



Mark Scheme (Results)

November 2024

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1) Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
 - M marks: method marks
 - A marks: accuracy marks
 - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
 - cao – correct answer only
 - ft – follow through
 - isw – ignore subsequent working
 - SC - special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - awrt – answer which rounds to
 - eeo – each error or omission
 - cas – Correct answer scores full marks (unless from obvious incorrect working)
 - wr working required
- **No working**

If no working is shown then correct answers normally score full marks
If no working is shown then incorrect (even though nearly correct) answers score no marks.
- **With working**

If there is a wrong answer indicated on the answer line always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used.
If there is no answer on the answer line then check the working for an obvious answer.
- **Ignoring subsequent work**

It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.

It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.

Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**

Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c| \text{ leading to } x = \dots$$

$$(ax^2 + bx + c) = (mx + p)(nx + q) \text{ where } |pq| = |c| \text{ and } |mn| = |a| \text{ leading to } x = \dots$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for a , b and c , leading to $x = \dots$

3. Completing the square:

$$x^2 + bx + c = 0: \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0 \quad \text{leading to } x = \dots$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration:

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show....")

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the is rule may allow the mark to be awarded before the final answer is given.

Question number	Scheme	Marks
1 (a)(i)	$3x + 4y = 24$ drawn	B1 [1]
(a)(ii)	$2x - 5y + 10 = 0$ drawn	B1 [1]
(b)	$x = -1$ drawn	M1 (B1 on ePen)
	<p>Correct region defined (see below)</p>	A1ft (B1 on ePen) [2]
Total 4 marks		

Part	Mark	Additional Guidance
(a) (i)	B1	Correct line – check the intersections with the axes, mark intention. The minimum length acceptable is reaching 2 sets of integer coordinates, for this mark: (0, 6) (4, 3) (8, 0)
(ii)	B1	Correct line – check the intersections with the axes, mark intention. The minimum length acceptable is reaching 2 sets of integer coordinates, for this mark: (-5, 0) (0, 2) (5, 4)
(b)	M1 (B1 on ePen)	$x = -1$ drawn ($y = 5$ doesn't need to be drawn) This line can be short, but must extend at least slightly above/below the x -axis. Note: this can be implied by further fully correct work
	A1ft (B1 on ePen)	Correctly shaded region – shaded in or out. Mark intention. The shading must reach below the x -axis, but doesn't need to extend to the bottom or right of grid, where it's unbounded. Accept dotted or solid lines. any ft must have all points such that $x > -1$ and $y < 5$ always. ft any line drawn with a positive gradient and positive y -intercept and any line drawn with a negative gradient and positive y -intercept where the region shaded is correct for their lines drawn. It is not sufficient to label R , with no shading.

Mark intention here means – if the candidate is clearly intending a line going through the correct points of intersection, the mark may be given.

A line with lots of 'fuzziness' or which is too wavy or misses the **integer coordinates** by obviously more than around a quarter of a square (examiners are not expected to measure) cannot be awarded the marks.

Question number	Scheme	Marks
2 (a)	$9(a + a + d) = a + 4d + a + 5d + a + 6d$ oe $(5a = 2d)$ oe $a + 2d = 12$ oe $5(12 - 2d) = 2d$ oe $d = 5 \quad a = 2$	M1 B1 M1 A1 A1 [5]
For the main scheme, ALT1 and ALT2, give the first 1 or 2 M marks, regardless of if students carry on to use the method for the 3 rd M mark. Favour the ALT which gives the most marks. ALT3 may not be used except in the circumstances described below – clarify with your team leader if this is not clear.		
(b)	$\sum_{n=1}^{60} u_n = \frac{60}{2} (2 \times "2" + (60 - 1) \times "5") (= 8970)$ $\sum_{n=1}^{14} u_n = \frac{14}{2} (2 \times "2" + (14 - 1) \times "5") (= 483)$ or $(u_{14}) = "2" + (14 - 1) \times "5" (= 67)$ "8970" – "483" = 8487	M1 M1 ddM1 A1 [4]
ALT1	(First term = u_{15} =) $"2" + "5" \times (15 - 1) (= 72)$ (Last term = u_{60} =) $"2" + "5" \times (60 - 1) (= 297)$ or $"72" + "5" \times (46 - 1) (= 297)$ $\frac{46}{2} ("72" + "297") = 8487$	M1 M1 ddM1 A1 [4]
ALT2	$(u_{14} =) "2" + "5" \times (14 - 1) (= 67)$ $(u_{60} =) "2" + (60 - 1) \times "5" (= 297)$ $\frac{60}{2} ("2" + "297") - \frac{14}{2} ("2" + "67") = 8487$	M1 M1 ddM1 A1 [4]
This following ALT, may <u>only</u> be used if candidates are clearly and obviously going on to use $\left(\sum_{r=15}^{60} u_r = \right) \frac{46}{2} (2 \times "72" + (46 - 1) \times "5")$ Allow $n = 45$ for first 3 method marks.		
ALT3	(First term =) $"2" + "5" \times (15 - 1) (= 72)$ $\left(\sum_{r=15}^{60} u_r = \right) \frac{46}{2} (2 \times "72" + (46 - 1) \times "5")$	M2 ddM1 A1 [4]
(c)	$2 \times \frac{n}{2} (2 \times 2 + (n - 1) \times 5) - 5(5n - 3) = 10$ oe $(4n + 5n^2 - 5n - 25n + 15 = 10)$ $5n^2 - 26n + 5 = 0$ $(5n - 1)(n - 5) = 0 \rightarrow n = \frac{1}{5}, 5$ $n = 5$	M1 A1 M1 A1 [4]
Total 13 marks		

Part	Mark	Additional Guidance
(a)	M1	Correctly forms the equation shown. Any equivalent unsimplified equation. Note: an alternative seen is (the mark is awarded for the second line below or any unsimplified equivalent) $(S_7 - S_4 = 9(a + a + d))$ $\frac{7}{2}(2a + (7 - 1)) - \frac{4}{2}(2a + (4 - 1)) = 9(a + a + d)$ $(\Rightarrow 15a = 6d)$
	B1	For the correct equation.
	M1	For a complete and valid method to solve the equations simultaneously, leading to either $a =$ or $d =$. Allow one processing error only . Can be awarded for a complete and valid method to solve any two linear equations, both in a and d , one processing error only. Must lead to either $a =$ or $d =$
	A1	For either the correct value of a or the correct value of d .
	A1	For the correct values of both a and d .
If candidates use a different method, follow the same principles to award marks or send to Review if this is not clear. Candidates may use any letters or symbols to denote a and d		
(b)	M1	Use of their a and d in the correct formula $\frac{n}{2}(2"a"+(n-1)"d")$ with $n = 60$
	M1	Use of their a and d in the correct formula $\frac{n}{2}(2"a"+(n-1)"d")$ with $n = 14$ Allow $n = 15$ as a concession.
	ddM1	For their " $\sum_{n=1}^{60} u_n$ " - " $\sum_{n=1}^{14/15} u_n$ ". Dependent on both previous method marks. Follow through their value from use of $n = 15$ if used. The presence of $8970 - 483$ will imply M3 here as will 8487 .
	A1	For 8487
ALT1	M1	For correct use of " $a"+(n-1)"d"$ with $n = 15$ with their a and their d
	M1	For correct use of " $a"+(n-1)"d"$ with $n = 60$ with their a and their d or correct use of " $a_{15}"+(n-1)"d"$ with $n = 46$ with their d and correctly using their term for a_{15} from the first M mark. Allow $n = 45$ as a concession.
	ddM1	For correct use of the formula $\frac{n}{2}(u_{15} + u_{60})$. Dependent on both previous method marks. Allow $n = 45$ as a concession. $\frac{46}{2}("72"+"297")$ or 8487 implies M3
	A1	For 8487
ALT2	M1	For correct use of " $a"+(n-1)"d"$ with $n = 14$ with their a and their d Allow $n = 15$
	M1	For correct use of " $a"+(n-1)"d"$ with $n = 60$ with their a and their d
	ddM1	For correct use of the formula $\frac{n}{2}(u_1 + u_{60}) - \frac{n}{2}(u_{14} + u_{60})$. Dependent on both previous method marks. Allow $n = 15$ The presence of $8970 - 483$ will imply M3 here as will 8487
	A1	For 8487

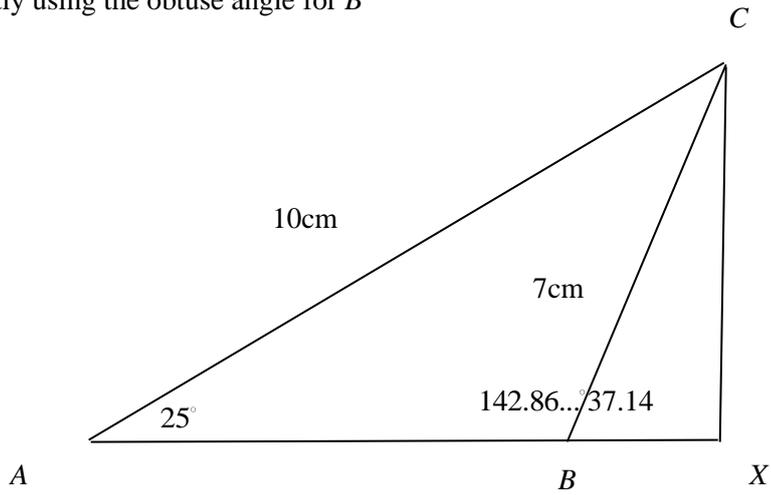
ALT3	M2	For correct use of " $a+(n-1)d$ " with $n = 15$ with their a and their d
	ddM1	For correct use of the formula $\frac{n}{2}(2 \times "u_{15}" + (n-1)d)$. Dependent on both previous method marks. Allow $n = 45$ as a concession.
	A1	8487
(c)	M1	For correct use of $2S_n - 5(an + b) = 10$ Substitution into S_n must be correct for their values of a and d . and candidates must correctly use their a in an expression of the form $an + b$ b can be any integer, $b \neq 0$ $2S_n - 5(an + b) = 10$ does not have to be simplified to be awarded this mark.
	A1	For the correct 3 term quadratic equation = 0. Equivalent coefficients are possible.
	M1	For any valid, complete method to solve their quadratic (see general guidance), which must be of the form $pn^2 + qn + r$ $p, q, r \neq 0$ leading to $n =$ $n = 5$ will imply this mark.
	A1	$n = 5$ $n = 0.2$ must be dismissed.

Q	Scheme	Marks
3	$\frac{\sin 25}{7} = \frac{\sin B}{10} \text{ oe } (B =) \sin^{-1}\left(\frac{10 \sin 25}{7}\right) \text{ oe } (B = 37.138(254541) \text{ or } 142.861(7455))$ $(C = 180 - [(180 - 37.138(25454)) + 25])$ $(C =) 12.138(25451)$ $(c^2 =) 7^2 + 10^2 - 2(7)(10) \cos "12.138(25451)" \text{ or}$ $\frac{c}{\sin "12.138(25451)" } = \frac{7}{\sin 25} \text{ oe eg } (c =) \frac{7}{\sin 25} \times \sin "12.138(25451)" \text{ or}$ $\frac{c}{\sin "12.138(25451)" } = \frac{10}{\sin "142.861(7455)" } \text{ oe}$ $\text{eg } (c =) \frac{10}{\sin "142.861(7455)" } \times \sin "12.138(25451)"$ <p>**allow students to follow through using their acute angle for B</p> $(c^2 =) 7^2 + 10^2 - 2(7)(10) \cos "117.861(7455)" \text{ or}$ $\frac{c}{\sin "117.861(7455)" } = \frac{7}{\sin 25} \text{ oe eg } (c =) \frac{7}{\sin 25} \times \sin "117.861(7455)" \text{ or}$ $\frac{c}{\sin "117.861(7455)" } = \frac{10}{\sin "37.138(254541)" } \text{ oe}$ $\text{eg } (c =) \frac{10}{\sin "37.138(254541)" } \times \sin "117.861(7455)"$ $(c =) 3.5(\text{cm})$	<p>M1dM1</p> <p>A1 (M1 on ePen)</p> <p>ddM1</p> <p>A1 [5]</p>
ALT1	$7^2 = 10^2 + (AB)^2 - 2(10)(AB) \cos 25$ $0 = (AB)^2 - 20 \cos 25(AB) + 100 - 49 \quad (20 \cos 25 = 18.12615574)$ $0 = (AB)^2 - 20 \cos 25(AB) + 51$ $(AB =) \frac{-(-20 \cos 25) \pm \sqrt{(-20 \cos 25)^2 - 4(1)(51)}}{2 \times 1} (= 3.48, 14.64)$ $(c =) 3.5(\text{cm})$	<p>M1 dM1 A1 (M1 on ePen)</p> <p>ddM1</p> <p>A1 [5]</p>
ALT2	<p>M1 M1 as main scheme (then students are working in right angled triangles).</p> $(AX =) 10 \cos 25 \text{ or } 9.(06307787)$ $(BX =) 7 \sin ("37.14.....") \Rightarrow 10 \cos 25 - 7 \sin ("37.14.....")$ <p>or if working with acute angle</p> $(BX =) 7 \sin ("37.14.....") \Rightarrow 10 \cos 25 + 7 \sin ("37.14.....")$ $= 3.5(\text{cm})$	<p>B1 (M1 on ePen)</p> <p>ddM1</p> <p>A1 [5]</p>
Total 5 marks		

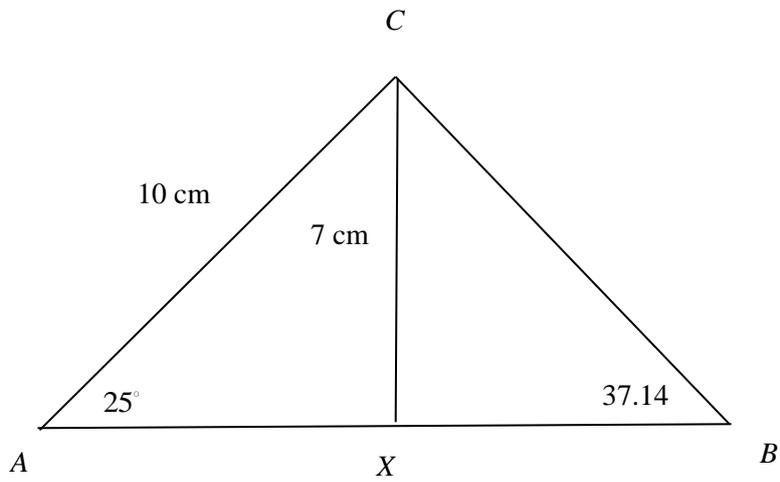
Part	Mark	Additional Guidance
	M1	For a fully correct substitution into the sine rule.
	dM1	For a correct rearrangement to find angle B , implied by sight of the correct acute or obtuse angle. Dependent on previous method mark.
	A1 (M1 on ePen)	For angle C , accept angles which round to 12.
	ddM1	Correct substitution into the cosine rule or sine rule using their angles B and/or C . Allow angle B to be an acute angle and angle C to be the angle that comes from this acute angle using the sum of angles in a triangle. This mark is dependent on the first 2 method marks and is for correct use of the cosine or sine rule, given they have found an angle for B and used this to find angle C .
	A1	For awrt 3.5 (cm)
ALT1	M1	For a fully correct substitution into the cosine rule. Allow AB to be denoted by any letter
	dM1	For a fully correct rearrangement = 0, dependent on the previous method mark.
	A1 (M1 on ePen)	For the correct 3 term quadratic equation = 0.
	ddM1	For a minimally acceptable attempt to solve their quadratic – see general guidance. Dependent on both previous method marks. Sight of 3.48..... or 14.64 may imply this mark
	A1	For awrt 3.5 (cm) Must reject any second solution.
ALT2	M2	As first 2 marks in main scheme
	B1 (M1 on ePen)	For AX either $10\cos 25$ or answers which round to 9
	ddM1	For finding BX correctly and a correct method to find AB , allowing use of acute B
	A1	For awrt 3.5 (cm)

Particularly to help with ALT2:

If correctly using the obtuse angle for B



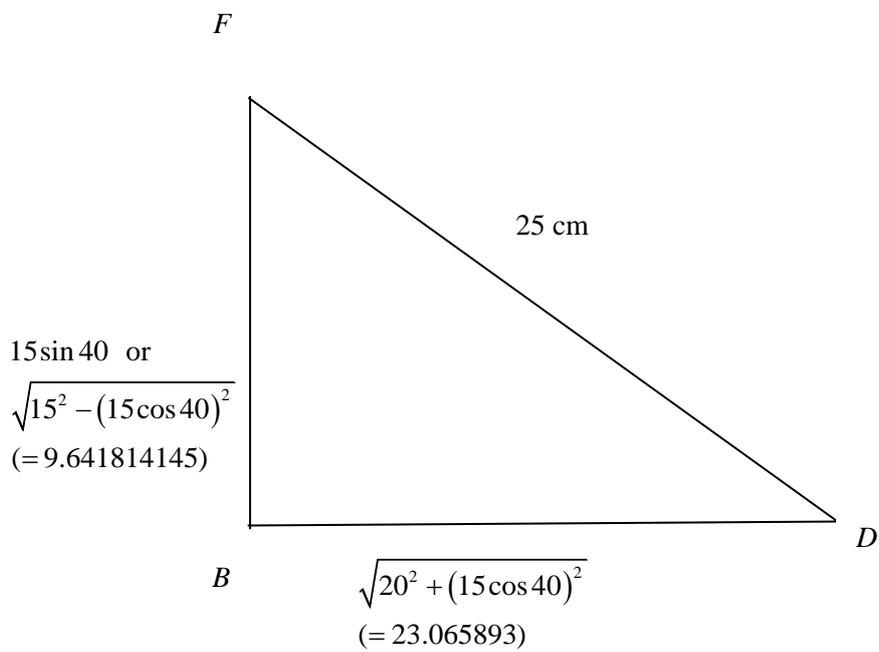
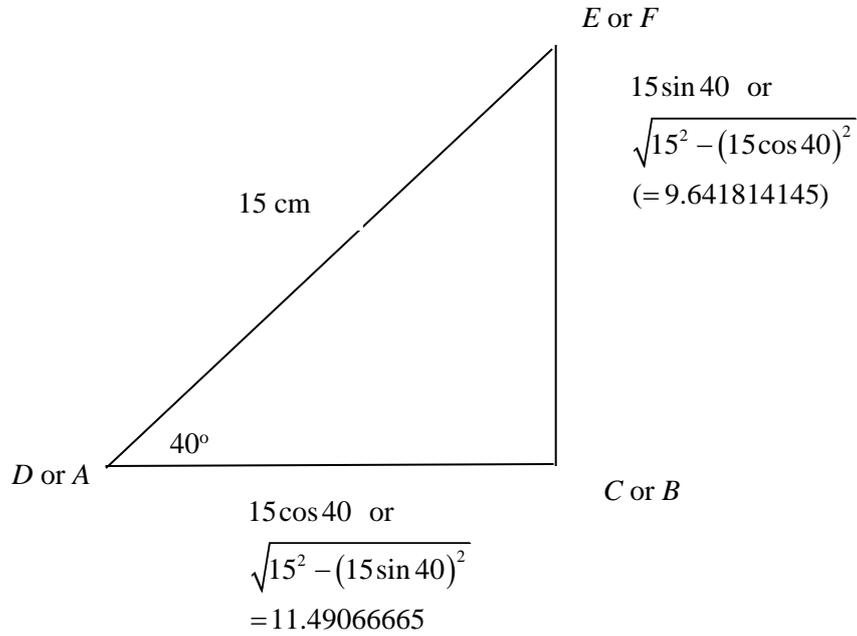
Using the acute angle for B



Question number	Scheme	Marks
Remember to look for work on the diagram.		
4	$(FD) = \sqrt{15^2 + 20^2}$ $(FD =) 25$ $(BF =) 15 \sin 40$ oe $(\Rightarrow (BF) = 9.641814145)$ $\sin BDF = \frac{"15 \sin 40"}{"\sqrt{15^2 + 20^2}"}$ (= 0.3856725658) or $\sin^{-1}\left(\frac{"9.641814145"}{"25"}\right)$ $(BDF) = 22.7^\circ$	M1 A1 M1 M1 dM1 A1 [6]
Example of an ALT	$(DC =) 15 \times \cos 40$ and $(BD =) \sqrt{20^2 + ("15 \cos 40")^2}$ $(BD =) \text{awrt } 23.1$ $(DC =) 15 \times \cos 40$ and $(FB =) \sqrt{15^2 - (15 \cos 40)^2}$ oe $(\Rightarrow (FB) = 9.641814145)$ $\tan BDF = \frac{"\sqrt{15^2 - (15 \cos 40)^2}"}{"\sqrt{20^2 + ("15 \cos 40")^2}"}$ (= 0.4180117433) or $\tan^{-1}\left(\frac{"9.641814145"}{"23.1"}\right)$ $(BDF) = 22.7^\circ$	M1 A1 M1 dM1 M1 A1 [6]
Total 6 marks		

Part	Mark	Additional Guidance
<p>$(BF =) 15 \sin 40$ oe $(\Rightarrow (BF) = 9.641814145)$ means does not need to be labelled BF but there mustn't be anything to contradict this. (General principle of marking)</p> <p>It is fine to label with equivalent sides eg $(EC = FB)$</p> <p>There are a number of ways candidates are completing this question.</p> <p>Please mark to the following general principles – send anything to review that can't be marked using these.</p>		
	M1	For a full and correct method to find FD or BD
	A1	For $FD = 25$ or $BD = \text{awrt } 23.1$
	M1	For a full and correct method to find a second side in triangle BDF
	M1	For correctly identifying the correct angle to be found. Dependent on previous method mark. Look for any clear identification that this is the angle they're finding, including on the diagram or in written work.
	dM1	<p>For the use of their values for two sides in triangle BDF in a correct trig equation or the correct expression to find the angle with the relevant inverse trig function.</p> <p>There must be a full, complete and correct method which leads to an equation or expression to find angle BDF, part methods should not be awarded this mark.</p> <p>Dependent on the previous method mark only ie the correct angle identified, which may be implied in a correct equation or expression.</p> <p>If it is clear candidates are using 2 of their values for BF, FD and BD, this mark can be awarded, even if the work to calculate either is incorrect.</p> $\sin BDF = \left(\frac{"BF"}{"DF"}\right) \text{ or } \cos BDF = \left(\frac{"BD"}{"DF"}\right) \text{ or } \tan BDF = \left(\frac{"BF"}{"BD"}\right)$ $\text{or } \sin^{-1}\left(\frac{"BF"}{"DF"}\right) \text{ or } \cos^{-1}\left(\frac{"BD"}{"DF"}\right) \text{ or } \tan^{-1}\left(\frac{"BF"}{"BD"}\right)$ <p>If incorrect lengths are found, you may allow $\left(\frac{"BF"}{"DF"}\right) \text{ or } \left(\frac{"BD"}{"DF"}\right) > 1$ for this mark</p> <p>Note: if students choose to use cosine rule, they must additionally use a full and correct method to find a third side of triangle BDF and have either of these</p> $\cos BDF = \frac{DF^2 + BD^2 - BF^2}{2 \times DF \times BD} \text{ or } \cos^{-1}\left(\frac{DF^2 + BD^2 - BF^2}{2 \times DF \times BD}\right)$
	A1	Any answer which rounds to (awrt) 22.7°

Useful triangles and sides.



Question number	Scheme	Marks
5 (a)	$3t^2 - 16t + 5 = 0 \rightarrow (3t - 1)(t - 5) \rightarrow t =$ $t = \frac{1}{3}$ oe $t = 5$	M1 M1 A1 [3]
(b)	$(a \Rightarrow) 6t - 16 > 0$ $t > \frac{8}{3}$ oe	M1 A1 [2]
(c)	$\int_{(1)}^{(4)} (3t^2 - 16t + 5) dt$ $[t^3 - 8t^2 + 5t]_{(1)}^{(4)}$ $(4^3 - 8(4)^2 + 5(4)) - (1^3 - 8(1)^2 + 5(1))$ -42 (distance) = 42	M1 A1 M1 A1 A1 [5]
Total 10 marks		

Part	Mark	Additional Guidance
(a)	M1	For setting $v = 0$ and attempting to solve, see general guidance for definition of a minimum attempt to solve leading to $t =$
	M1	For a correct value of t . Any correct value will imply M2.
	A1	For both correct values of t Both correct values will imply M2 A1
(b)	M1	For an attempt to differentiate and setting their expression > 0 (allow placing $= 0$ as a concession) See general guidance for definition of a valid attempt. Additionally for this mark, no power of t to increase, at least one term differentiated correctly.
	A1	For $t > \frac{8}{3}$ oe Accept rounding to 1 dp or better eg 2.7, 2.67, 2.667 etc Or indication of recurring e.g. $2.\dot{6}$ or 2.6^r or $2.6\dots$ minimum 3 dots
(c)	M1	For an attempt to integrate the given expression for v , see general guidance for the definition of a valid attempt. Additionally, no power of t to decrease. Limits do not need to be present.
	A1	For a correct integration. Limits do not need to be present.
	M1	For correct substitution of limits into their changed expression. Can be any changed expression with a minimum of 2 terms. This mark may be implied by sight of -42 or 42 or $-44 - - 2$ or $44 - 2$. If the final answer is incorrect, must see substitution of both limits correctly at least once. For this mark, students may assume an arbitrary constant (typically $C = 0$) or leave an arbitrary constant in their working and have the equivalent $(4^3 - 8(4)^2 + 5(4) + C) - (1^3 - 8(1)^2 + 5(1) + C)$ Candidates cannot attain this mark for just -44 and -2 without a subtraction.
	A1	For -42 or 42 .
	A1	Correct positive distance given.
Any candidate not showing the step of integration – 0 marks for this part of the question.		

Question number	Scheme	Marks
6(a)	eg $\frac{a}{\sqrt{4\left(1+\frac{bx}{4}\right)}}$ or $\frac{a}{\sqrt{4}\sqrt{\left(1+\frac{bx}{4}\right)}}$ or $\frac{a}{4^{\frac{1}{2}}\left(1+\frac{bx}{4}\right)^{\frac{1}{2}}}$ or $a\left(4^{\frac{1}{2}}\left(1+\frac{bx}{4}\right)^{\frac{1}{2}}\right)^{-\frac{1}{2}}$ or $a\left(4\left(1+\frac{bx}{4}\right)\right)^{-\frac{1}{2}}$ $=\frac{a}{2}\left(1+\frac{bx}{4}\right)^{-\frac{1}{2}}$	M1 A1*cso [2]
(b)	$\left(\frac{a}{2}\right)\left[1+\left(-\frac{1}{2}\right)\left(\frac{bx}{4}\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}\left(\frac{bx}{4}\right)^2+\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{3!}\left(\frac{bx}{4}\right)^3\right]$ oe $\left(\left(\frac{a}{2}\right)\left[1+\left(-\frac{bx}{8}\right)+\frac{3}{128}b^2x^2+\left(-\frac{5}{1024}b^3x^3\right)\right]\right)$ Not necessary - but must be checked if seen All intermediate steps must be checked for errors $\frac{a}{2}-\frac{ab}{16}x+\frac{3ab^2}{256}x^2-\frac{5ab^3}{2048}x^3 \quad P=\frac{a}{2}, Q=-\frac{ab}{16}^*, R=\frac{3ab^2}{256}, S=-\frac{5ab^3}{2048}^*$	M1 M1 A1 A1*cso [4]
(c)	$-\frac{ab}{16}=\frac{128}{5}\cdot\left(-\frac{5ab^3}{2048}\right)$ or $\frac{5}{128}\cdot\left(-\frac{ab}{16}\right)=-\frac{5ab^3}{2048}$ or $-\frac{5ab}{2048}=-\frac{5ab^3}{2048}$ or $-\frac{ab}{16}=-\frac{ab^3}{16}$ $(b^2=1)b=1^*$ $\frac{3a \times 1^2}{256}=\frac{9}{256} \Rightarrow a=3^*$	M1 A1* B1*cso [3]
(d)	$\frac{\sqrt{6}}{2}=\frac{3}{2}\left(1+\frac{x}{4}\right)^{\frac{1}{2}}$ or $\frac{\sqrt{6}}{2}=\frac{3}{\sqrt{4+x}}$ or $\frac{3}{\sqrt{6}}=\frac{3}{\sqrt{4+x}}$ or $\sqrt{6}=\sqrt{4+x}$ oe $\Rightarrow x=(2)$ $\frac{3}{2}-\frac{3 \times 1}{16} \times 2+\frac{3 \times 3 \times 1^{(2)}}{256} \times 2^2-\frac{5 \times 3 \times 1^{(3)}}{2048} \times 2^3$ 1.207	M1 dM1 A1 [3]
Total 12 marks		

Part	Mark	Additional Guidance
(a)	M1	For correctly taking out 4 as a factor (must be explicitly seen as in the mark scheme).
	A1*cs0	For the correct answer, minimum steps as shown, no errors seen or omissions .
(b)	M1	For an attempt to find the binomial expansion for the given expression. <ul style="list-style-type: none"> The expansion must begin with 1 The denominators must be correct (ie. 2! And 3! oe) on the third and fourth terms. The power of x must be correct (ie. Must see $\frac{bx}{4}$, $\left(\frac{bx}{4}\right)^2$ and $\left(\frac{bx}{4}\right)^3$ with the correct corresponding denominators). Simplification not necessary.
	M1	For any two terms correct and unsimplified in the binomial expansion. It is not necessary to see the term in a .
	A1	For a fully correct unsimplified binomial expansion. It is not necessary to see the term in a .
	A1*cs0	For the correct P , Q , R and S , stated explicitly or embedded, with no omissions or errors seen.
<p>It is not necessary to see this step to gain the mark, but if seen, must be checked to ensure no errors:</p> $\left(\frac{a}{2}\right)\left[1 + \left(-\frac{bx}{8}\right) + \frac{3}{128}b^2x^2 + \left(-\frac{5}{1024}b^3x^3\right)\right] \quad \text{oe}$ <p>As must any intermediate steps, as this is a show question.</p>		
(c)	M1	For $(Q) = \frac{128}{5}(S)$ or $\frac{5}{128}(Q) = (S)$ in terms of a and b .
	A1*	For b correct, minimum steps as shown, no errors or omissions.
	B1*cs0	For a correct equation as shown leading to the correct value of a , no errors or omissions. Although B marks are independent of method, this is a show question and this mark won't be awarded if there are errors in the work.
(d)	M1	For equating either form to $\frac{\sqrt{6}}{2}$ or $\frac{3}{\sqrt{6}}$, leading to a value for x (ie an incorrect rearrangement may be seen, but candidate must reach a value of x)
	dM1	For correctly substituting their value of x into their expansion with the given values of a and b , the given expressions for Q and S and their expressions for P and R Dependent on previous method mark. 1.207 will imply this mark.
	A1	For awrt 1.207 (note calculator value is 1.2247...)

Question number	Scheme	Marks
7 (a)	$\frac{1}{4}\pi r^2 + 2xr = 100 \quad \left(100 - \frac{1}{4}\pi r^2 = 2xr\right) \Rightarrow x = \frac{50}{r} - \frac{1}{8}\pi r$ <p>oe for example $\frac{100 - \frac{\pi r^2}{4}}{2r} = \frac{400 - \pi r^2}{8r}$</p> $(P =) \frac{1}{4}(2\pi r) + 4x + 2r \quad \text{oe for example} \quad (P =) \frac{1}{2}\pi r + 2x + 2x + 2r$ $(P =) \frac{1}{2}\pi r + 4\left(\frac{50}{r} - \frac{1}{8}\pi r\right) + 2r \left(= \frac{1}{2}\pi r + \frac{200}{r} - \frac{1}{2}\pi r + 2r\right) \quad \text{oe}$ $P = \frac{200}{r} + 2r \quad *$	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1*cs0 [4]</p>
(b)	$\left(\frac{dP}{dr}\right) = \frac{200}{r^2} + 2$ $0 = \frac{200}{r^2} + 2 \Rightarrow r =$ $(r =) 10$ $\left(\frac{d^2P}{dr^2}\right) = \frac{400}{r^3}$ $r = 10, \frac{d^2P}{dr^2} > 0 \text{ therefore minimum or } \frac{d^2P}{dr^2} = \frac{400}{10^3} \text{ or } \frac{2}{5} > 0 \text{ therefore minimum}$	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 [5]</p>
(c)	$P = \frac{200}{(10)} + 2(10) = 40$	<p>M1 A1 [2]</p>
Total 11 marks		

Part	Mark	Additional Guidance
(a)	M1	Writes the correct (unsimplified) equation for the area and rearranges to give $x =$ The rearrangement may contain up to 2 errors and does not have to be simplified.
	M1	Writes the correct formula or expression for the perimeter. It is not necessary to see $P =$ at this point and the expression does not need to be simplified.
	M1	Correctly substitutes their expression for x into their formula for the perimeter Not a dependent mark but x must be of the form $\frac{a}{r} + b\pi r$ $a, b \neq 0$ P must be of the form $d\pi r + ex + fr$ $d, e, f \neq 0$ Not necessary to see $P =$ for this mark.
	A1*cso	For the given result, minimum steps as shown, with no errors seen. It is not necessary to see the steps in brackets. They are present to help examiners check intermediate work as there shouldn't be incorrect work for this mark. It is necessary to see $P =$ at some point for this mark to be awarded.
(b)	M1	For attempting to differentiate P wrt r At least one term must be fully correct, the other – follow general guidance.
	M1	For setting their derivative (must involve a changed expression and have 2 terms) $= 0$ and a completely correct rearrangement to find the value of r
	A1	For $(r =) 10$, ignore $r = - 10$
	M1	For an attempt to find the second derivative to give an expression of the form $\pm \frac{a}{r^3}$
	A1	For a correct second derivative and correct justification that this is a minimum, with some form of conclusion. All work to be correct for this mark. i.e it is acceptable, as this is a simple substitution to state (if $r = 10$) $\frac{d^2P}{dr^2} > 0$, but if a substitution of $r = 10$ is made, the value for the second derivative must be given as 0.4 Work with $r = - 10$ to find a maximum may be ignored.
ALT final A1	A1	Testing and substituting into the correct first derivative with appropriate values either side of $r = 10$. For correct justification that this is a minimum, with some form of conclusion. All work to be correct for this mark.
(c)	M1	For correct substitution of their r into the given formula for P .
	A1	40

Question number	Scheme	Marks
8 (i) (a)	$(\tan 2A = \tan(A + A) =) \frac{\tan A + \tan A}{1 - \tan A \tan A}$ $= \frac{2 \tan A}{1 - \tan^2 A} \quad *$	M1 A1cso* [2]
(b)	$\tan A - \frac{2 \tan A}{1 - \tan^2 A} = 0$ $\tan A(1 - \tan^2 A) - 2 \tan A = 0$ $\tan A - \tan^3 A - 2 \tan A = 0$ $\tan^3 A + \tan A = 0$ $\tan A(\tan^2 A + 1) = 0$ $\tan A = 0 \quad A = 0, 180$ $(\tan^2 A = -1 \quad \text{no solutions})$	M1 M1 M1 A1, A1 [5]
ALT	$\tan A - \frac{2 \tan A}{1 - \tan^2 A} = 0$ $\tan A \left(1 - \frac{2}{1 - \tan^2 A} \right) = 0 \Rightarrow 1 - \tan^2 A - 2 = 0 \quad \text{or} \quad 2 = 1 - \tan^2 A$ $\tan^2 A = -1$ $\tan A = 0 \quad A = 0, 180$ $(\tan^2 A = -1 \quad \text{no solutions})$	
(ii)	$\cos x \cos \frac{\pi}{6} + \sin x \sin \frac{\pi}{6} (= \sin x)$ $\sin x + \sqrt{3} \cos x = 2 \sin x \Rightarrow \sqrt{3} = \tan x$ $x = -\frac{2\pi}{3}, \frac{\pi}{3}, \frac{4\pi}{3}$	M1 dM1 A1 A1 [4]
Total 11 marks		

Part	Mark	Additional Guidance
(i) (a)	M1	For $\frac{\tan A + \tan A}{1 - \tan A \tan A}$
	A1cso*	For the given result shown with minimum steps as shown in the MS, no errors or omissions seen. Extra steps must be correct and checked.
(b)	M1	For the correct substitution of the given result from part (a).
	M1	For correctly multiplying by $1 - \tan^2 A$ or for sight of a correct single fraction (which could possibly afterwards be simplified incorrectly) and elimination of $1 - \tan^2 A$ as the denominator.
	M1	For a correct rearrangement of their equation, factorising correctly, leading to a quadratic factor and a linear factor = 0
	A1	For $A = 0$ or 180 must follow M3
	A1	For $A = 0$ & 180 with no other values in range. Ignore values outside range. Must follow M3 Note if you see any solutions based on trial and improvement, for this question, this will score 0 marks.
	eg	$\frac{\tan A - \tan^3 A - 2 \tan A}{1 - \tan^2 A} = 0$ is M1 M1 M0 $3 \tan A - \tan^3 A = 0$
ALT	M1	For the correct substitution of the given result from part (a).
	M1	For a correct factorisation and correctly multiplying by $1 - \tan^2 A$
	M1	For a correct rearrangement of their equation leading to $\tan^2 A = P$ $P \neq 0$
	A1	For $A = 0$ or 180 must follow M3
	A1	For $A = 0$ & 180 with no other values in range. Ignore values outside range. Must follow M3 Note if you see any solutions based on trial and improvement, for this question, this will score 0 marks.
(ii)	M1	For correctly using the formula for $\cos(A - B)$ as shown.
	dM1	For simplifying and rearranging to $\tan x = a$, allow errors in rearrangement, a must be an exact value. Dependent on previous method mark.
	A1 A1	A1 for one value or for any correct value(s) with extra value in range. A2 for all three values with no extra values in range. Ignore extra values out of range correct or otherwise.

Part	Mark	Additional Guidance
	M1	For correctly placing the 2 equations equal to each other and any attempt to form a quadratic = 0 in x or y This attempt doesn't need to be correct, but they must reach an unsimplified quadratic
	A1	For the correct $3TQ = 0$
	M1	For any method to solve their quadratic – see general guidance for minimally acceptable attempt. $x = 2$ will imply this mark.
	A1	For $x = 2$
	B1	For $x = 1$ or $x = 4$
	B1	For $x = 1$ and $x = 4$
	ddM1	For $\pi \int_{"1"}^{"2"} (x-1) dx + \pi \int_{"2"}^{"4"} \left(2 - \frac{x}{2}\right)^2 dx$ or $\pi \int_{"1"}^{"2"} (x-1) dx + \frac{1}{3} \pi \times "1" \times ("4" - "2")^2$ Dependent on both previous method marks. Also dependent on having attempted to find the intersection of the curve and the line with the x -axis. Allow their "1" and their "4" limits as long as it's clear the candidate has attempted to find the intersections of the line and curve with the x axis
	M1	For integrating (minimum attempt – see general guidance and also, no power of x to decrease) their terms in their volume of revolution. There must be a minimum of 2 terms. This is not a dependent mark, the mark is given for integration, π and limits do not need to be present for this mark to be awarded. If students have written an incorrect formula for the volume of the cone, this mark may still be awarded for integrating. If expressions have been combined incorrectly, this mark for integration of a minimum of 2 terms may be given.
	M1	For correct substitution of limits into any changed expression. Also not dependent, any substitution of what candidates consider to be their limits into their changed expression for the volume of revolution may be awarded this mark. If students have written an incorrect formula for the volume of the cone, this mark may still be awarded for integrating. If expressions have been combined incorrectly, this mark is for substitution of limits into any changed expression(s), minimum 2 terms, with each limit substituted correctly at least once and may be given.
	A1	For $\frac{7}{6} \pi$ oe

Question number	Scheme	Marks
10 (a)	$\left(3 - 9\left(\frac{4}{9}x + x^2\right) \text{ or } -9\left(-\frac{1}{3} + \frac{4}{9}x + x^2\right)\right)$ $3 - 9\left[\left(x + \frac{2}{9}\right)^2 - \frac{4}{81}\right] \text{ or } -9\left[\left(x + \frac{2}{9}\right)^2 - \frac{4}{81} - \frac{1}{3}\right]$ $\text{or } -1\left[9\left(x + \frac{2}{9}\right)^2 - \frac{36}{81} - 3\right] \text{ or } 3 + (1)\left[-9\left(x + \frac{2}{9}\right)^2 + \frac{36}{81}\right]$ $\frac{31}{9} - 9\left(x + \frac{2}{9}\right)^2 \quad A = \frac{31}{9} \quad B = 9 \quad C = \frac{2}{9} \text{ oe}$	M1 A1 A1 A1 [4]
(b)	„ $\frac{31}{9}$ “	B1ft [1]
(c)	$\alpha + \beta = -\frac{4}{9} \quad \alpha\beta = -\frac{3}{9} \text{ oe}$ $\frac{3\alpha}{\beta} + \frac{3\beta}{\alpha} \left(= \frac{3(\alpha^2 + \beta^2)}{\alpha\beta} \right) = \frac{3((\alpha + \beta)^2 - 2\alpha\beta)}{\alpha\beta}$ $\frac{3\left[\left(-\frac{4}{9}\right)^2 - 2\left(-\frac{3}{9}\right)\right]}{-\frac{3}{9}} \quad \left(= -\frac{70}{9} \right)$ $\left(\frac{3\alpha}{\beta} \times \frac{3\beta}{\alpha} = 9\right)$ $x^2 + \frac{70}{9}x + 9 = 0$ $9x^2 + 70x + 81 = 0$	B1 M1 A1ft B1 M1 A1cso [6]
(d)	$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3 = x^3 + y^3 + 3xy(x + y)$	B1*cso [1]
(e)	$(\alpha^2 - \beta + \beta^2 - \alpha =)$ $(\alpha + \beta)^2 - 2\alpha\beta - (\alpha + \beta) \quad \left((\alpha + \beta)^2 = (\alpha + \beta)^2 - 2\alpha\beta = \frac{70}{81} \right)$ $\left(-\frac{4}{9}\right)^2 - 2\left(-\frac{3}{9}\right) - \left(-\frac{4}{9}\right) \quad \left(= \frac{106}{81} \right)$ <p>candidates may also transfer a value for $\alpha^2 + \beta^2$ from working in part (c)</p> $\left[(\alpha^2 - \beta)(\beta^2 - \alpha) = \alpha^2\beta^2 - \beta^3 - \alpha^3 + \alpha\beta \right]$ $= (\alpha\beta)^2 + \alpha\beta - (\alpha^3 + \beta^3) \text{ or } (\alpha\beta)^2 + \alpha\beta - \alpha^3 - \beta^3$ $\left[(\alpha + \beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3 = \alpha^3 + \beta^3 + 3\alpha\beta(\alpha + \beta) \right]$ $= (\alpha\beta)^2 + \alpha\beta - (\alpha + \beta)^3 + 3\alpha\beta(\alpha + \beta) \text{ or } (\alpha\beta)^2 + \alpha\beta - \left((\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) \right)$ $\left[= \left(-\frac{3}{9}\right)^2 + \left(-\frac{3}{9}\right) - \left(-\frac{4}{9}\right)^3 + 3\left(-\frac{3}{9}\right)\left(-\frac{4}{9}\right) \left(= \frac{226}{729} \right) \right]$ $\frac{106}{81} = -\frac{q}{3} \quad \text{or} \quad \frac{226}{729} = \frac{r}{3}$ $q = -\frac{106}{27} \quad \text{and} \quad r = \frac{226}{243}$	M1 A1ft M1 M1 dddM1 A1 [6]

Total 18 marks

Part	Mark	Additional Guidance
(a)	M1	For a method to complete the square to achieve as a minimum $3 \pm 9 \left[\left(x \pm \frac{2}{9} \right)^2 \pm p \right]$ or $\pm 9 \left[\left(x \pm \frac{2}{9} \right)^2 \pm q \right]$ or $\pm 1 \left[9 \left(x \pm \frac{2}{9} \right)^2 \pm r \right]$ or $3 \pm (1) \left[-9 \left(x \pm \frac{2}{9} \right)^2 + s \right]$ where p, q, r or s are constants > 0 .
	A1	For one of A, B or C correct.
	A1	For two of A, B or C correct.
	A1	For all of A, B or C correct For all accuracy marks, A, B and C may be either explicitly stated or embedded and if embedded correctly then stated incorrectly, we may isw. Correct A, B and C will imply 4 marks. 1 or 2 values correct, will need M1 to be gained first.
ALT	M1	For an attempt to expand $A - B(x + C)^2$ AND equate coefficients to the given $f(x) \Rightarrow A - Bx^2 - 2BCx - BC^2 = 3 - 4x - 9x^2$ Allow $A \pm Bx^2 \pm 2BCx \pm BC^2$ for the expansion of $A - B(x + C)^2$ Must be an attempt to correctly equate at least one coefficient using their expansion. Eg $-B = -9 \Rightarrow B = \dots$ $-2BC = -4 \Rightarrow C = \dots$ $A - BC^2 = 3 \Rightarrow A = \dots$
	A1A1A1	As main scheme.
(b)	B1ft	For their $\frac{31}{9}$ Note, strict 'hence' question, if $\frac{31}{9}$ is written with incorrect work in (a), B0. It is the candidate's final answer in part (a) which will be used to mark part (b) ft must be from $A - B(x + C)^2$ If $\frac{31}{9}$ is written with no work in (a), award the mark.
(c)	B1	Correct values for $\alpha + \beta$ and $\alpha\beta$. Award if and look for values not seen explicitly but embedded in the sum/product calculations for the new equation.
	M1	Reaches a correct expression ready for substitution of their values of $\alpha + \beta$ and $\alpha\beta$
	A1ft	Correctly substitutes their values for $\alpha + \beta$ and $\alpha\beta$ into a correct expression for the sum of roots. Simplification is not necessary.
	B1	For 9
	M1	Use $x^2 - (\text{their sum of their roots})x + \text{their product of roots}$ " $= 0$ " may be missing. This mark is not dependent and may be awarded for a clear substitution of anything we can see is their product and sum of roots.
	A1cso	For the given equation oe with integer coefficients. Must have $= 0$.
<p>Note. It is possible to get the correct equation from using $\alpha + \beta = +\frac{4}{9}$</p> <p>This will always achieve final A0 as it's a correct equation from incorrect working.</p>		

(d)	B1*cso	Complete and full algebra to show the given identity. Minimum steps as shown in MS, no errors or omissions. Extra steps checked. For this question, where students have chosen to do full expansion of brackets, we will allow them to recover brackets if following work is correct.
(e)		Look carefully for students transferring their work in (c) on $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ as this can get the first 2 marks in (e), implied M1, A1ft. This work must be correct for their original sum and product of roots and must be shown. Examiners are not expected to check work if their substitution isn't shown.
	M1	Reaches a correct expression ready for substitution of their values of $\alpha + \beta$ and $\alpha\beta$
	A1ft	Correctly substitutes their values for $\alpha + \beta$ and $\alpha\beta$ into a correct expression for sum of roots.
	M1	For the correct expression shown. Any equivalent, ready for substitution of $\alpha\beta$
	M1	For the correct expression shown. Any equiv ready for sub of $\alpha\beta$ and $\alpha + \beta$ For this mark and the previous M mark, we're looking for expressions preparing for substitution of candidates' values. You may see completion of this work in stages, eg when the values are subbed in later. Eg, you may see $\alpha^2\beta^2 + \alpha\beta - (\alpha^3 + \beta^3) \rightarrow \alpha^2\beta^2 + \alpha\beta - (\alpha + \beta)^3 + 3\alpha\beta(\alpha + \beta)$ then $\left(-\frac{3}{9}\right)^2 + \left(-\frac{3}{9}\right) - \left(-\frac{4}{9}\right)^3 + 3\left(-\frac{3}{9}\right)\left(-\frac{4}{9}\right)$ which is M2, $(\alpha\beta)^2$ is implied.
	dddM1	For equating their sum of roots to $-\frac{q}{3}$ or their product of roots to $\frac{r}{3}$. Dep on all method marks. This mark may sometimes be implied by a correct p or q
	A1	For both correct values for q and r

In general, A marks (or M marks) cannot come from incorrect working (though a correct method followed by incorrect simplification can usually go on to gain M marks).

In this question, you are looking for any of the forms given in the notes, at any point (and disregarding incorrect work around it) to award M1.

Only when this is awarded, you may award following A marks. The exception being a fully correct answer may be awarded 4 marks without M1 being present.

