Mark Scheme (Results)

November 2023

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 2

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme - not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed out work should be marked **unless** the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.
- **With working**

If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking
(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to $x=....$

$(ax^2+bx+c)=(mx+p)(nx+q)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=....$

2. Formula:
Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=....$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n\to x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:
The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."**Exact answers:**
When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)
Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

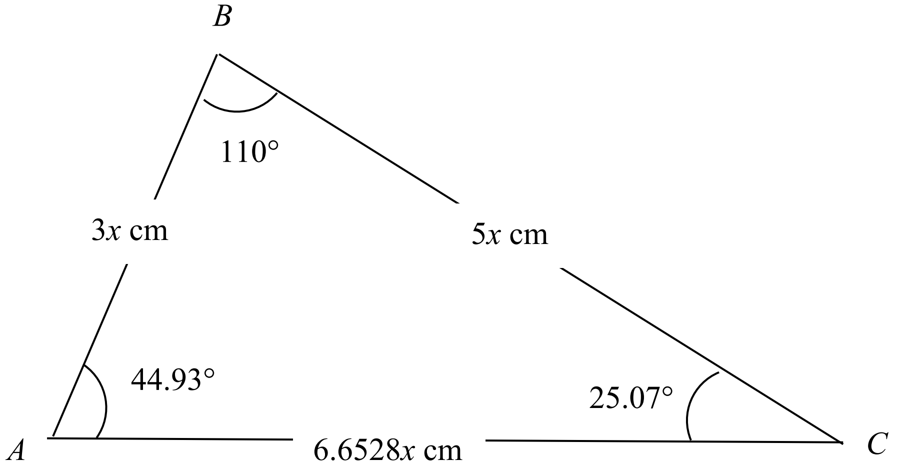
2311
4PM1 Paper 2
Mark Scheme

Question	Scheme	Marks
1	$b^2 - 4ac > 0 \Rightarrow 8^2 - 4 \times k \times 3k > 0$	M1
	$64 - 12k^2 > 0$	
	cvs: $k = \pm\sqrt{\frac{64}{12}} = \left[\pm\frac{4\sqrt{3}}{3}\right]$ or $\left[\pm\frac{4}{\sqrt{3}}\right]$	M1A1
	$\Rightarrow -\frac{4\sqrt{3}}{3} < k < \frac{4\sqrt{3}}{3}$	M1A1 [5]
		Total 5 marks

Mark	Notes
M1	Applies the correct values, with the correct inequality to $b^2 - 4ac > 0$
M1	Attempts to find two critical values by solving the quadratic equation, which must be of the form $k^2 = \text{constant}$ [oe] using a correct method. Accept as a minimum solution $k = \pm\sqrt{\text{constant}}$ Allow simplified or unsimplified. Ignore any inequalities, equal signs etc
A1	For the correct critical values simplified or unsimplified Award this mark for correct critical values. Ignore $=, <, >$ or even $\leqslant, \geqslant$ M0M1A1 is a possible marking pattern.
M1	Simplifies the critical values to the required form and writes down the **inside** region for **their TWO** critical values. If they solve a linear equation for *k* this mark is not available. Allow use of *x* for this mark and also allow $\leqslant$ in place of $<$.
A1	For the correct region specified correctly in either of the two forms specified in terms of *k*. That is, a continuous inside region. Accept $-\frac{4}{\sqrt{3}} < k < \frac{4}{\sqrt{3}}$ oe Accept also for example $k > -\frac{4}{\sqrt{3}}$ AND $k < \frac{4}{\sqrt{3}}$

Question	Scheme	Marks
2(a)	$AC=\sqrt{(3x)^2+(5x)^2-2\times 3x\times 5x\times\cos 110^{\circ}}=6.6528...x$	M1A1
	$\sin\angle BCA=\dfrac{3x\sin 110^{\circ}}{'6.6528'x}=0.4237...\Rightarrow\angle BCA=25.07...^{\circ}$	M1
	Accept awrt 25.1°	A1 [4]
	ALT (for 2nd M1 A1)	
	$\cos\angle BCA=\dfrac{(5x)^2+('6.6528'x)^2-(3x)^2}{2\times 5x\times '6.6528'x}=0.9057...\Rightarrow\angle BCA=25.07^{\circ}$	[M1
	Accept awrt 25.1°	A1]
(b)	$24=\dfrac{1}{2}\times 3x\times 5x\times\sin 110^{\circ}$	M1
	$\Rightarrow x=\sqrt{\dfrac{24\times 2}{3\times 5\times\sin 110^{\circ}}}=(1.84536...)$	M1
	Accept awrt 1.85	A1 [3]
		Total 7 marks

Useful sketch

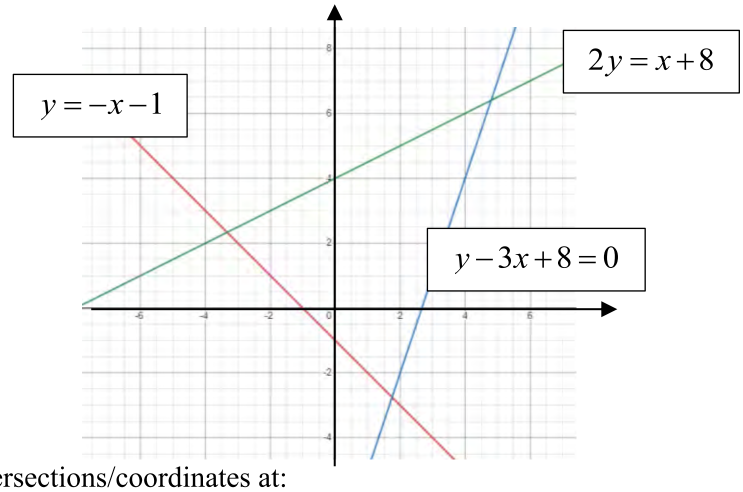
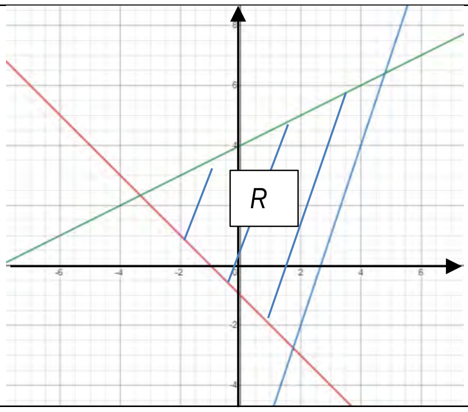


Part	Mark	Notes
(a)	M1	For using a correct cosine rule for AC or AC^2. Note, this is given in the formula sheet so must be correct for this mark.
	A1	For finding a length of $AC = 6.6528...x$ or $AC = \sqrt{44.26....x^2}$ accept awrt $6.7x$ or $\sqrt{44.3x^2}$ **Note:** Allow the missing x here and throughout their working This mark can be implied by sight of $AC = \sqrt{34x^2 - 30x^2 \cos 110}$ if they carry it through to find the angle in the next step. For example: $\sin \angle BCA = \dfrac{3x \sin 110^\circ}{\sqrt{34x^2 - 30x^2 \cos 110}}$
	M1	For using any appropriate trigonometry to find the size of angle BCA. For example; **Sine Rule** $\sin \angle BCA = \dfrac{3x \sin 110^\circ}{'6.6528'x} = 0.4237... \Rightarrow \angle BCA = 25.07...^\circ$ **Cosine Rule** $\cos \angle BCA = \dfrac{(5x)^2 + (6.6529x)^2 - (3x)^2}{2 \times 5x \times 6.6529x} \Rightarrow \angle BCA = 25.07$ Allow a missing x from their working provided it is consistent. Do not allow for example $\sin \angle BCA = \dfrac{3\boldsymbol{x} \sin 110^\circ}{'6.6528'}$
	A1	For awrt 25.1°
	ALT – Uses sine rule	
	M1	Use of sine rule: $\dfrac{\sin(70-\theta)}{5x} = \dfrac{\sin\theta}{3x} \Rightarrow \left[\dfrac{\sin(70-\theta)}{\sin\theta} = \dfrac{5}{3}\right]$
	A1	Expands $\sin(70-\theta) = \sin 70 \cos\theta - \cos 70 \sin\theta$
	M1	Uses the tan identity and rearranges to make $\tan\theta$ the subject. $\dfrac{\sin 70 \cos\theta - \cos 70 \sin\theta}{\sin\theta} = \dfrac{5}{3}$ $\Rightarrow \dfrac{\sin 70}{\tan\theta} - \cos 70 = \dfrac{5}{3} \Rightarrow \tan\theta = \dfrac{\sin 70}{\cos 70 + \dfrac{5}{3}}$
	A1	For awrt 25.1°
(b)	M1	For using the correct formula for the area of a triangle with the correct angle and side lengths. e.g., Can use AB and AC with $\angle ABC = 110^\circ$ or BC and AC with their $\angle BCA$ [25.1°] or AB and AC with their $(180^\circ - 110^\circ - 25.07^\circ = 44.93^\circ)$

	M1	For finding a value of x using their value for AC and BCA or BAC The processing must be correct for this mark and they must be finding the square root of x for this mark.
	A1	For awrt 1.85

Question	Scheme	Marks
3(a)(i)	$v = 6t^2 - 16t + c$	M1
	$t = 0, \quad v = 12$	A1
	$v = \frac{6t^2}{2} - 16t + 12 = \left[3t^2 - 16t + 12\right]$	
(ii)		M1
	$s = \frac{3t^3}{3} - \frac{16t^2}{2} + 12t + k$	
	$\left[t = 0, \quad s = 0\right] \Rightarrow k = 0$	A1 [4]
	$s = t^3 - 8t^2 + 12t$	
(b)	At the origin, $s = 0$	
	$t^3 - 4t^2 + 12t = 0$	M1
	$\Rightarrow t(t-2)(t-6) = 0$	dM1
	$\Rightarrow t = 2, 6, (0)$	
	P first returns to the origin when $t = 2$ seconds	A1 [3]
		Total 7 marks

Part	Mark	Notes
(a)(i)	M1	For integrating the given expression. (+*c*) not required for this mark See General Guidance for the definition of an attempt. No term to be differentiated.
	A1	Finds the value of *c* **and** the correct expression for *v*
(ii)	M1	For an acceptable attempt to integrate **their** *v*. See General Guidance for the definition of an attempt. (+*k*) not required for this mark. No term to be differentiated.
	A1	For the correct expression for the distance, which must have come from establishing **explicitly** that $k = 0$ from the integration. Without $+ k$ in their integration this mark is automatically lost.
(b)	M1	For setting their expression [which must be a cubic] for $s = 0$
	dM1	For solving the cubic Minimally acceptable attempt is $\Rightarrow t\left(t \pm A\right)\left(t \pm B\right) = 0$ where $\lvert AB \rvert = 12$ or also allow $\left(t \pm A\right)\left(t \pm B\right) = 0$ where $\lvert AB \rvert = 12$ as candidates are told that one value of $t = 0$ Any method to solve the quadratic is acceptable. Formula or completing the square. This is a dependent mark. <u>Use of a calculator</u> If there is no working seen and either their values of *t* are incorrect or their *s* is incorrect, only award this mark if a correct method is used/seen to solve the equation.
	A1	For the correct value of $t = 2$ explicitly stated.

Question	Scheme	Marks
4(a)	 Intersections/coordinates at: $y = -x - 1 \Rightarrow (-1, 0)$ and $(0, -1)$ $2y = x + 8 \Rightarrow (-2, 3)$ and $(0, 4)$ $y - 3x + 8 = 0 \Rightarrow (3, 1)$ and $(2, -2)$	B3 [3]
(b)		B1ft [1]
(c)	<table><tr><td>Intersection</td><td>$\left(-3\frac{1}{3}, 2\frac{1}{3}\right)$</td><td>$\left(1\frac{3}{4}, -2\frac{3}{4}\right)$</td><td>$\left(4\frac{4}{5}, 6\frac{2}{5}\right)$</td></tr><tr><td>$P$</td><td>$\frac{44}{3}$</td><td>$-10.75$</td><td>$-1.6$</td></tr></table> $P_{max} = \frac{44}{3}$ $P_{min} = -10.75$	M1A1 dM1A1 [4]

	Total 8 marks

Part	Mark	Notes
(a)	B1	For at least one line drawn correctly. Please check their intersections/coordinates carefully, these lines are to be drawn accurately.
	B1	For at least two lines drawn correctly
	B1	For all three lines drawn correctly.
(b)	B1ft	For an enclosed region shaded in or out. Ft their lines from (a) provided the region is closed. *R* does not need to be written in the region.
(c)	M1	For an attempt to find the coordinates of at least one point of intersection either by simultaneous equations or by reading off from their graph. The question does not specify 'using your graph' so either method is fine.
	A1	For at least one set of correct coordinates. If the candidate uses the graph, then allow all values awrt: $(-3.3\pm0.1,\ 2.3\pm0.1)$ for $\left(-3\frac{1}{3}, 2\frac{1}{3}\right)$ $(1.7\pm0.1,\ -2.7\pm0.1)$ for $\left(1\frac{3}{4}, -2\frac{3}{4}\right)$ $(4.8\pm0.1,\ 6.4\pm0.1)$ for $\left(4\frac{4}{5}, 6\frac{2}{5}\right)$
	dM1	For using at least one set **their** coordinates of their intersections to find any value of *P* Explicit substitution need not be seen provided it is clear candidates are using the expression for *P*. Accept awrt: 14.6 ± 0.1 or -10.7 ± 0.1 or -1.6 ± 0.1 **Note** this is dependent on the previous M mark If their values are incorrect and you see no working this is M0 If their values are correct, you see no working and no value of *P* is correct M0
	A1	For both the correct minimum and maximum values. $P_{max} = 14.66$ accept awrt 14.7 ± 0.3 $P_{min} = -10.75$ accept awrt -10.7 ± 0.3

Question	Scheme	Marks
5(a)	$y = Q(x-6)(x+2)$ where Q is a constant Using the coordinates $(4,-6)$	M1
	$-6 = Q(4-6)(4+2) \Rightarrow Q = \dfrac{-6}{-12} = \dfrac{1}{2}$	M1
	$y = \dfrac{1}{2}(x-6)(x+2) \Rightarrow y = \dfrac{x^2}{2} - 2x - 6$ *	A1 cso [3]
(b)	$\dfrac{dy}{dx} = \dfrac{2x}{2} - 2$	M1
	$x = 4,\quad \dfrac{dy}{dx} = 4 - 2 = 2$	M1
	Gradient of normal is $-\dfrac{1}{2}$	B1ft
	Equation of l: $y - (-6) = -\dfrac{1}{2}(x-4)$	M1
	$\Rightarrow y = -\dfrac{x}{2} - 4 \Rightarrow 2y + x + 8 = 0$ *	A1 cso [5]
(c)	$\dfrac{x^2}{2} - 2x - 6 = -\dfrac{x}{2} - 4 \Rightarrow \dfrac{x^2}{2} - \dfrac{3}{2}x - 2 = 0$ $\Rightarrow (x-4)(x+1) = 0 \Rightarrow x = -1, 4$	M1 M1 A1
	Area $= \int_{-1}^{4}\left(\dfrac{x^2}{2} - 2x - 6\right)dx - \int_{-1}^{4}\left(-\dfrac{x}{2} - 4\right)dx$	M1
	$= \left[\dfrac{x^3}{6} - \dfrac{3}{4}x^2 - 2x\right]_{-1}^{4}$	M1
	$= \left(\dfrac{4^3}{6} - \dfrac{3}{4}\times 4^2 - 2\times 4\right) - \left(\dfrac{(-1)^3}{6} - \dfrac{3}{4}\times(-1)^2 - 2\times(-1)\right)$	M1
	$= \left(\dfrac{4^3}{6} - \dfrac{3}{4}\times 4^2 - 2\times 4\right) - \left(\dfrac{(-1)^3}{6} - \dfrac{3}{4}\times(-1)^2 - 2\times(-1)\right)$ $= -\dfrac{125}{12} \Rightarrow \text{Area} = \dfrac{125}{12}$ (units^2) oe	A1 [7]
		Total 15 marks

Part	Mark	Notes
(a)	M1	Uses the intersections with the x-axes to form a quadratic equation of the form $y = Q(x \pm 6)(x \pm 2)$

	M1	Uses their Quadratic with the coordinates (4, – 6) to find the value of Q Allow just one processing error here.
	A1 cso	For the correct equation in the required form. Note this equation is given to candidates. Both above steps must be complete and correct for the award of this mark.
	ALT – Uses simultaneous equations	
	M1	Sets up all three equations with the given coordinates. These must be correct. $y = px^2 + qx + r$ $0 = 4p - 2q + r$ **1** $0 = 36p + 6q + r$ **2** $-6 = 16p + 4q + r$ **3**
	M1	Attempts to solve their three simultaneous equations to find the values of p, q and r At least one correct value is evidence of correct method. **2-1** $0 = 32p + 8q$ **4** **3-2** $6 = 20p + 2q$ **5** **5**$\times 4$ $24 = 80p + 8q$ **6** **6 - 4** $24 = 48p \Rightarrow p = \frac{1}{2}$, $24 = 40 + 8q \Rightarrow q = -2$ $0 = 2 + 4 + r \Rightarrow r = -6$
	A1 cso	For the correct equation in the required form. Note this equation is given to candidates. All of the above steps must be complete and correct for the award of this mark.
(b)	M1	For differentiating the given expression. This must be correct, simplified or unsimplified for this mark.
	M1	For substituting $x = 4$ into their $\frac{dy}{dx}$ to find the gradient of the tangent.
	B1ft	For finding the gradient of the normal. Ft their gradient.
	M1	For forming an equation for l using the equation of the normal which must have come from use of calculus. If they use $y = mx + c$ then they must find c and form an equation for the award of this mark. For example; $c = -4 \Rightarrow y = -\frac{x}{2} - 4$
	A1 cso	For the correct equation in the required form. Accept the terms in any order. For example: even $0 = -8 - x - 2y$
(c)	M1	For equating the equation of S to their l and forming a 3TQ
	M1	For attempting to solve the 3TQ [see General Guidance for the definition of an attempt] to find two points of intersection.
	A1	For **both** correct values of x
	M1	For a correct statement for the area using **their two points of intersection** correctly. **Do not accept limits of x = – 2 and 4 or 6** They may complete these two areas separately and combine at the end. Check to the end of their work before you score this mark. Accept either $\int_{'-1'}^{'4'} \text{Curve} - \int_{'-1'}^{'4'} \text{Line}$ or $\int_{'-1'}^{'4'} \text{Line} - \int_{'-1'}^{'4'} \text{Curve}$ for this mark.
	M1	For an attempt to integrate either the expression for the line or the curve.

		See General guidance – but no power of x is to decrease.
	M1	For substituting in their values correctly **AND** subtract the two integrals. Explicit substitution must be seen if the final area is incorrect, or the limits are incorrect. A final correct area which follows correct integration is adequate evidence.
	A1	For the correct area

Question	Scheme	Marks
6	$\frac{dV}{dt}=12$	B1
	$\frac{dV}{dh}=9h^2$	B1
	$1536=3h^3 \Rightarrow h=\sqrt[3]{\frac{1536}{3}}=8$	M1A1
	$\frac{dh}{dt}=\frac{dV}{dt}\times\frac{1}{\frac{dV}{dh}}=12\times\frac{1}{9h^2}=12\times\frac{1}{9\times8^2}=\frac{1}{48}$ (cm/s) oe	M1dM1A1
		[7]
		Total 7 marks

Mark	Notes
B1	For stating $\frac{dV}{dt}=12$ This must be clearly labelled $\frac{dV}{dt}$
B1	For differentiating the **given** expression for the volume. This must be clearly labelled $\frac{dV}{dh}$ **It must be correct for this mark.**
M1	For using the given formula, rearranged correctly to find the height of oil when the volume = 1536 That is, unless you see h = 8, you must see this expression. $h=\sqrt[3]{\frac{1536}{3}}$
A1	For the correct value of h = 8 Sight of h = 8 without working is M1A1
M1	For sight of a **correct** chain rule involving $\frac{dh}{dt}, \frac{dV}{dh}$ and $\frac{dV}{dt}$ only. Accept in any order. For example, accept $\frac{dh}{dt}\times\frac{dV}{dh}=\frac{dV}{dt}$ that is, $\frac{dh}{dt}$ does not need to be the subject. This mark can be implied by a correct next step.
dM1	For substituting in their values/expressions with their value of h into a correct chain rule. Note, this mark is dependent on the previous M mark.
A1	For the correct rate of increase. This question asks for an exact value. Do not accept a decimal estimate or accept $0.0208\dot{3}$ unless you see a recurring sign.

	Accept any fraction that simplifies to $\frac{1}{48}$. You will see $\frac{12}{576}$ which is completely acceptable.

Question	Scheme	Marks
7(a)	$S = 5x^3 + (3x-4)^2$ $\Rightarrow S = 5x^3 + 9x^2 - 24x + 16$ *	M1 A1 cso [2]
(b)	$\frac{dS}{dx} = 15x^2 + 18x - 24 = 0$ $\Rightarrow (5x-4)(x+2) = 0 \Rightarrow x = \frac{4}{5}, -2$ $\frac{d^2S}{dx^2} = 30x + 18 = 30\left(\frac{4}{5}\right) + 18 \Rightarrow +\text{ve}$ hence minimum	M1 M1A1 M1A1 [5]
(c)	$S = 5\left(\frac{4}{5}\right)^3 + 9\left(\frac{4}{5}\right)^2 - 24\left(\frac{4}{5}\right) + 16 = \frac{128}{25}$ or 5.12	M1A1 [2]
		Total 9 marks

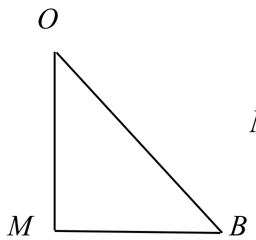
Part	Mark	Notes
(a)	M1	For substituting y into the given S Substituting x will not yield the required expression.
	A1 cso	For obtaining the given expression with no errors. You must check every line of their working.
(b)	M1	For an attempt to differentiate **the given** expression for S wrt x, Accept **at least** two terms fully correct with no power of x to increase.
	M1	Sets their differentiated expression = 0 and attempts to solve, **provided** it is a quadratic. See General Guidance for the definition of an attempt to solve a QE
	A1	For the correct **two** values of x.
	M1	Attempts to differentiate again. Minimally acceptable attempt is $\left(\frac{d^2S}{dx^2}\right) = Ax + B$
	A1	Conclusion: Concludes that the positive value of x $\left(\frac{4}{5}\right)$ will give a positive $\frac{d^2S}{dx^2}$ hence will be a minimum. For example, positive + positive = positive hence minimum. OR Substitutes either value of x, with the appropriate conclusion and correctly concludes that $x = \frac{4}{5}$ gives a minimum.

		NOTE: If they evaluate $\frac{d^2S}{dx^2}$ it must be correct [= 42] An incorrect value is A0.
(c)	M1	Uses **their** minimum value of x in the **given** expression, even if they identify $x = -2$ as the value giving a minimum S. If their minimum is not explicitly identified, then award M0. If they substitute both values without identifying which is which, award M0.
	A1	For the correct value of S

Question	Scheme	Marks
8(a)	$42 = \frac{4}{2}\left(2a + (4-1)d\right) \Rightarrow 42 = 4a + 6d$ oe $23 = a + 4d$ $\Rightarrow a = 3, \quad d = 5$ $n\text{th term} = 3 + (n-1)5 = 5n - 2 \Rightarrow S_n = \sum_{r=1}^{n}(5r-2)$ $[P = 5, \; Q = 2]$	B1 B1 M1A1 M1A1 [6]
(b)	$S_{2n} - 3U_n = 1062$ $\Rightarrow \frac{2n}{2}\left(2\times'3' + (2n-1)'5'\right) - 3\left['3' + (n-1)'5'\right] = 1062$ $\Rightarrow 10n^2 + n - 15n + 6 = 1062 \Rightarrow 10n^2 - 14n - 1056 = 0$ $10n^2 - 14n - 1056 = (5n+48)(2n-22) = 0$ $\Rightarrow n = 11$	 M1 M1 M1 A1 [4]
		Total 10 marks

Part	Mark	Notes
(a)	B1	Forms a correct equation in a and d for **either** the sum of the first 4 terms or for the 5th term This must be correct.
	B1	Forms correct equations in a and d for **both** the sum of the first 4 terms and for the 5th term Both must be correct.
	M1	Solves their two equations simultaneously by any method. Accept a pair of equations as follows: $pa+qd=21$ or 42 $ra+sd=23$ where $p\neq r$ and $q\neq s$ Allow a maximum of one arithmetical error in the solution of their SE.
	A1	For **both** $a=3$ and $d=5$
	M1	Attempts to form the required expression using their values of a and d They must use the nth term for this.
	A1	For the correct expression as written. This must be exactly as written in the question with the inclusion of the $\sum$. However, allow the omission of $S_n=......$
	ALT for last 2 marks	
	M1	$S_n=\sum_{r=1}^{n}(Pr-Q)\Rightarrow 3=P-Q \quad 5=(2P-Q)-(P-Q)=P$ $\Rightarrow 3=5-Q\Rightarrow Q=2$
	A1	For the correct expression as written. This must be exactly as written in the question with the inclusion of the $\sum$. However, allow the omission of $S_n=......$ Allow also $S_n=\sum_{r=1}^{n}(Pr-Q)\quad(\Rightarrow)\quad P=5, Q=2$ for this mark.
(b)	M1	Applies the summation formula and the nth term formula correctly to form an equation in n using their values for a and d For this mark you must see an equation, e.g., $\frac{2n}{2}(2\times3+(2n-1)5)-3[3+(n-1)5]=1062$ However, allow recovery in the next step.
	M1	Forms a 3TQ $\quad 10n^2-14n-1056=0$ o.e. Condone missing = 0 if it is clear they are solving this equation.
	M1	Attempts to solve their 3TQ Accept any method. If their 3TQ is incorrect, only award this mark when you can see their method. If a calculator is used with an incorrect 3TQ and no method is seen, this mark is not available.

		Note: We do not need to see $-\frac{48}{5}$. Some candidates will automatically reject this solution.
	A1	For the correct value of n Do not award if $-\frac{48}{5}$ is included as a value.

Question	Scheme	Marks
9(a)	$\angle AOB = \frac{1.8x}{x} = 1.8$	B1
	 $MB = x\sin 0.9 \Rightarrow AB = 2x\sin 0.9$	M1
	Hence diameter $AC = 2x\sin 0.9 + 2x$ [or Radius = $x\sin 0.9 + x$]	dM1A1
	Arc length of semicircle: $\frac{\pi D}{2} = \frac{\pi\left(2x\sin 0.9 + 2x\right)}{2} = \pi x\left(1 + \sin 0.9\right)$	M1
	Perimeter of logo: $P = \pi x\left(1 + \sin 0.9\right) + 2x + 1.8x \Rightarrow P = x\left(\pi + \pi\sin 0.9 + 3.8\right)$ [a=1, b =3.8]	M1A1 [7]
(b)	$D = 2\times 10\times \sin 0.9 + 2\times 10 = 35.6665...$	M1
	Area of semicircle $= \frac{\frac{\pi\times 35.6665...^2}{4}}{2} = 499.553...$	M1
	Area of sector $= 1.8\times\frac{10^2}{2} = 90$	B1
	Area of segment $= 90 - \frac{1}{2}\times 10\times 10\times \sin 1.8 = 41.307...$ OR Area of triangle = $\frac{1}{2}\times 10\times 10\times \sin 1.8 = 48.692$	B1
	Area of logo $= 499.553 - 41.307 = 458.245... \approx$ awrt $458\left(\text{cm}^2\right)$	M1A1 [6]
		Total 13 marks

Part	Mark	Notes
(a)	B1	For finding angle AOB = 1.8 in radians only seen anywhere.
	M1	For find the length AB or $\frac{1}{2}AB$ using their angle.

		$AB = 2x\sin 0.9 \quad \frac{1}{2}AB = x\sin 0.9$ [Allow $AB = 1.57x$] Please check their diagram for their labelling/notation.
	dM1	For attempting to find the diameter (or radius) of the semi-circle *ADC* Allow $2x + 1.57x$ or $3.57x$ [Dependent on the previous M mark]
	A1	For the correct expression for the diameter $[2x\sin 0.9 + 2x]$ or the radius $[x\sin 0.9 + x]$ of the semi-circle.
	M1	For finding the arc length of the semi-circle. This can be simplified or unsimplified. [Allow also $1.79\pi x$]
	M1	For finding the perimeter of the logo using their values **in terms of *x* and sin 0.9 only** That is: $P = 1.8x + 2x +$ their arc length of *ADC*
	A1	For the correct expression exactly as written.
	ALT – Uses cosine rule	
	B1	For finding angle $AOB = 1.8$
	M1	Finds the length *AB* or $\frac{1}{2}AB$ using their angle. [They must sqrt to find *AB*] $AB = \sqrt{x^2 + x^2 - 2\times x\times x\cos 1.8}$
	dM1	For attempting to find the diameter (or radius) of the semi-circle *ADC* $AC = 2x + \text{'}x\sqrt{2-2\cos 1.8}\text{'}$
	A1	For the correct diameter or radius of the circle.
	M1	For finding the arc length of the semi-circle. $P = \dfrac{\pi\left(x\sqrt{2-2\cos 1.8}\right)}{2}$
	M1	For finding the perimeter of the logo using their values. That is: $P = 1.8x + 2x +$ ‘$\pi x(1+\sin 0.9)$’ They must change cosine to sine for this mark. $\frac{1}{2}\left(\sqrt{2-2\cos 1.8}\right) = \sin 0.9$ must be explicitly seen **NOTE:** $\left[\sin\theta = \sqrt{\dfrac{1-\cos 2\theta}{2}}\right]$
	A1	For the correct expression exactly as written.
(b)	**Allow work in degrees in this part as it makes no difference.**	
	M1	For attempting to find the diameter or radius of the semicircle. Accept: $D = 2\times 10\times\sin\angle\text{their } AOB + 2\times 10 = (35.6665...)$ $r = \dfrac{2\times 10\times\sin\angle\text{their } AOB + 2\times 10}{2} = (17.833...)$
	M1	For using their *D* ,[where $D \neq 10$] to find the area of the **semicircle**. $\text{Area}_{semicircle} = \dfrac{\dfrac{\pi\times('35.6665...')^2}{4}}{2}$ OR $\dfrac{\pi\times 17.8333...^2}{2} = (499.553...)$ **Note: This is an A mark in Epen**
	B1	For finding the area of the sector If they work in degrees, accept awrt 90. **This is not a ft mark.**

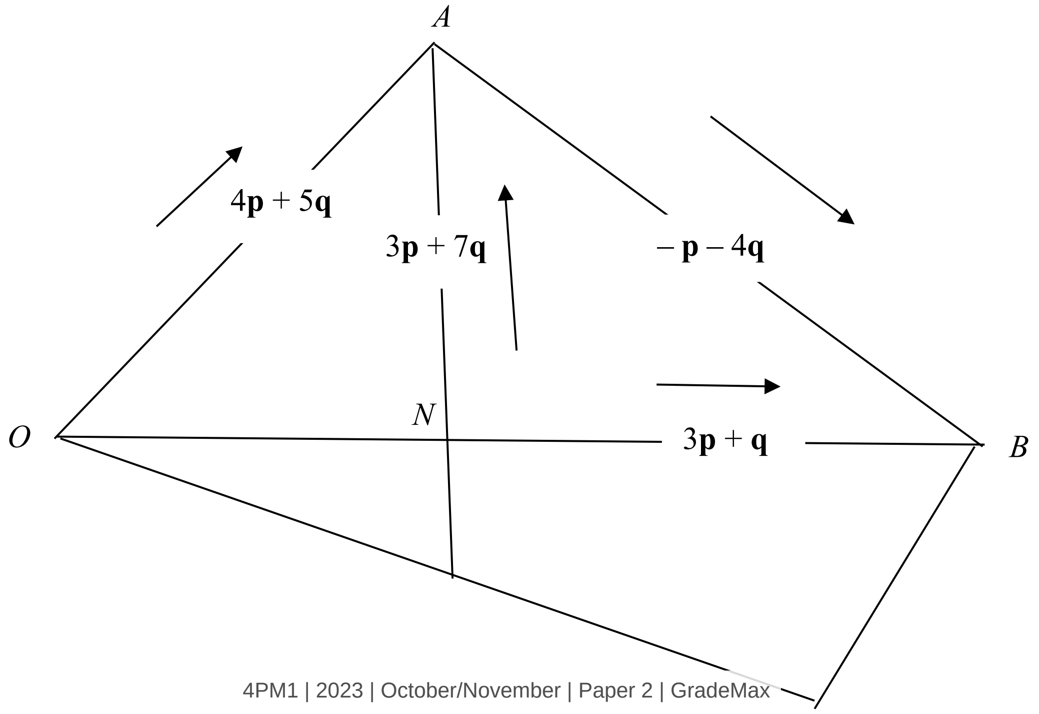
	B1	For finding the area of the segment, awrt 41 OR for finding the area of the triangle OAB $=\frac{1}{2}\times10^2\times\sin1.8=[48.692...]$ **This is not a ft mark.**
	M1	For finding the area of the logo Area = Their area of semicircle – their area of the segment. Or Area = Area of whole shape – area of sector
	A1	For the correct value of the area of the logo.

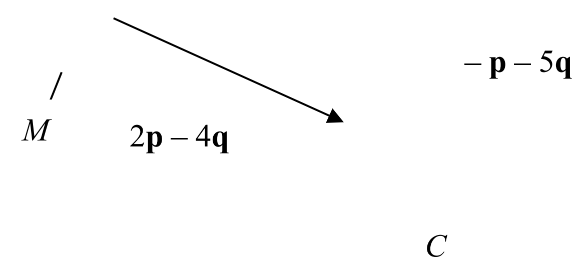
Question	Scheme	Marks
10(a)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$	
	$\frac{115}{8}=\left(-\frac{5}{2}\right)^3-3\alpha\beta\left(-\frac{5}{2}\right)$	M1
	$\Rightarrow\alpha\beta=\dfrac{-\frac{115}{8}+\left(-\frac{5}{2}\right)^3}{3\times\left(-\frac{5}{2}\right)}$	dM1
	$\Rightarrow\alpha\beta=4$	A1 cso [3]
(b)	**Sum:** $\frac{\alpha^2+1}{\beta}+\frac{\beta^2+1}{\alpha}=\frac{\alpha^3+\beta^3+\alpha+\beta}{\alpha\beta}$	
	$\Rightarrow\frac{\alpha^2+1}{\beta}+\frac{\beta^2+1}{\alpha}=\dfrac{\frac{115}{8}+\left(-\frac{5}{2}\right)}{4}=\frac{95}{32}$	M1A1
	Product: $\frac{\alpha^2+1}{\beta}\times\frac{\beta^2+1}{\alpha}=\frac{\alpha^2\beta^2+\alpha^2+\beta^2+1}{\alpha\beta}$	M1
	$\Rightarrow\frac{\alpha^2+1}{\beta}\times\frac{\beta^2+1}{\alpha}=\frac{\alpha^2\beta^2+\left[(\alpha+\beta)^2-2\alpha\beta\right]+1}{\alpha\beta}$	M1
	$\Rightarrow\frac{\alpha^2+1}{\beta}\times\frac{\beta^2+1}{\alpha}=\dfrac{4^2+\left(-\frac{5}{2}\right)^2-2\times4+1}{4}=\frac{61}{16}$	A1
	Equation: $x^2-\left(\frac{95}{32}\right)x+\left(\frac{61}{16}\right)=0\Rightarrow32x^2-95x+122=0$ oe	M1A1 [7]
		Total 10 marks

Part	Mark	Notes
(a)	Part (a) is a 'Show that' question. You must see sufficient work for the award of **both** M marks for the award of the A mark	
	M1	For the correct algebra **and** substitution of the given values into a correct expression for $\alpha^3+\beta^3$ There is more than one acceptable form of this expansion. They must be able to substitute the given values of $\alpha+\beta$ and $\alpha^3+\beta^3$ with $\alpha\beta$ as the value to find in any version they use. For example: $\alpha^3+\beta^3=(\alpha+\beta)\left[(\alpha+\beta)^2-3\alpha\beta\right]$
	dM1	For an attempt to solve the linear equation to find a value for $\alpha\beta$ Allow one processing error for this mark. Note, this is a dependent M mark.
	A1 cso	For $\alpha\beta=4$ This is a show question, you must check their algebra carefully.
(b)	M1	For the **correct** algebra **and** substitution of the given values to find the sum.
	A1	For the correct sum $=\frac{95}{32}$
	M1	For the correct algebra **and** substitution to find $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\left[-\frac{7}{4}\right]$
	M1	For the **correct** algebra **and** substitution of the given values to find the product
	A1	For the correct product $=\frac{61}{16}$
	M1	For forming an equation using **their** sum and product correctly. $x^2-(\text{their sum})x+(\text{their product})=(0)$ Allow missing = 0 for this mark
	A1	For a correct equation with integer coefficients. For example: $64x^2-190x+244=0$

Question	Scheme	Marks
11(a)	$\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OC} = \mathbf{p} - 2\mathbf{q}$	B1
	$\overrightarrow{MA} = -\overrightarrow{OM} + \overrightarrow{OA}$	M1
	$\overrightarrow{MA} = -(\mathbf{p} - 2\mathbf{q}) + (4\mathbf{p} + 5\mathbf{q}) = 3\mathbf{p} + 7\mathbf{q}$	A1 [3]
(b)	$\overrightarrow{MN} = \lambda(3\mathbf{p} + 7\mathbf{q})$	M1
	$\overrightarrow{MN} = -(\mathbf{p} - 2\mathbf{q}) + \mu(3\mathbf{p} + \mathbf{q})$	M1
	$\lambda(3\mathbf{p} + 7\mathbf{q}) = -(\mathbf{p} - 2\mathbf{q}) + \mu(3\mathbf{p} + \mathbf{q})$	M1
	$\Rightarrow 3\lambda\mathbf{p} + 7\lambda\mathbf{q} = (-1 + 3\mu)\mathbf{p} + (2 + \mu)\mathbf{q}$	
	$3\lambda = -1 + 3\mu$	dM1
	$7\lambda = 2 + \mu$	A1
	$\Rightarrow 18\lambda = 7 \Rightarrow \lambda = \frac{7}{18}$	A1
	$MN : NA = 7 : 11$	[6]
		Total 9 marks

Useful sketch





Part	Mark	Notes
(a)	B1	For finding either vector $\overrightarrow{OM} = \mathbf{p} - 2\mathbf{q}$ or $\overrightarrow{MO} = -\mathbf{p} + 2\mathbf{q}$ This may be embedded in their working for $\overrightarrow{MA}$
	M1	For the correct vector statement $\overrightarrow{MA} = -\overrightarrow{OM} + \overrightarrow{OA}$ or $\overrightarrow{MA} = -\frac{1}{2}\overrightarrow{OC} + \overrightarrow{OA}$
	A1	For the correct simplified vector
(b)	**General principle of marking part (b)** • First two M marks are for two vector statements for $\overrightarrow{MN}$ or $\overrightarrow{AN}$ that will allow them to be equated. Note that they can use any path that involves either of these two vectors. • The third M mark is for equating coefficients and forming a pair of simultaneous equations. • The final M mark is for solving their simultaneous equations.	
	M1	For one vector for $\overrightarrow{MN}$ or $\overrightarrow{AN}$ involving a constant One example is: $\overrightarrow{AN} = K\,\overrightarrow{AM} = K(-3\mathbf{p} - 7\mathbf{q})$
	M1	For a second vector for $\overrightarrow{MN}$ or $\overrightarrow{AN}$ following a different path involving a different constant. One example is: $\overrightarrow{AN} = \overrightarrow{AO} + L\,\overrightarrow{OB} = -4\mathbf{p} - 5\mathbf{q} + L(3\mathbf{p} + \mathbf{q})$
	M1	For equating the two vectors and forming a pair of simultaneous linear equations, both of which must be in terms of λ and μ
	dM1	For an attempt to solve their linear equations. Allow up to one processing error. This mark is dependent on the previous M mark.
	A1	For the value of $\lambda = \frac{7}{18}$ μ is not required but the value is $\frac{13}{18}$
	A1	For the correct ratio $MN : NA = 7 : 11$

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