coordinates of A and B
	A1	For the correct equation of the line in the required form – any equivalent where a, b and c are integers.

Question	Scheme	Marks
5	$\sqrt{2x+6}=\frac{x}{3}+1\Rightarrow 2x+6=\frac{x^2}{9}+\frac{2x}{3}+1\Rightarrow\frac{x^2}{9}-\frac{4x}{3}-5=0$	M1
	$\Rightarrow x^2-12x-45=0\Rightarrow(x+3)(x-15)=0\Rightarrow x=-3,15$	M1A1
	$\left[\text{When } x=15,\quad y=6\right]$	
	$(V=)\pi\int_{'-3'}^{'15'}(2x+6)\ \mathrm{d}x-\frac{1}{3}\times\pi\times'6'^2\times('15'-'-3')$	M1M1
	$(V=)(\pi)\left[\frac{2x^2}{2}+6x\right]_{'-3'}^{'15'}-216\pi$	M1
	$(V=)\pi\left[\left(\frac{2\times'15'^2}{2}+6\times'15'\right)-\left(\frac{2\times('-3')^2}{2}+6\times('-3)'\right)\right]-216\pi(=324\pi-216\pi)=108\pi$	M1A1 [8]
ALT last 5 marks	OR	
	$(V=)\pi\int_{'-3'}^{'15'}(2x+6)\ \mathrm{d}x-\pi\int_{'-3'}^{'15'}\left(\frac{x^2}{9}+\frac{2x}{3}+1\right)\mathrm{d}x$ or $\pi\int_{'-3'}^{'15'}(2x+6)\ \mathrm{d}x-\pi\int_{'-3'}^{'15'}\left(\frac{x}{3}+1\right)^2\ \mathrm{d}x$	M1M1
	$(V=)(\pi)\left[\frac{2x^2}{2}+6x\right]_{'-3'}^{'15'}-\pi\left[\frac{x^3}{9\times3}+\frac{2x^2}{3\times2}+x\right]_{'-3'}^{'15'}$	M1
	$V=\pi\left[\left(\frac{2\times'15'^2}{2}+6\times'15'\right)-\left(\frac{2\times('-3')^2}{2}+6\times('-3')\right)\right]$	
	$-\pi\left[\left(\frac{'15'^3}{9\times3}+\frac{2\times'15'^2}{3\times2}+15\right)-\left(\frac{('-3')^3}{9\times3}+\frac{2\times('-3')^2}{3\times2}'-3'\right)\right]=108\pi$	M1A1
		Total 8 marks

Mark	Notes
M1	Sets the equation of the curve = equation of line and attempts to form a 3TQ, which must include an attempt to expand $\left(\frac{1}{3}x+1\right)\left(\frac{1}{3}x+1\right)$. Allow only 1 error in processing overall. Accept as a minimum for this mark: $\frac{x^2}{9} \pm Jx - K = 0$ or $x^2 \pm Lx - M = 0 \quad J, K, L, M > 0$
M1	Solves their 3TQ using a full and minimally acceptable method (see general guidance) to find two values. They must show their method if using an incorrect 3TQ. Note this is not a dependent mark, so the candidate can solve a 3TQ not of the form required for the first M mark.
A1	$x = -3, 15$ M1 M1 A1 may be awarded for both correct values given without working.
M1	For the correct expression for the volume of rotation under the curve including π and ft their $x = -3, 15$.
M1	For the correct expression for the volume of the cone. Ft their $x = -3, 15$ and their value of y only (must be clear they are using a value of y. '6' cannot be 15 or – 3). $\frac{1}{3}\times\pi\times'6'^2\times('15'-'-3')$
M1	For an attempt to integrate their expression for the curve. Minimally acceptable attempt (see general guidance) – no power of x to decrease. π and limits don't need to be present.
M1	For substitution of their limits into any changed expression representing their volume of the curve and evaluation of the final volume using their volume of the cone. Not a dependent mark but substitution **must** be into their changed expression. This mark may be awarded for the work being done in parts and subtracted in any order at any point, but **there must be a subtraction.** This M mark may be implied by a correct final answer.
A1	For $V = 108\pi$ **This mark may not be awarded if the subtraction is the incorrect way around and the sign of the answer is simply changed at the end.**
ALT last 5 marks	
M1	For the correct expression for the volume of rotation under the curve including π and ft their $x = -3, 15$
M1	For the correct expression for the volume of rotation under the straight line including π and ft their $x = -3, 15$
Where candidates have combined the 2 expressions, M1 M1 can be awarded for $\pm\pi\int_{-3}^{15}\left(\frac{x^2}{9}-\frac{4x}{3}-5\right)\mathrm{d}x$ **or** for a clear attempt to combine the correct two expressions, even if the following simplification is incorrect. [Note for final A mark correct integral is $\pi\int_{-3}^{15}\left(-\frac{x^2}{9}+\frac{4x}{3}+5\right)\mathrm{d}x$	
M1	For an attempt to integrate the expression for **either** the curve **or** the line **or** what they intend to be **<u>their</u>** combined expression. Minimally acceptable attempt (see general guidance) – no power of x to decrease. π and limits do not need to be present for this mark.
M1	For substitution of their limits into any changed expressions representing their volume of the line and the curve and evaluation of the final volume or their combined expression. There must be at least one correct substitution of both limits into each expression. Not a dependent mark but substitution **must** be into changed expressions. This mark may be awarded for the work being done in parts and subtracted in any order at any point, but **there must be a subtraction.** This M mark may be implied by a correct final answer.
A1	For $V = 108\pi$ **This mark may not be awarded if the subtraction is the incorrect way around and the sign of the answer is simply changed at the end.**

Question	Scheme	Marks
PLEASE REMEMBER TO CHECK DIAGRAM FOR RELEVANT LENGTHS AND WORKING **Throughout the question where lengths are in brackets eg (*AC* =), this means you don't have to see *AC* =.** **But the mark cannot be awarded for a clearly incorrect method eg *VO* = $\sqrt{x^2+x^2}$ for the first method mark in (a) scores M0**		
6(a)	$\left(AC=\right)\sqrt{x^2+x^2}\left(=\sqrt{2}x\right)$ **or** $\left(AC=\right)\frac{x}{\sin 45}$ **or** $\left(AC=\right)\frac{x}{\cos 45}$ $\left(AO=\right)\sqrt{\left(\frac{x}{2}\right)^2+\left(\frac{x}{2}\right)^2}\ \left(=\sqrt{\frac{x^2}{2}}=\frac{x}{\sqrt{2}}\right)$ **or** $\left(AO=\right)\frac{\frac{x}{2}}{\sin 45}$ **or** $\left(AO=\right)\frac{\frac{x}{2}}{\cos 45}$	M1
	$\left(VO=\right)\sqrt{x^2-\left(\frac{\text{"}\sqrt{2}x\text{"}}{2}\right)^2}$ **or** $\sqrt{x^2-\left(\frac{1}{2}\sqrt{x^2+x^2}\right)^2}$ **or** $\sqrt{x^2-\frac{x^2}{2}}\ \left(=\sqrt{\frac{1}{2}x^2}\right)$ **or** $\left(VO=\right)\frac{\sqrt{2}}{2}\tan 45$	M1
	$=\frac{\sqrt{2}}{2}x$ (cm)*	A1*cso [3]
ALT	Any indication the candidate realises triangle *AVC* is right angled and isosceles.	M1
	$\sin 45=\frac{VO}{x}$ **or** $x\sin 45=VO$	M1
	$VO=\frac{\sqrt{2}}{2}x$ *	A1*cso [3]
(b)	$\left(\angle AVC=\right)\cos^{-1}\left(\frac{x^2+x^2-\left(\sqrt{2}x\right)^2}{2\times x\times x}\right)$ **or** $\left(\angle AVC=\right)2\times\sin^{-1}\left(\frac{\frac{\sqrt{2}}{2}x}{x}\right)$ **or** $\left(\angle AVC=\right)2\times\cos^{-1}\left(\frac{\frac{\sqrt{2}}{2}x}{x}\right)$ **or** $\left(\angle AVC=\right)2\times\tan^{-1}\left(\frac{\frac{\sqrt{2}}{2}x}{\frac{\sqrt{2}}{2}x}\right)$	M1
	$=90^{\circ}$	A1 [2]

(c)	Let M be the midpoint of AB and let N be the midpoint of DC	
	$\left(VM=VN=\right)\sqrt{x^2-\left(\frac{x}{2}\right)^2}=\left(\frac{\sqrt{3}}{2}x\right)$	M1
	$\cos MVN=\dfrac{\left("\frac{\sqrt{3}}{2}x"\right)^2+\left("\frac{\sqrt{3}}{2}x"\right)^2-x^2}{2\times\left("\frac{\sqrt{3}}{2}x"\right)\times\left("\frac{\sqrt{3}}{2}x"\right)}$ **or** $\left(\angle MVN=\right)\cos^{-1}\left(\dfrac{\left("\frac{\sqrt{3}}{2}x"\right)^2+\left("\frac{\sqrt{3}}{2}x"\right)^2-x^2}{2\times\left("\frac{\sqrt{3}}{2}x"\right)\times\left("\frac{\sqrt{3}}{2}x"\right)}\right)$ **or** $\left(\angle MVN=\right)2\times\cos^{-1}\left(\dfrac{\frac{\sqrt{2}}{2}x}{"\frac{\sqrt{3}}{2}x"}\right)$ **or** $\left(\angle MVN=\right)2\times\sin^{-1}\left(\dfrac{\frac{1}{2}x}{"\frac{\sqrt{3}}{2}x"}\right)$ **or** $\left(\angle MVN=\right)2\times\left(\angle MVO=\right)$ **and** $\cos MVO=\dfrac{\frac{\sqrt{2}}{2}x}{"\frac{\sqrt{3}}{2}x"}$	M1
	$70.528...^\circ\approx 70.5^\circ$	A1 [3]
	ALT $OM=\frac{x}{2}$	M1
	$\left(\angle MVN=\right)2\tan^{-1}\left[\dfrac{\frac{x}{2}}{\frac{\sqrt{2}x}{2}}\right]=\left(2\tan^{-1}\left[\frac{1}{\sqrt{2}}\right]=\right)70.528...^\circ\approx 70.5^\circ$	M1A1 [3]
(d)	Volume $=\frac{1}{3}\times\left(\frac{\sqrt{2}}{2}x\right)\times x^2\left(=\frac{\sqrt{2}}{6}x^3\right)$ oe	M1
	$200=\frac{1}{3}\times\left(\frac{\sqrt{2}}{2}x\right)\times x^2\Rightarrow x=$	dM1
	$x=9.4672...\approx 9.47(\text{cm})$	A1 [3]
		Total 11 marks

Part	Mark	Notes
For part (a), we must see the algebraic proof, including *x* throughout.		
(a)	M1	For correct use of Pythagoras theorem or any correct appropriate trigonometry and a complete method to find the length *AC* or *AO* ie candidates must rearrange correctly – minimum steps as shown in mark scheme and no errors must be present.
	M1	For a full, correct and complete method to find *VO*, using their *AC/AO*.
	A1 cso	For $\frac{\sqrt{2}}{2}x$ (cm) – correct solution only, no errors or omissions, minimum steps as shown.
ALT	M1	Any indication the candidate realises triangle *AVC* is right angled and isosceles. This mark may be implied by correct working for the next method mark or present on the diagram.
	M1	For the correct trig minimum steps as shown and no errors.
	A1 cso	For $\frac{\sqrt{2}}{2}x$ (cm) – correct solution only, no errors or omissions.
(b)	M1	For any correct appropriate trigonometry to find angle *AVC.* $(\angle AVC =)$ If finding $\angle AVO$, must be doubled for this mark. *AC* must be $\sqrt{2}x$ oe, *AO* must be $\frac{\sqrt{2}}{2}x$ oe.
	A1	For 90°
	Allow M1A1 for candidates writing down the correct answer by inference.	
Consistent omission of *x* is permissible for all marks in part (b).		
(c)	M1	For correct Pythagoras theorem to find the lengths of *VM* or *VN*
	M1	For any correct trigonometry to find angle *MVN*, such as the statements shown in the mark scheme. The candidate can state, for example, cos(*MVN*) = fully correct expression (using their *VN* and *VM*) or, for example, $\cos^{-1}$(fully correct expression), where they do not have to write angle *MVN* = i.e – this can involve any fully correct method to reach an equation which would allow them to use inverse trig to find *MVN* or if they use inverse trig, a fully correct expression must be seen with the correct inverse trig function. Send any items where allocation of marks is unclear (potentially using other methods to the main and ALT) to Review. or fully correct **method** to find angle *MVO* which must then be doubled for this mark.
	A1	For any answer stated which rounds to 70.5° (ie more decimal places is condoned).
Consistent omission of *x* is permissible for all marks in part (c).		
ALT	M1	States or uses *OM* = $\frac{x}{2}$
	M1	For fully correct **method** to find angle *MVO* which must then be doubled for this mark.
	A1	For any answer stated which rounds to 70.5° (ie more decimal places is condoned).
(d)	M1	For a correct unsimplified expression for the volume of the pyramid as shown.
	dM1	For placing their expression = 200, which must involve a term in x^3, and a fully correct method/rearrangement to solve. Note this mark is dependent on the first method mark, so they must have got the correct unsimplified expression even if then simplified incorrectly.
	A1	For any final answer stated which rounds to 9.47 (ie more decimal places is condoned).

Question	Scheme	Marks
7(a)	$\left(\frac{ar^2}{a}=r^2=\right)\frac{\frac{2704}{625}}{16}$ oe	M1
	$(r=)\pm\frac{13}{25}$ oe	A1 [2]
(b)	$(S_\infty=)\frac{16}{1-"\frac{13}{25}"}=\frac{100}{3}$ oe	M1A1 [2]
(c)	$\frac{16\left(1-\left("\frac{13}{25}"\right)^n\right)}{1-"\frac{13}{25}"}>33\left(\Rightarrow 1-\left(\frac{13}{25}\right)^n>0.99\right)$ oe	M1
	$\Rightarrow\left("\frac{13}{25}"\right)^n<0.01$ oe	dM1
	$n\lg\left("\frac{13}{25}"\right)<\lg(0.01)$ oe	M1
	$\Rightarrow n>\frac{\lg(0.01)}{\lg\left("\frac{13}{25}"\right)}\Rightarrow n>7.04...\text{oe}\Rightarrow(n=)8$	dddM1A1 [5]
		Total 9 marks

Part	Mark	Notes
(a)	M1	For any correct unsimplified expression for *r* or r^2.
	A1	For $r = \pm\frac{13}{25}$ oe
(b)	M1	Uses the correct formula for the sum to infinity of a geometric series using their positive $r < 1$ and the given first term.
	A1	For $\frac{100}{3}$ oe. Allow 33.33……, 33.3^r, $33.\dot{3}$ etc. The question demand is for an exact answer so there must be some indication of recurrence. This could be min 1 dp with dots after ie 33.3….., but cannot be a terminating decimal.
(c)	M1	Uses the correct formula for the sum of a geometric series to set up an **inequality or equation** in terms of *n*, *f*t their **positive** $r < 1$, must be using $a = 16$ Allow < or > or =
	dM1	For simplifying (allow errors in simplification) their **inequality or equation** in *n* to the form $\left("\frac{13}{25}"\right)^n < d \quad d \neq 0$. Allow $"\frac{13}{25}"$ to be their positive $r < 1$. Allow < or > or =. Dependent on the 1st method mark.
	M1	Takes logarithms (any base) of their **exponential equation or inequality <u>correctly</u>** on both sides and correctly uses the power law to reach $n\lg(a) < \lg(b)$. or 'de-logs' their **exponential equation or inequality <u>correctly</u>** to get $\log_c d < e$ This isn't a dependent mark, but the candidate must be able to correctly take logs of both sides and use the power law or correctly de-log their equation/inequality. *r* can be any value carried through from part (a) for this mark. Allow < or > or =, the inequality doesn't need to have been reversed at this point.
	dddM1	For finding a value for *n* setting up the correct inequality (with their positive $r < 1$) from the beginning and where the inequality sign has been reversed at the appropriate point during their work. Dependent on all previous method marks. If candidates give a final answer of (*n* =) 8 – this mark can be implied even if the inequality sign is not correctly reversed.
	A1	For (*n* =) 8

Question	Scheme	Marks
8(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{5x^2\times2\times3\times\mathrm{e}^{3x+1}-10x\times2\mathrm{e}^{3x+1}}{\left(5x^2\right)^2}$ oe $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\frac{x\mathrm{e}^{3x+1}\left(30x-20\right)}{25x^4}$ or $\frac{10x\mathrm{e}^{3x+1}\left(3x-2\right)}{25x^4}$ $=\left[\frac{10\mathrm{e}^{3x+1}\left(3x-2\right)}{25x^3}\right]=\frac{2\mathrm{e}^{3x+1}\left(3x-2\right)}{5x^3}$	M1A1A1 dM1 A1 [5]
ALT – use of product rule	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)2\mathrm{e}^{3x+1}(-1)\times2\times5x\times(5x^2)^{-2}+2\times3\mathrm{e}^{3x+1}(5x^2)^{-1}$ oe $=x\times(5x^2)^{-2}\mathrm{e}^{3x+1}(-20+30x)$ or $10x\times(5x^2)^{-2}\mathrm{e}^{3x+1}(-2+3x)$ $=\left[\frac{10\mathrm{e}^{3x+1}\left(3x-2\right)}{25x^3}\right]=\frac{2\mathrm{e}^{3x+1}\left(3x-2\right)}{5x^3}$	M1A1A1 dM1 A1 [5]
(b)	$\left(\frac{\delta y}{y}\times100=\right)$"$\frac{2x\mathrm{e}^{3x+1}\left(3x-2\right)}{5x^4}$"$\times\delta x\times\frac{100}{y}$ or "$\frac{2\mathrm{e}^{3x+1}\left(3x-2\right)}{5x^3}$"$\times\delta x\times\frac{100}{y}$ oe or $\left(\delta y=\right)$"$\frac{2xe^{3x+1}\left(3x-2\right)}{5x^4}$"$\times\delta\mathrm{x}$ or "$\frac{2e^{3x+1}\left(3x-2\right)}{5x^3}$"$\times\delta x$ or $y\times\frac{\left(3x-2\right)}{5x}\times\delta x$ δx may also be written as $0.02x$ $\left(\frac{\delta y}{y}\times100=\right)\frac{2\mathrm{e}^{3x+1}\left(3x-2\right)}{5x^3}\times0.02x\times\frac{100}{\frac{2\mathrm{e}^{3x+1}}{5x^2}}$ oe or $y\times\frac{\left(3x-2\right)}{x}\times0.02x\times\frac{100}{y}$ $\left(\%\text{ change in }y=\right)6x-4$	M1 dM1 A1 [3]
		Total 8 marks

Part	Mark	Notes
(a)	M1	For attempting to use the quotient rule. The definition of an attempt is as follows: • The denominator must be correct. • The terms in the numerator subtracted either way around The expression **must** be of the form: $\frac{K\times x^2\times e^{3x+1}-L\times x\times e^{3x+1}}{\left(5x^2\right)^2}$ $K\neq 10$ or 15 and $L\neq 10$ and $K,L>0$ Note K and L may not be simplified and will often need to be checked.
	A1	Following M1 (general principle of marking), at least one fully correct simplified or unsimplified term on the numerator.
	A1	Fully correct **unsimplified** derivative
	dM1	For correctly taking out common factors of x and e^{3x+1} and attempting to simplify. A numerical factor does not need to be taken out at this point. Dependent on the 1st method mark.
	A1	For the correct derivative in the required form.
ALT use of product rule	M1	For attempting to use the product rule to give an expression of the form $Me^{3x+1}x\times(5x^2)^{-2}+Ne^{3x+1}(5x^2)^{-1}$ $M<0$ and $\neq 4,10$ $N>0$ and $\neq 2,3$
	A1	Following M1 (general principle of marking), at least one term is fully correct.
	A1	Fully correct **unsimplified** derivative
	M1	For correctly taking out common factors of x and $(5x^2)^{-2}$ and e^{3x+1} and attempting to simplify. A numerical factor does not need to be taken out at this point. Dependent on the 1st method mark.
	A1	For the correct derivative in the required form.
For the final M mark in particular – any work felt to be of credit not fitting the mark scheme – send to Review.		
(b)	M1	For any of the statements shown, using their answer from part (a), which must be of the form: $\frac{Ae^{3x+1}\left(Bx-C\right)}{Dx^3}$ Note, we'll allow candidates to use an expression here that is not quite of the form given in part (a). This mark may be implied by further correct working.
	dM1	For substituting in the expression for y **and** a correct attempt to simplify their expression to the required form. You must check their working here, do not allow an answer that has come from incorrect working (general principle anyway, but extra care needed for this question). Condone 100 written as 100% if the candidate is correctly multiplying by 100. Dependent on previous method mark.
	A1	For the correct expression as shown.

Question	Scheme	Marks
9(a)	$\left(1-8x^2\right)^{-\frac{1}{2}}=1+\left(-\frac{1}{2}\right)\left(-8x^2\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-8x^2\right)^2}{2!}$ $+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)\left(-8x^2\right)^3}{3!}$	M1
	$\left(1-8x^2\right)^{-\frac{1}{2}}=1+4x^2+24x^4+160x^6+\ldots$	A1A1 [3]
(b)	$\left(\left(a+bx\right)"\left(1+4x^2+24x^4+160x^6\right)"=\right)\left(a+bx+4ax^2\right)+4bx^3+24ax^4$	M1A1ft
	$4b=20\Rightarrow b=5$	A1
	$24a=48\Rightarrow a=2$	A1 [4]
(c)	$\left(\int_0^{0.2}\mathrm{g}\left(x\right)\ \mathrm{d}x=\int_0^{0.2}\left(\frac{2+5x}{\sqrt{1-8x^2}}\right)\ \mathrm{d}x=\right)$	
	$\int_0^{0.2}\left("2"+"5"x+"8"x^2+"20"x^3+"48"x^4\right)\ \mathrm{d}x$	M1
	$\left(\int_0^{0.2}\mathrm{g}\left(x\right)\ \mathrm{d}x=\right)\left["2x+\frac{5x^2}{2}+\frac{8x^3}{3}+\frac{20x^4}{4}+\frac{48x^5}{5}"\right]_0^{0.2}$	M1
	$\left(\int_0^{0.2}\mathrm{g}\left(x\right)\ \mathrm{d}x=\right)"2\times0.2+\frac{5\times0.2^2}{2}+\frac{8\times0.2^3}{3}+\frac{20\times0.2^4}{4}+\frac{48\times0.2^5}{5}"$	M1
	$=0.532405333\approx0.5324$	A1
	Note: The calculator value is 0.5347698	[4]
		Total 11 marks

Part	Mark	Notes
(a)	M1	For an attempt at the binomial expansion. The expansion must: • Begin with 1 • The denominators must be correct • The powers of $-8x^2$ must be correct eg $\left(-8x^2\right)^2$ Do not allow missing brackets unless recovered later – this is a general point of marking. Ignore any terms with powers higher than 3. Allow a misread of $\frac{1}{2}$ for $-\frac{1}{2}$ but **not** $8x^2$ for $-8x^2$ - see gen guidance for misreads.
	A1	Following M1 (this is a general point of marking, A marks can only follow M marks). All conditions above met and at least one **algebraic** term correct and simplified.
	A1	Fully correct and simplified
(b)	M1	For attempting to expand $(a+bx)\left(1+cx^2+dx^4+......\right)$ to reach at least the correct terms in x^3 and x^4 for their expansion from part (a) ie $4bx^3$ and $24ax^4$ or cbx^3 and dax^4 Ignore terms with powers higher than 4. It is only necessary to see the correct terms (for their expansion) in x^3 and x^4. They must be using an expansion from (a) of the form $1+cx^2+dx^4+.....$ This mark may be implied from any following working.
	A1ft	For a fully correct expansion in terms of x, a and b,up to the term in x^4 or for $4b = 20$ AND $24a = 48$. **or** if using an incorrect expansion from part (a) of the form $1+cx^2+dx^4+.....$ for a fully correct expansion **or** $cb = 20$ AND $da = 48$. Ignore terms with powers higher than 4.
	A1ft	Either a or b correct ft their expansion from (a) of the form $1+cx^2+dx^4+.....$ **and** $cb = 20$ AND $da = 48$.
	A1ft	Both a and b correct. ft their expansion from (a) of the form $1+cx^2+dx^4+.....$ **and** $cb = 20$ AND $da = 48$.
(c)	M1	For using their expansion from (b) with the correct limits. Although their expression does not need to be fully correct, it must be clear it has arisen from attempting to find a and b in part (b) and using these values. The expression must include the **first** 5 terms in their expansion of the form $p+qx+rx^2+sx^3+tx^4$ from part (b) using their a and b and may include more for this mark. It is not necessary to see the integral sign, if the candidate clearly integrates later. Similarly, if the limits are not present until substitution, this mark can be awarded.
	M1	For an attempt to integrate an expression (see general guidance) – no power of x to decrease. This is not a dependent method mark and the mark is for an attempt to integrate, **but** they must integrate all their terms and there must be a minimum of 2 algebraic terms.
	M1	For substitution of the given limits the correct way around. There must be at least one correct substitution of limits into their **changed** expression with a minimum of 2 algebraic terms (ie this again is not a dependent method mark). 0 need not be substituted if substitution into the changed expression gives an evaluation of 0. This may be implied by a correct answer, but must be shown if the final answer is not correct.
	A1	For awrt 0.5324 ie condone more decimal places.
The question does not demand that students use algebraic integration or show their working clearly and therefore **a correct answer of 0.5324….** may imply the method marks and full marks should be awarded. A value of 0.5347…..will score 0 marks.		

Question	Scheme	Marks
10(a)(i) **(ii)**	$\sin(A+A)=\sin A\cos A+\cos A\sin A=2\sin A\cos A*$ $\cos(A+A)=\cos A\cos A-\sin A\sin A$ $\Rightarrow \cos 2A=\cos^2 A-\sin^2 A$ $=\cos^2 A-\left(1-\cos^2 A\right)$ $=2\cos^2 A-1\ *$	B1 cso M1 A1*cso [3]
(b)	$\left(\frac{2\tan\theta}{1+\tan^2\theta}=\right)\frac{2\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$ $=\frac{2\frac{\sin\theta}{\cos\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$ or $\frac{2\frac{\sin\theta}{\cos\theta}}{1+\frac{1}{\cos^2\theta}-\frac{\cos^2\theta}{\cos^2\theta}}$ or $\frac{2\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}\times\frac{\cos^2\theta}{\cos^2\theta}$ or $\frac{2\sin\theta\cos\theta}{\cos^2\theta+\sin^2\theta}$ $=\frac{2\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}}\left(=2\frac{\sin\theta}{\cos\theta}\times\cos^2\theta\right)$ or $\left(\frac{2\sin\theta\cos\theta}{\cos^2\theta+\sin^2\theta}\right)=2\sin\theta\cos\theta$ $=\sin 2\theta *$	M1 dM1 ddM1 A1*cso [4]
ALT	$\left(\frac{2\tan\theta}{1+\tan^2\theta}=\right)\frac{2\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$ $=\frac{2\frac{\sin\theta}{\cos\theta}}{1+\tan^2\theta}$ $\cos^2\theta+\sin^2\theta=1\Rightarrow\tan^2\theta+1=\frac{1}{\cos^2\theta}\ \Rightarrow\frac{2\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}}\left(=2\frac{\sin\theta}{\cos\theta}\times\cos^2\theta\right)=2\sin\theta\cos\theta$ $=\sin 2\theta *$	M1 dM1 ddM1 A1*cso [4]
(c)	$\frac{5\tan\left(x+\frac{\pi}{6}\right)}{1+\tan^2\left(x+\frac{\pi}{6}\right)}=\left(1-2\cos^2\left(x+\frac{\pi}{6}\right)\right)\Rightarrow\frac{5}{2}\sin\left(2\left(x+\frac{\pi}{6}\right)\right)=\left(1-2\cos^2\left(x+\frac{\pi}{6}\right)\right)$ $1-2\cos^2\left(x+\frac{\pi}{6}\right)=-\cos\left(2x+\frac{\pi}{3}\right)$ $\left(\frac{5}{2}\sin\left(2x+\frac{\pi}{3}\right)=-\cos\left(2x+\frac{\pi}{3}\right)\Rightarrow\right)\tan\left(2x+\frac{\pi}{3}\right)=-\frac{2}{5}$ $\left(\Rightarrow 2x+\frac{\pi}{3}=\right)-0.38050,\quad 2.76108$ $(\Rightarrow x=)-0.714,\quad 0.857$	M1 M1 M1 M1 A1A1 [6]

(d)	$I=\int_0^{\frac{\pi}{2}}\left(2\sin 2\theta-\cos 5\theta+2\right)\ \{\mathrm{d}\theta\}$	M1
	$=\left[\frac{-2\cos 2\theta}{2}-\frac{\sin 5\theta}{5}+2\theta\right]_0^{\frac{\pi}{2}}$	M1
	$=\left(-\cos 2\left(\frac{\pi}{2}\right)-\frac{\sin 5\left(\frac{\pi}{2}\right)}{5}+2\times\left(\frac{\pi}{2}\right)\right)-\left(-\cos 2(0)-\frac{\sin 5(0)}{5}+2\times 0\right)$	M1
	$\left(\quad=\left(1-\frac{1}{5}+\pi\right)-\left(-1-0+0\right)\right)=\frac{9}{5}+\pi$	A1 [4]
		Total 17 marks

Part	Mark	Notes
(a)	B1 cso	For $\sin(A+A)$ or $\sin 2A=\sin A\cos A+\cos A\sin A=2\sin A\cos A*$ No steps missing or incorrect work. Condone sin A + A with brackets missing if all other work correct.
	M1	For $\cos A\cos A-\sin A\sin A\Rightarrow\cos^2 A-\sin^2 A$
	A1* cso	For $\cos(A+A)$ or $\cos 2A=\cos A\cos A-\sin A\sin A\Rightarrow\cos^2 A-\sin^2 A=\cos^2 A-\left(1-\cos^2 A\right)$ $=2\cos^2 A-1$ No steps missing, no incorrect work. Condone cos A + A with brackets missing if all other work correct.
Students who miss ‘A’ in their working, (eg coscos – sinsin) may be awarded M mark, but not A or B marks.		
(b)	M1	Uses $\tan\theta=\frac{\sin\theta}{\cos\theta}$ and $\tan^2\theta=\frac{\sin^2\theta}{\cos^2\theta}$ in the given identity. Allow the 2 to be missing for this mark **but not the final A mark.**
	dM1	For $1+\frac{\sin^2\theta}{\cos^2\theta}=\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}$ or multiplying by $\frac{\cos^2\theta}{\cos^2\theta}$ to give $\left(\frac{2\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}\right)=\frac{2\sin\theta\cos\theta}{\cos^2\theta+\sin^2\theta}$ The $\times\frac{\cos^2\theta}{\cos^2\theta}$ does not need to be explicitly written. Dependent on previous method mark.
	ddM1	For minimum steps (and dependent on previous method marks): $\frac{2\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}}$ or $\frac{2\sin\theta\cos\theta}{\cos^2\theta+\sin^2\theta}=2\sin\theta\cos\theta$
	A1*cso	Achieves the required result – minimum steps as shown, no errors. **Steps in brackets do not need to be shown.**
ALT	M1	Uses $\tan\theta=\frac{\sin\theta}{\cos\theta}$ and $\tan^2\theta=\frac{\sin^2\theta}{\cos^2\theta}$ in the given identity.
	dM1	For replacing $1+\frac{\sin^2\theta}{\cos^2\theta}$ with $1+\tan^2\theta$ Dependent on previous method mark.
	ddM1	For replacing $\tan^2\theta+1=\frac{1}{\cos^2\theta}\Rightarrow\frac{2\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}}=2\sin\theta\cos\theta$ Dependent on both previous method marks.
	A1*cso	Achieves the required result – minimum steps as shown, no errors. **Steps in brackets do not need to be shown.**
Students who miss θ in their working, may be awarded M marks, but not the A mark.		

Allow work throughout where candidates have let, for example $\theta = x+\frac{\pi}{6}$ and worked in θ and 2θ throughout in (c)		
(c)	M1	Rearranges the given equation into $\frac{5\tan\left(x+\frac{\pi}{6}\right)}{1+\tan^2\left(x+\frac{\pi}{6}\right)}=\left(1-2\cos^2\left(x+\frac{\pi}{6}\right)\right)$ **and** changes the left hand side into $P\sin\left(2\left(x+\frac{\pi}{6}\right)\right)$ where P is any constant.
	M1	Correctly states or uses $1-2\cos^2\left(x+\frac{\pi}{6}\right)=-\cos\left(2x+\frac{\pi}{3}\right)$ (condone being replaced with – cos2A)
	M1	For $\tan\left(2x+\frac{\pi}{3}\right)=Q$ where Q is any constant.
	M1	For –0.38050 or 2.76108 This may be awarded for either value being seen, ignore any extra values seen. Accept any values with at least 3 significant figures.
	A1	For $(x=)-0.714$ or $(x=)0.857$ This may be awarded for either value being seen, ignore any extra values seen. Accept any value with more than 3 significant rounding to those given.
	A1	For $(x=)-0.714$ and $(x=)0.857$ Ignore any extra values **out of range**. Extra values in range will mean A0. Accept any value with more than 3 significant rounding to those given.
Students must use θ or $x+\frac{\pi}{6}$ and 2θ or $2\left(x+\frac{\pi}{6}\right)$		
(d)	M1	For changing the integral into $(I=)\int_0^{\frac{\pi}{2}}\left(2\sin 2\theta-\cos 5\theta+2\right)\ \{\mathrm{d}\theta\}$ $\mathrm{d}\theta$ may be omitted. If the integral sign and limits are not present, these can be implied by a clear attempt in later work to integrate all the terms and a substitution of limits. Students must use 2θ and 5θ in their working.
	M1	For a minimum of 2 out of the 3 terms shown above correctly integrated. Students must use 2θ and 5θ in their working.
	M1	For substituting in the given limits the correct way around into their changed function with at least two terms, there must be at least one explicit substitution of each limit. If the candidate has a constant term as part of the function, they can omit this from this step (as the terms cancel out anyway). This mark may be implied by a correct answer. If the candidate is working with an incorrect function and substitution of 0 clearly gives 0, this substitution does not need to be shown.
	A1	For $\frac{9}{5}+\pi$ The approximate decimal value $4.941\ldots$will **usually** indicate correct working (as a guide), but a substitution of limits will be needed to get the 3rd method mark.
In this question, students are directed to use calculus, so must show a correct integration. The penultimate method mark may be implied from a fully correct answer, but if the answer is not correct, we must see substitution of limits as stated.		

Question	Scheme	Marks
As with many log questions, there are a number of ways of reaching a correct answer. The first scheme shows 3 common routes, with a set of general principles in the notes. Not all variations in methods will have been captured, please mark to general principles, referring <u>ANY work where this is not possible and worthy of marks to REVIEW.</u> This includes work that converts everything to completely new bases.		
11 **Eg1**	$2\log_4 x = \log_3 3y^2$ $\dfrac{2\log_2 x}{\log_2 4} = \log_3 3 + \log_3 y^2 \Rightarrow \dfrac{2\log_2 x}{\log_2 4} - 2\log_3 y = 1$ $\Rightarrow \log_2 x - 2\log_3 y = 1$ oe $\log_2 x^3 + 8\log_9 y = 13$ $3\log_2 x + \dfrac{8\log_3 y}{\log_3 9} = 13 \Rightarrow 3\log_2 x + 4\log_3 y = 13$ oe	M1B1 M1M1
	Solves the simultaneous equations by any method to find a value for $\log_2 x$ or $\log_3 y$ $3\log_2 x + 4\log_3 y = 13$ oe $\log_2 x - 2\log_3 y = 1$ oe	dddM1
	$\Rightarrow \log_2 x = \dfrac{15}{5}$ or $\log_3 y = \dfrac{10}{10}$ oe	A1
	$x = 2^{\frac{15}{5}} = 8 \quad y = 3^{\frac{10}{10}} = 3$	M1 A1 [8]
Eg2	$2\log_4 x = \log_3 3y^2$ $2\log_4 x = \log_3 3 + \log_3 y^2 \Rightarrow 2\log_4 x = 1 + 2\log_3 y$ oe $\log_2 x^3 + 8\log_9 y = 13$ $\dfrac{\log_4 x^3}{\log_4 2} + \dfrac{8\log_3 y}{\log_3 9} = 13 \Rightarrow 2\log_4 x^3 + 4\log_3 y = 13$ $\Rightarrow 6\log_4 x + 4\log_3 y = 13$ oe	M1B1 M1M1
	Solves the simultaneous equations by any method to find a value for $\log_4 x$ or $\log_3 y$ $2\log_4 x = 1 + 2\log_3 y$ oe $6\log_4 x + 4\log_3 y = 13$ oe	dddM1
	$\Rightarrow \log_4 x = \dfrac{15}{10}$, or $\log_3 y = \dfrac{10}{10}$ oe	A1
	$x = 4^{\frac{15}{10}} = 8 \quad y = 3^{\frac{10}{10}} = 3$	M1 A1 [8]

<table>
<tr><td>Eg3</td><td>$2\log_4 x = \log_3 3y^2$
$2\log_4 x = \dfrac{\log_9 3y^2}{\log_9 3} \Rightarrow \log_4 x = \log_9 3y^2 \Rightarrow \log_4 x = \log_9 3 + \log_9 y^2$
$\Rightarrow \log_4 x = 0.5 + 2\log_9 y$ oe</td><td>M1B1
M1M1</td></tr>
<tr><td></td><td>$\log_2 x^3 + 8\log_9 y = 13$
$\dfrac{\log_4 x^3}{\log_4 2} + 8\log_9 y = 13 \Rightarrow \log_4 x^3 + 4\log_9 y = \dfrac{13}{2}$
$\Rightarrow 3\log_4 x + 4\log_9 y = \dfrac{13}{2}$ oe
Solves the simultaneous equations by any method to find a value for $\log_4 x$ or $\log_9 y$
$3\log_4 x + 4\log_9 y = \dfrac{13}{2}$ oe</td><td>dddM1</td></tr>
<tr><td></td><td>$\log_4 x = 0.5 + 2\log_9 y$ oe
$\Rightarrow \log_4 x = \dfrac{15}{10}$ or $\log_9 y = \dfrac{5}{10}$ oe</td><td>A1</td></tr>
<tr><td></td><td>$x = 4^{\frac{15}{10}} = 8 \qquad y = 9^{\frac{5}{10}} = 3$</td><td>M1 A1
[8]</td></tr>
<tr><td colspan="3">An aid to marking (please also see more detailed notes).</td></tr>
<tr><td colspan="2">Correct application of addition/subtraction law, anywhere in working.
Once seen, ignore any other incorrect applications.</td><td>M1</td></tr>
<tr><td colspan="2">Any correct evaluation of a $\log_a a^b = b$ seen.
Look for implicit evaluation during change of base of log. a, b rational.</td><td>B1</td></tr>
<tr><td colspan="2">Correct application of the power law seen once anywhere.
Once seen, ignore any other incorrect applications.</td><td>M1</td></tr>
<tr><td colspan="2">Any correct change of base seen once anywhere.
Once seen, ignore any other incorrect applications.</td><td>M1</td></tr>
<tr><td colspan="2">Solve a pair of simultaneous equations – one error permitted, see detailed notes for required form.</td><td>dddM1</td></tr>
<tr><td colspan="2">One correct log equation following from this solution.</td><td>A1</td></tr>
<tr><td colspan="2">Undoing a log of the required form – see detailed notes.</td><td>M1</td></tr>
<tr><td colspan="2">Both correct values of <i>x</i> and <i>y</i>.</td><td>A1</td></tr>
<tr><td colspan="2">Note also that:
$2\log_4 x = \log_2 x$ or $8\log_9 y = 4\log_3 y$ or $\log_2 x^3 = 2\log_4 x^3$ or $\log_3 3y^2 = 2\log_9 3y^2$
will normally imply B1 and 3rd M1</td><td></td></tr>
</table>

ALT1	$2\log_4 x = \log_3 3y^2$	
	$\dfrac{2\log_2 x}{\log_2 4} = \log_3 3y^2 \Rightarrow \log_2 x = \log_3 3y^2$	M1
	$\log_2 x^3 + 8\log_9 y = 13 \Rightarrow 3\log_2 x + 8\log_9 y = 13$	B1 M1
	$3\log_3 3y^2 + 8\log_9 y = 13$	M1
	$\log_3 27y^6 + 8\dfrac{\log_3 y}{\log_3 9} = 13$	dddM1
	$\log_3 27y^6 + 4\log_3 y = 13$	
	$\log_3 27y^6 + \log_3 y^4 = 13$	
	$\log_3 27y^{10} = 13$	A1
	$27y^{10} = 3^{13}$	
	$y = \sqrt[10]{\dfrac{3^{13}}{27}} = 3$	M1
	$\log_2 x = \log_3 3\times 3^2 \Rightarrow \log_2 x = 3$	
	$x = 2^3 = 8$	A1 [8]
ALT2	$2\log_4 x = \log_3 3y^2$	
	$3\log_4 x = \dfrac{3}{2}\log_3 3y^2 \Rightarrow \log_4 x^3 = \dfrac{3}{2}\log_3 3y^2$	M1
	$\dfrac{\log_2 x^3}{\log_4 2} = \dfrac{3}{2}\log_3 3y^2 \Rightarrow \log_4 x^3 = 3\log_3 3y^2$	B1 M1
	$3\log_3 3y^2 + 8\log_9 y = 13$	M1
	$\log_3 27y^6 + 8\dfrac{\log_3 y}{\log_3 9} = 13$	
	$\log_3 27y^6 + 4\log_3 y = 13$	
	$\log_3 27y^6 + \log_3 y^4 = 13$	dddM1
	$\log_3 27y^{10} = 13$	A1
	$27y^{10} = 3^{13}$	
	$y = \sqrt[10]{\dfrac{3^{13}}{27}} = 3$	M1
	$\log_2 x = \log_3 3\times 3^2 \Rightarrow \log_2 x = 3$	
	$x = 2^3 = 8$	A1 [8]
		Total 8 marks

Mark	Notes
M1	For any correct application of the addition (or subtraction) law eg $\log_3 3y^2 = \log_3 3 + \log_3 y^2$ Seen anywhere in their working once, ignore any other incorrect applications.
B1	For $\log_3 3 = 1$ **or** if students do a change of base first, this mark can also be awarded for example for $\log_2 4 = 2$ or $\log_3 9 = 2$ or $\log_4 2 = 0.5$ or $\log_9 3 = 0.5$ - note that this evaluation may not be explicitly seen, but seen in the simplification of an equation, particularly following a change of base of logs. **This mark may be awarded for any correct use of $\log_a a^b = b$ seen. *a, b* rational.**
M1	For applying the power law correctly with **either** $\log_p x^3$ **or** $\log_q y^2$ $p = 2$ or 4 $q = 3$ or 9 Seen anywhere in their working once, ignore any other incorrect applications.
M1	For changing the base of any log in either equation. eg $2\log_4 x = \dfrac{2\log_2 x}{\log_2 4}$ **or** $8\log_9 y = \dfrac{8\log_3 y}{\log_3 9}$ **or** $\log_2 x^3 = \dfrac{\log_4 x^3}{\log_4 2}$ **or** $\log_3 3y^2 = \dfrac{\log_9 3y^2}{\log_9 3}$ **This mark may be awarded for any correct change of base of logs seen.** Seen anywhere in their working once, ignore any other incorrect applications.
dddM1	For solving their simultaneous equations by any valid complete method. Allow one error. Candidates must reach a value for $\log_p x^n$ or $\log_q y^m$ $p = 2$ or 4 $q = 3$ or 9 m, n constants Dependent on all 3 previous method marks. The equations must be in a form with $\log_p ax^n$ and $\log_q by^m$ in $p = 2$ or 4 $q = 3$ or 9 All methods of solving (substitution and elimination being the most common) are valid.
A1	For the correct values of $\log_p x^n$ or $\log_q y^m$ $p = 2$ or 4 $q = 3$ or 9 m, n constants Markers will need to check the solution of candidates' simultaneous equations, but more than one error seen will result in dddM0 and A0. **The question asks students to show their working clearly.** **This accuracy mark would generally only be gained after clearly seeing the work for the previous 4 method marks. If there is a solution where it is clear the student is working more concisely and fully correctly, perhaps taking shortcuts or missing the odd step – please send this to Review.**
M1	For a correct method to 'undo' the logs on any equation of the form $\log_p ax^n = E$ or $\log_q by^m = F$ $p = 2$ or 4 $q = 3$ or 9 , *E, m* and *n* are non-zero constants.
A1	For the correct values of *x* and *y*

ALTS	**For any similar methods mark using the same principles, but send to Review, if not clear how to implement these.**
ALT1 M1	For change of base of log eg $2\log_4 x = 2\dfrac{\log_2 x}{\log_2 4}$ or $8\log_9 y = 8\dfrac{\log_3 y}{\log_3 9}$ **This mark may be awarded for any correct change of base of logs seen.**
B1	For $\log_2 4 = 2$ or $\log_3 9 = 2$. **This mark may be awarded for any correct use of** $\log_a a^b = b$ **seen. *a, b* rational.**
M1	For applying the power law correctly with $\log_2 x^3$ or in reverse $3\log_3 3y^2 \Rightarrow \log_3 27y^6$
M1	For correctly using $\log_2 x = \log_3 3y^2$ in $3\log_2 x + 8\log_9 y = 13$
dddM1	For using the addition (or subtraction) law in reverse $\log_3 27y^6 + \log_3 y^4 \Rightarrow \log_3 27y^{10}$
A1	For $\log_3 27y^{10} = 13$
M1	For a correct method to undo logs.
A1	For the correct values of *x* and *y*
ALT2 M1	For applying the power law correctly in reverse $3\log_2 x = \log_2 x^3$
B1	For $\log_2 4 = 2$ or $\log_3 9 = 2$. **This mark may be awarded for any correct use of** $\log_a a^b = b$ **seen. *a, b* rational.**
M1	For change of base of log eg $2\log_4 x = 2\dfrac{\log_2 x}{\log_2 4}$ or $8\log_9 y = 8\dfrac{\log_3 y}{\log_3 9}$ **This mark may be awarded for any correct change of base of logs seen.**
M1	For correctly using $\log_4 x^3 = 3\log_3 3y^2$ in $3\log_3 3y^2 + 8\log_9 y = 13$
dddM1	For using the addition (or subtraction) law in reverse $\log_3 27y^6 + \log_3 y^4 \Rightarrow \log_3 27y^{10}$
A1	For $\log_3 27y^{10} = 13$
M1	For a correct method to undo logs.
A1	For the correct values of *x* and *y*

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