Pearson Edexcel

Mark Scheme (Results)

Summer 2025

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1) Paper 02R

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks – can only be awarded when relevant M marks have been gained
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - cso – correct solution only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**
  If no working is shown, then correct answers may score full marks
  If no working is shown, then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

  If a candidate misreads a number from the question: e.g. uses 252 instead of 255; follow through their working and deduct 2A marks from any gained provided the work has not been simplified. (Do not deduct any M marks gained.)

  If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used

  Examiners should send any instance of a suspected misread to review (but see above for simple misreads).

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: e.g. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect e.g. algebra.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. <u>Factorisation</u>:

   $\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right)$ where $|pq|=|c|$ leading to $x=\dots$

   $\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=\dots$

2. <u>Formula</u>:

   Attempt to use the **correct** formula (shown explicitly or implied by working) with values for a, b and c, leading to $x=\dots$

3. <u>Completing the square:</u>

   $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=\dots$

Method marks for differentiation and integration:

1. <u>Differentiation</u>

   Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. <u>Integration:</u>

   Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either**
quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values
or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks". General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this e.g. in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - i.e. giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

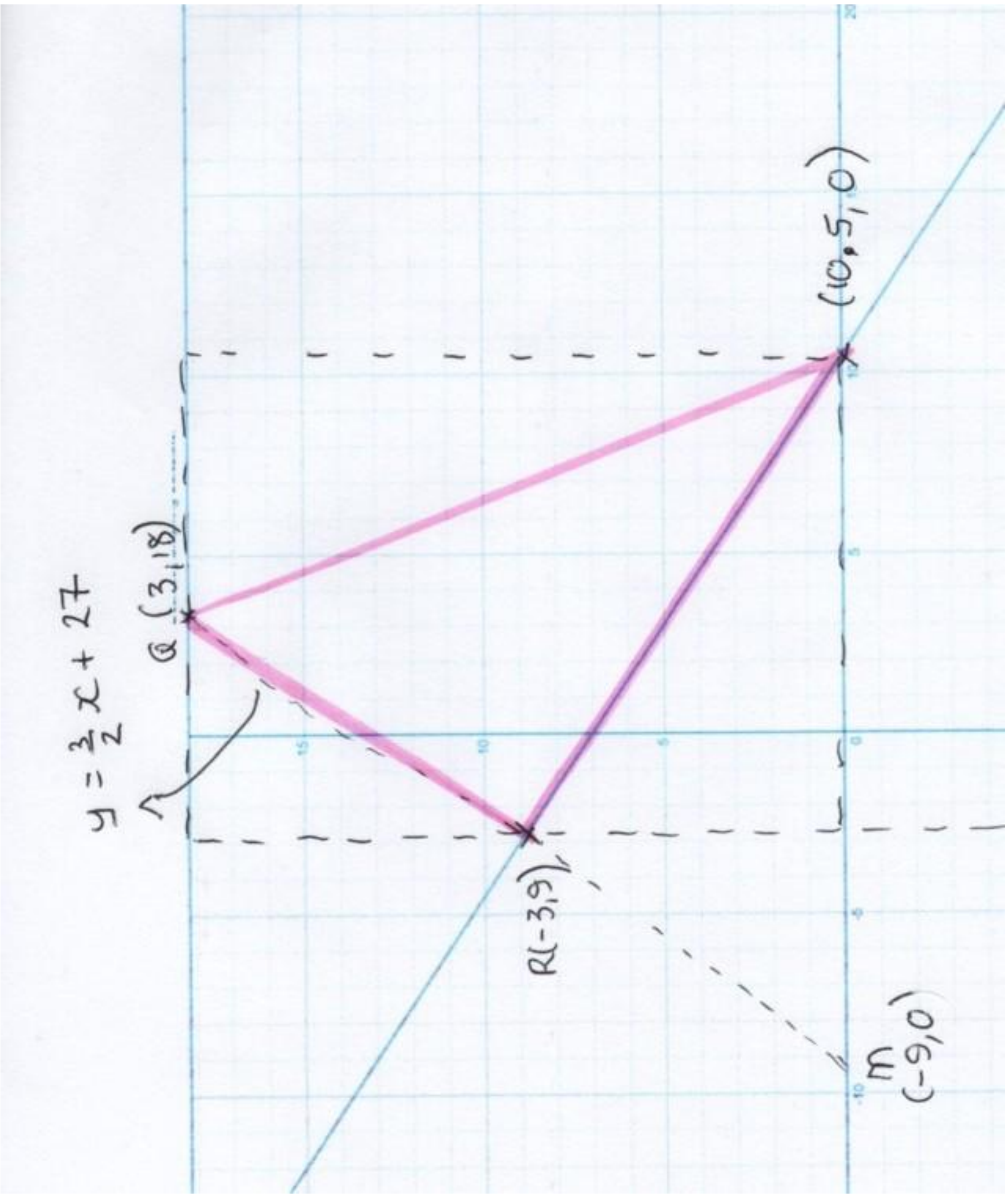
Question number	Scheme	Marks
1	$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC} = -3\mathbf{a} - 2\mathbf{b} + 5\mathbf{a} - 3\mathbf{b}$ oe $\overrightarrow{DC} = \overrightarrow{DA} + \overrightarrow{AC} = -2\mathbf{a} + 5\mathbf{b} + 5\mathbf{a} - 3\mathbf{b}$ oe	M1 M1
	$\overrightarrow{BC} = 2\mathbf{a} - 5\mathbf{b}$ $\overrightarrow{DC} = 3\mathbf{a} + 2\mathbf{b}$ $\overrightarrow{BC} = \overrightarrow{AD}$ therefore parallelogram * or $\overrightarrow{DC} = \overrightarrow{AB}$ therefore parallelogram *	A1 A1 cso* [4]
		Total 4 marks

Part	Mark	Additional Guidance
	M1	For a correct vector path for at least one of $\overrightarrow{BC}$ or $\overrightarrow{DC}$ or $\overrightarrow{CB}$ or $\overrightarrow{CD}$ Eg $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$ or $\overrightarrow{DC} = \overrightarrow{DA} + \overrightarrow{AC}$ This mark may be implied by the next method mark – ie by correctly adding vectors in terms of **a** and **b**. Accept poor notation Eg for $\overrightarrow{BC}$ accept BC or $\lvert \overrightarrow{BC} \rvert$ or $\lvert BC \rvert$
	M1	For a correct unsimplified vector in terms of **a** and **b** for $\overrightarrow{BC}$ or $\overrightarrow{CB}$ or $\overrightarrow{DC}$ or $\overrightarrow{CD}$ Don't accept $\overrightarrow{BC} = +3\mathbf{a} + 2\mathbf{b} - 5\mathbf{a} + 3\mathbf{b}$ (as this is $\overrightarrow{CB}$) **Don't accept unlabelled vectors** Accept poor notation Eg for $\overrightarrow{BC}$ accept BC or $\lvert \overrightarrow{BC} \rvert$ or $\lvert BC \rvert$
	A1	For a correct simplified vector in terms of **a** and **b** for $\overrightarrow{BC}$ or $\overrightarrow{CB}$ or $\overrightarrow{DC}$ or $\overrightarrow{CD}$ In order to gain this accuracy mark we must see the first **or** second M mark (ie the adding together of the paths in vector notation or in terms of the vectors in **a** and **b**) **Do not allow poor notation.** **A correct simplified vector alone does not imply previous marks.**
	A1 cso*	For a correct simplified vector in terms of **a** and **b** for $\overrightarrow{BC}$ or $\overrightarrow{CB}$ or $\overrightarrow{DC}$ or $\overrightarrow{CD}$ and a clear (brief) conclusion. Eg $\overrightarrow{BC} = -\overrightarrow{DA}$ $\overrightarrow{BC} = \overrightarrow{AD}$ $\overrightarrow{DC} = -\overrightarrow{BA}$ $\overrightarrow{DC} = \overrightarrow{AB}$ shown or # * or Candidates may also work with 2 correctly paired vectors and **state *DC* = *AB* and *BC* = *AD*** or $\overrightarrow{DC}$ **is parallel to** $\overrightarrow{AB}$ **and** $\overrightarrow{BC}$ **is parallel to** $\overrightarrow{AD}$ to gain this mark, with also a brief conclusion, if all their work is correct for the vectors involved. Minimum steps shown, **ignore any incorrect vectors** that they are not using to draw their conclusion.

Question number	Scheme	Marks
2 (a)	$(-3,9)$	B1
	$\left(m_{PQ}=\right)\dfrac{18-3}{3--7}\left(=\dfrac{3}{2}\right)$ oe	M1
	$\left(m_l=\right)=-\dfrac{1}{"\dfrac{18-3}{3--7}"}=\left(-\dfrac{1}{"\dfrac{3}{2}"}\right)\left(=-\dfrac{2}{3}\right)$ oe	M1
	$y-"9"="-\dfrac{2}{3}"(x+"3")$	M1
	$3y+2x-21=0$ oe *eg* $-6y-4x+42=0$	A1 [5]
(b)	$3\times 0+2x-21=0\Rightarrow x=(10.5)$	M1
	$(QR^2=)(3-"-3")^2+(18-"9")^2\,(=117)$ or $(QR=)\sqrt{(3-"-3")^2+(18-"9")^2}\,\left(=3\sqrt{13}\right)$	M1
	$(RS^2=)("10.5"-"-3")^2+(0-"9")^2\left(=\dfrac{1054}{4}\right)$ or $(RS=)\sqrt{("10.5"-"-3")^2+(0-"9")^2}\left(=\dfrac{9\sqrt{13}}{2}\right)$	M1
	$\dfrac{1}{2}\times "3\sqrt{13}"\times"\dfrac{9\sqrt{13}}{2}"$	ddM1
	$\dfrac{351}{4}$ oe (87.75)	A1 [5]
ALT1	$3\times 0+2x-21=0\Rightarrow x=(10.5)$	M1
	$\left(\dfrac{1}{2}\right)\begin{vmatrix}3 & "-3" & "10.5" & 3\\ 18 & "9" & 0 & 18\end{vmatrix}$ oe eg $\left(\dfrac{1}{2}\right)\begin{vmatrix}"10.5" & "-3" & 3 & "10.5"\\ 0 & "9" & 18 & 0\end{vmatrix}$ oe	M1 M1
	$\dfrac{1}{2}\left((3\times 9+-3\times 0+10.5\times 18)-(3\times 0+10.5\times 9+-3\times 8)\right)$ oe	ddM1
	$\dfrac{351}{4}$ oe (87.75)	A1
	Finding the area of a rectangle and subtracting area of triangles	
ALT2a	$3\times 0+2x-21=0\Rightarrow x=0$	M1
	$("10.5"-"-3")\times 18=243$	M1
	$\dfrac{1}{2}\times(18-"9")\times(3-"-3")(=27)$ $\dfrac{1}{2}\times("10.5"-3)\times(18(-0))\left(=\dfrac{135}{2}\right)$ $\dfrac{1}{2}\times("10.5"-"-3")\times("9"(-0))\left(=\dfrac{243}{4}\right)$	M1 M1
	$243-"27"-"\dfrac{135}{2}"-"\dfrac{243}{4}"$	ddM1
	$\dfrac{351}{4}$ oe (87.75)	A1

ALT2b	Let M be the intersection of PQ with the x-axis $3\times 0+2x-21=0\left(\Rightarrow x=10.5\right)$ $\left(y=\frac{3}{2}x+13.5\Rightarrow\ x=\right)-9$ $\left(\text{Area of triangle } SQM\ =\right)\frac{1}{2}\times\left(\text{"}10.5\text{"}-\text{"}-9\text{"}\right)\times 18\left(=\frac{351}{2}\right)$ $\left(\text{Area of triangle } SRM\ =\right)\frac{1}{2}\times\left(\text{"}10.5\text{"}-\text{"}-9\text{"}\right)\times 9\left(=\frac{351}{4}\right)$ $\text{"}\frac{351}{2}\text{"}-\text{"}\frac{351}{4}\text{"}$ $\frac{351}{4}$ oe (87.75)	M1 M1 M1 M1 ddM1 A1
ALT3	Usually with a good diagram, the candidate can work out that triangles *SRM* and *QRS* are congruent (SAS) If spotting this, this is full marks or 0 marks (because without the correct coordinates, this is not so) $\frac{1}{2}\times 19.5\times 9=\frac{351}{4}$	M1M1 M1M1 ddM1A1
		Total 10 marks

Part	Mark	Additional Guidance
(a)	B1	For $(-3,9)$
	M1	For the method for gradient of *PQ* in unsimplified form
	M1	For finding the negative reciprocal of their gradient of *PQ*, does not need to be simplified. This mark is not dependent, so may be awarded for finding the negative reciprocal of any gradient believed to be their gradient of *PQ*
	M1	For a full and correct method to find the equation of the line. Doesn't need to be simplified. Ft their grad for l, so long as it involves a changed gradient from *PQ* and ft their point *R*. If using $y = mx + c$, a full method includes correct rearrangement to find c. If the equation is not the correct one and they're working with their values, this rearrangement must be shown or check their c is correct for their values. $c = 7$ implies this mark. Full equation needs to be seen.
	A1	For the equation shown oe integer coefficients.
(b)	M1	For substituting $y = 0$ into their equation for l and correctly solving to find a value of x This mark may be implied by $(x =)$ 10.5
	M1	For a correct method to find *QR* or QR^2 using their coordinates for point *R*
	M1	For a correct method to find *RS* or RS^2 using their coordinates for point *R* and their value for the x coordinate of *S*
	ddM1	For a correct method to find the area of the triangle. Dependent on both previous method marks.
	A1	Correct answer as shown.
ALT1	M1	For substituting $y = 0$ into their equation for l and solving to find a value of x This mark may be implied by $(x =)$ 10.5
	M1	For a correct matrix array, condone round or brackets, using their coordinates for *R* and their 10.5, the 0.5 doesn't need to be present. Be aware of all the equivalents, including reverse order or 'upside down'. May be implied by correct working
	M1	For a correct determinant array, using their coordinates for *R* and their 10.5, with 0.5 outside. May be implied correct working.
	ddM1	For a correct calculation of the determinant do need to see multiplication by 0.5. May be implied by a correct answer. If the candidate is using their values rather than the correct values, they must show the products (or at the minimum each product evaluated) and subtraction correctly. Dependent on both previous method marks.
	A1	Correct answer as shown.
ALT2a	M1	For substituting $y = 0$ into their equation for l and solving to find a value of x This mark may be implied by $(x =)$ 10.5
	M1	For a correct method to find the area of the rectangle, using their *R* and their 10.5.
	M1	For a correct method to find the area of the surrounding triangles
	ddM1	For a correct method to find the area of triangle *QRS*
	A1	Correct answer as shown.
ALT2b	M1	For substituting $y = 0$ into their equation for l and solving to find a value of x This mark may be implied by $(x =)$ 10.5
	M1	For -9 This is the intersection of *PQ* with the *x-axis* The mark is for the number **not** the method
	M1	For a correct method to find the area of triangle *SQM*
	ddM1	For a correct method to find the area of triangle *SPM*
	A1	Correct answer as shown.

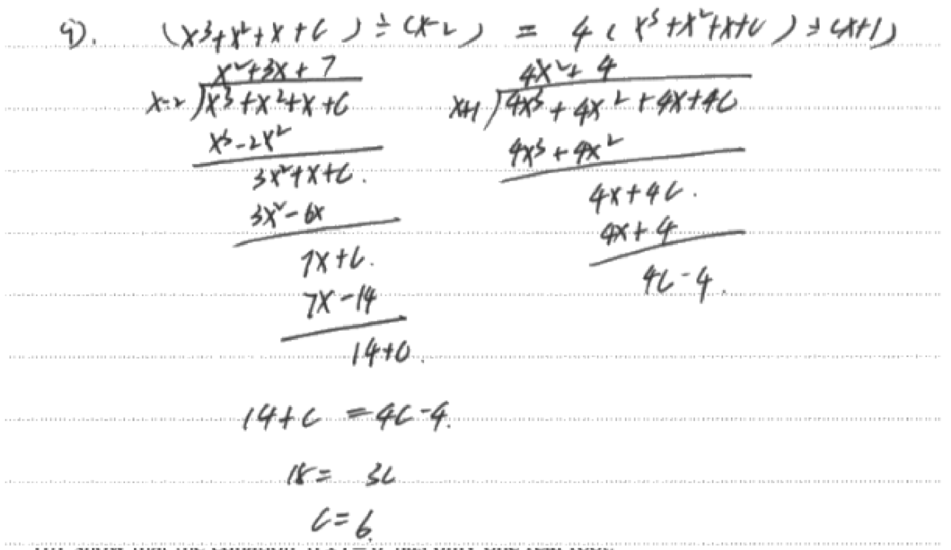
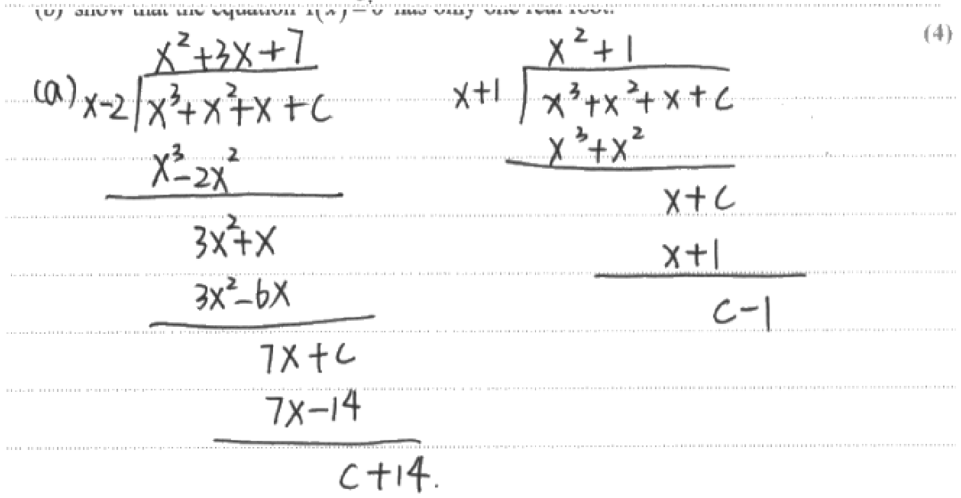

y = 3/2 x + 27
Q (3,18)
(10.5,0)
R(-3,9)
m
(-9,0)

Question number	Scheme	Mark
3	$(l = r\theta =)6\theta$ or $\left(l = \frac{\theta}{360}\times 2\pi r =\right)\frac{\theta}{360}\times 2\pi\times 6$ oe	M1
	$6+6+"6\theta"=12+\pi$ or $6+6+"\frac{\theta}{360}\times 2\pi\times 6"=12+\pi$ oe	M1
	$(6\theta=\pi)\Rightarrow \theta=\frac{\pi}{6}$ or $\theta=30^\circ$	A1
	$\left(A=\frac{1}{2}r^2\theta-\frac{1}{2}r^2\sin\theta=\right)\frac{1}{2}6^2"\frac{\pi}{6}"-\frac{1}{2}6^2\sin"\frac{\pi}{6}"$ *or* $\frac{"30"}{360}\times\pi\times 6^2-\frac{1}{2}6^2\sin"30"$	M1 M1
	$=3\pi-9$ oe	A1 [6]
		Total 6 marks

Part	Mark	Additional Guidance
Students may work in radians or degrees. If using both, marks may be given for accurate use of formulas.		
	M1	Correct use of formula for length of an arc.
	M1	Allow use of their length of arc, ie for following through their length of arc and equating an equation of the form $6+6+a\theta=12+\pi$ or $6+6+\frac{\theta}{360}\times 2\pi\times a=12+\pi$ a doesn't have to be an integer, can be irrational. This may be done in stages. Take care to look out for equivalences, where stages have been missed, this is not a show that question. For example, $"6\theta"=\pi$ would imply both method marks as the candidate could have realised the 12 cancels from each side. As can often be the case, a correct angle will imply method marks (unless incorrect method shown). General marking principle.
	A1	$\theta=\frac{\pi}{6}$
	M1	Correct use and substitution into the formula for area of sector using their θ
	M1	Correct use and substitution into the of formula for area of triangle, using their θ
	A1	$3\pi-9$ oe (correct decimal is 0.4247…. and is likely to imply correct method)

Question number	Scheme	Marks
4 (i)	$a=3$ or $r=2$	B1
	$\left(\sum_{k=1}^{17}3(2)^{k-1}\right)=\frac{3\left(1-(2)^{17}\right)}{1-2}=(393213)$ $\quad \left(\sum_{k=1}^{5}3(2)^{k-1}\right)=\frac{3\left(1-(2)^{5}\right)}{1-2}=(93)$	M1
	"393213" − "93"	dM1
	393120	A1 [4]
ALT	$a=3\times2^5\,(=96),\,r=2$	B1
	$\left(\sum_{k=6}^{17}3(2)^{k-1}\right)=\frac{\left(3\times2^5\right)\left(1-(2)^{\text{"11 or 12"}}\right)}{1-2}$	M1
	$\frac{\left(3\times2^5\right)\left(1-(2)^{12}\right)}{1-2}$	M1
	393120	A1 [4]
(ii)	$ar^2=\frac{7}{13}$ or $ar^8=\frac{448}{9477}$	M1
	$\frac{ar^8}{ar^2}=\frac{\frac{448}{9477}}{\frac{7}{13}}\left(=\frac{64}{729}\right)\Rightarrow r^6=$	dM1
	$r=\sqrt[6]{\left(\frac{\frac{448}{9477}}{\frac{7}{13}}\right)}$	M1
	$r=\frac{2}{3}$ $\quad a=\frac{63}{52}$	A1 A1
	$S_\infty=\frac{\text{"}\frac{63}{52}\text{"}}{1-\text{"}\frac{2}{3}\text{"}}$	M1
	$=\frac{189}{52}$ oe	A1 [7]
		Total 11 marks

Part	Mark	Additional Guidance
(i)	B1	For either value of a or r correct, stated or implicit in working.
	M1	For correctly substituting into the formula for either sum. Allow use of their a and their r with any numbers it is clear they intend to be a and r
	dM1	For their 2 sums subtracted, the correct way round. Dependent on the previous method mark. **They must be using $a = 3$ for this mark.**
	A1	For 393120
ALT	B1	For either value of a or r correct, stated or implicit in working.
	M2	For correctly substituting into the formula for the sum to n terms. **Must be using $n = 12$ and $a = 96$, may use their r.** For M1 students may use $n = 11$ or 12 and allow use of their a and their r with any numbers it is clear they intend to be a and r
	A1	For 393120
	Students who complete this by adding every term B1 for the first term correct M2 for all terms written out and added (we are not expected to evaluate them), there is no M1 mark A1 correct answer and as usual, can imply the M and B marks. **A fully correct answer with little or no working will score 4 marks**	
(ii)	M1	For either correct equation.
	dM1	For correctly using their equations to eliminate a (or r) We will condone $\frac{ar^9}{ar^3}\left(=\frac{\frac{448}{9477}}{\frac{7}{13}}=\frac{64}{729}\right) \Rightarrow r^6 =$ For this mark if the candidate works correctly to find r
	M1	Rearranges their equation, allowing one processing error, to find a value for r (or a). Do not award the mark if a student moves from r^6 to r without showing what they are doing if r is incorrect. We will condone correct r from $\frac{ar^9}{ar^3}\left(=\frac{\frac{448}{9477}}{\frac{7}{13}}=\frac{64}{729}\right) \Rightarrow r =$ if the candidate works correctly to find r
	A1	Either r or a correct.
	A1	Both r and a correct.
	M1	Correct substitution of their values for a and r into a correct formula for S_∞
	A1	$\frac{189}{52}$ oe

Question number	Scheme	Marks
5 (a)	$4[(-1)^3+1^2+(-1)+c]=2^3+2^2+2+c$ $-4+4c=14+c$ oe $c=6$ *	M1 M1 ddM1 A1*cso [4]
	  	
(b)	$\big[(x+2)\big](x^2+Ax+3)$ or $\big[(x+2)\big](x^2-x+B)$ $\big[(x+2)\big](x^2-x+3)$ $\left(b^2-4ac=\right)(-1)^2-4(1)(3)<0$ therefore only one (real) root * or $\left(x-\dfrac{1}{2}\right)^2=-\dfrac{11}{4}$ and $-\dfrac{11}{4}<0$ therefore only one (real) root	M1 A1 M1 A1*cso [4]
		Total 8 marks

Part	Mark	Additional Guidance
(a)	M1	For correct substitution of – 1 **or** 2 into f(x)
	M1	For correct substitution of – 1 **and** 2 into f(x)
		For the first two marks: Where candidates employ simultaneous equations, mark the work for the substitution Accept eg -1 for $(-1)^3$ 4 for 2^2 Allow – 1 + c **with no errors** to imply substitution of – 1 Allow 14 + c **with no errors** to imply substitution of 2
	ddM1	For $4[(-1)^3+1^2+(-1)+c]=2^3+2^2+2+c$ or condone the 4 on the 'wrong side'. $(-1)^3+1^2+(-1)+c=4[2^3+2^2+2+c]$ Dependent on both previous method marks. For forming simultaneous equations eg $4R = 14 + c$ $R = -1 + c$ Condone $R = 14 + c$ $4R = -1 + c$ The work doesn't have to be simplified for this mark.
	A1*cso	Correct solution, no errors or omissions, minimum steps as shown. The steps will need to be checked. Note there are different notations for long division.
		For a route that uses long division – examples in mark scheme. M1 For completing a correct long division, giving a correct remainder M1 for completing 2 correct long divisions, giving both correct remainders ddM1 for the correct equation or condone the 4 on the wrong side. A1 Correct solution, no errors or omissions, minimum steps as shown. The steps will need to be checked.
(b)	M1	For $[(x+2)](x^2+Ax+3)$ or $[(x+2)](x^2-x+B)$ **using any method** – there must be a quadratic to award this mark. Mark the quadratic seen, not the method.
	A1	For x^2-x+3
	M1	For calculating the discriminant correctly for their 3 term quadratic. If working with the correct quadratic – 11 will imply this mark. If not, they must show at least partial substitution (their b^2 and $4ac$ evaluated will be fine) And condone/allow $(1)^2-4(1)(3)<0$ since $(-1)^2=(1)^2$ Or full correct completion of square and $-\frac{11}{4}<0$ oe. The discriminant may be calculated in the quadratic formula.
	A1*cso	Fully correct solution and short conclusion State < 0 and (can be "no (real) roots", (x =) "– 2 is the only (real) root", "shown" or #), no errors. Minimum steps as shown.
		Send any work on complex numbers to Review.

Question number	Scheme		Marks
6	Correctly identifies the angle *VXO* where *X* is the midpoint of *CD* and *O* is the foot of the perpendicular from *V* oe		B1
	If y is the length of *AD*. $VC = 3y \quad CX = \dfrac{y}{2}$ $OX = \dfrac{3y}{2}$		M1
	$(VX) = \sqrt{(3y)^2 - \left(\dfrac{y}{2}\right)^2}$	$(OC) = \sqrt{\left(\dfrac{3y}{2}\right)^2 + \left(\dfrac{y}{2}\right)^2} = \left(\dfrac{\sqrt{10}}{2}y\right)$ and	dM1
	$VX = \dfrac{\sqrt{35}y}{2}$ oe	$VO = \sqrt{(3y)^2 - \left("\dfrac{\sqrt{10}}{2}y"\right)^2} = \left(\dfrac{\sqrt{26}}{2}y = \sqrt{\dfrac{13}{2}}y\right)$	A1
	$\cos\theta = \dfrac{\frac{3y}{2}}{"\frac{\sqrt{35}y}{2}"}\left(= \dfrac{OX}{VC} = 0.507062.....\right)$ oe eg $\tan\theta = \dfrac{"\frac{\sqrt{26}}{2}y"}{\frac{3y}{2}}\left(= \dfrac{OV}{OX} = 1.699673....\right)$ $\sin\theta = \dfrac{"\frac{\sqrt{26}}{2}y"}{"\frac{\sqrt{35}y}{2}"}\left(= \dfrac{OV}{VC} = 0.861891.......\right)$ (Cosine rule) $\cos\theta = \dfrac{\left("\frac{\sqrt{35}y}{2}"\right)^2 + (3y)^2 - \left("\frac{\sqrt{35}y}{2}"\right)^2}{2\times\frac{\sqrt{35}}{2}y\times 3y}$ oe $\cos\theta = \dfrac{\left("\frac{\sqrt{35}y}{2}"\right)^2 + \left(\frac{3y}{2}\right)^2 - \left("\frac{\sqrt{26}y}{2}"\right)^2}{2\times\frac{\sqrt{35}}{2}y\times 3y}$ $\left[\cos\theta = \dfrac{3}{\sqrt{35}} \quad \text{oe} \quad \text{leading to } \theta =\right]$		M1
	59.5		A1 [6]
			Total 6 marks

Part	Mark	Additional Guidance
	B1	Angle identified in written work **or on diagram**. Allow labelling to be any letters. This mark should be permitted for any work where it is clear the candidate is identifying or using the correct angles. If ambiguous or unclear, cannot be awarded.
	M1	Denotes correct and appropriate letters for **any two** relevant lengths on the pyramid to make the next stage of progress. The sides are shown in the MS, but we will accept sides with different labels elsewhere in the triangle. Eg $DX = \frac{y}{2}$ is the same as CX This can be implied by written work or on the diagram. The two sides can be any **values** or expressions in the correct ratio which are necessary to form part of the working. Watch out for equivalences. Eg $AD = VC = y$ $DC = \frac{y}{3}$ $CX = \frac{y}{6}$ or $AD = VC = 9$ $DC = \frac{9}{3}$ $CX = \frac{9}{6}$ May be implied by correct working. For this mark, the sides used must have the correct algebraic expressions, (including multiples of those shown) or numbers in the correct proportion.
	dM1	Correct full method to find *VX* or *VO*. Correct full method means they can't use a side from an incorrect method in another method for *VX* or *VO* (so for example if they use an incorrect method to find *VX* and then use this to find *VO* they can't have the mark for finding *VO*) Dependent on the previous mark.
	A1	Correct expression **or values** for *VO* or *VX* ie all sides that they will be using.
	M1	Working in triangle *VXO* (or other valid triangle) with their values or expressions from previous working, using any appropriate trigonometry. The candidate can use sides which are labelled with the correct lengths for the trig ratio used, even if calculated incorrectly in previous work.
	A1	awrt 59.5 (59.52964053) Note: 60 is likely to have come from an incorrect angle identified And it is possible to get an answer close without a correct method. The method must always be checked through (in any question)

Question number	Scheme	Marks
7 (a)	$2x^2y$	M1
	$72 = 2x^2y \Rightarrow y = \frac{36}{x^2}$ oe or $xy = \frac{36}{x}$ oe	M1
	$(A =) 2\times 2xy + 2\times xy + 2\times 2x^2 \left(= 6xy + 4x^2\right)$	
	$= 6x\left(\frac{36}{x^2}\right) + 4x^2$ or $= 6\left(\frac{36}{x}\right) + 4x^2$	dM1
	$A = \frac{216}{x} + 4x^2$ *	A1*cso [4]
(b)	$\left(\frac{dA}{dx} =\right) - \frac{216}{x^2} + 8x$	M1
	$0 = -\frac{216}{x^2} + 8x \rightarrow x =$ or $x^3 =$	dM1
	$x = 3$	A1
	$(A =) \frac{216}{3} + 4\times 3^2$	dM1
	$= 108$	A1
	$\left(\frac{d^2A}{dx^2} =\right) \frac{432}{x^3} + 8$	M1
	$= \frac{432}{3^3} + 8 (= 24) > 0$ therefore minimum	A1 [7]
		Total 11 marks

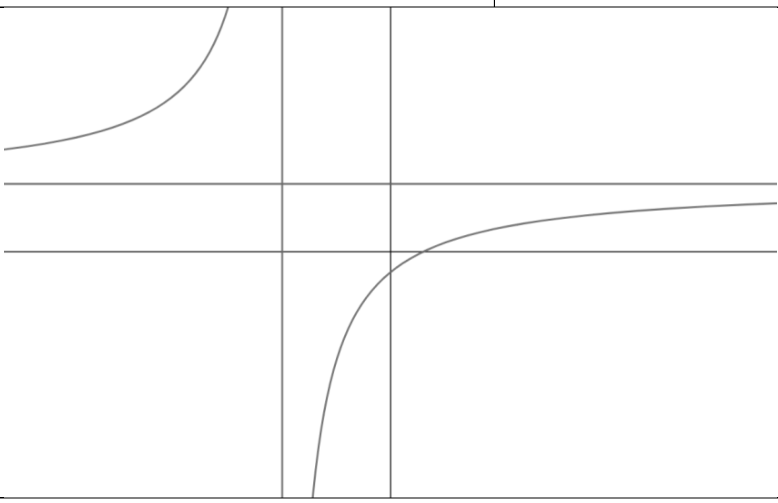
Part	Mark	Additional Guidance
(a)	M1	Correct expression for volume.
	M1	Equates their expression of the form ax^2y to 72 and rearranges correctly for y or xy Simplification is not required to be awarded this mark.
	dM1	Substitutes their expression for y or xy into a correct (unsimplified) formula or expression for area.. Dep on previous M mark
	A1*	Correct solution only, minimum steps as shown. A = must be present at least once.
(b)	M1	Differentiates to get an expression of the form $\pm\frac{a}{x^2}+8x \quad a\neq 0$
	dM1	Places their expression of the correct form = 0 and rearranges to give x or x^3 = Allow errors in rearrangement. Dependent on previous method mark. So long as differentiation shown, this mark can be implied by correct value.
	A1	$x = 3$ dependent on having gained 1st method mark – the question directs the candidates to use calculus.
	dM1	Substitutes their value for x into the **correct** expression for A. Dependent on 1st method mark. So long as differentiation shown, this mark can be implied by a correct value.
	A1	For 108 dependent on having gained 1st method mark – the question directs the candidates to use calculus.
	M1	Differentiates their expression for $\frac{dA}{dx}$ to give an expression of the form $\pm\frac{b}{x^3}+8 \; b\neq 0$
	A1	Correctly substitutes $x = 3$ into the correct second derivative or argues the correct second derivative is > 0 and draws a conclusion. It is not necessary to see an evaluation, but the evaluation must be correct if carried out. Second derivative **must be stated to be > 0 or positive.** As always with A1, must have gained all previous marks. **Note: it is highly unusual to have a final A1, when any of the preceding marks have been 0. The structure of this question means, this is possible, if they either don't find the minimum area or calculate it incorrectly.** **So for the final 4 marks dM1 A0 M1 A1 and dM0 A0 M1 A1**

Question number	Scheme	Marks
8	$3e^{3x}(9x+2)^{\frac{1}{3}}+\frac{1}{3}.9.e^{3x}(9x+2)^{-\frac{2}{3}}$	M1 A1 A1
	$3e^{3}(9+2)^{\frac{1}{3}}+\frac{1}{3}.9.e^{3}(9+2)^{-\frac{2}{3}}$	M1
	$=146$	A1 [5]
		Total 5 marks

Part	Mark	Additional Guidance
	M1	Use of product rule to give an expression of the form $ke^{3x}(9x+2)^{\frac{1}{3}}+le^{3x}(9x+2)^{-\frac{2}{3}}$ where k = 1 or 3, $l \neq 1$. There must be a + sign between terms.
	A1	For $+\frac{1}{3}.9.e^{3x}(9x+2)^{-\frac{2}{3}}$ or $+3e^{3x}(9x+2)^{\frac{1}{3}}$ simplification not necessary.
	A1	Both terms correct – simplification not necessary.
	M1	Substitution of x = 1 into any changed expression Allow substitution into an expression which has been simplified incorrectly after an expression of the required form. Can be implied by a correct answer or by 12.18…… + 134.0…….
	A1	Awrt 146

Question number	Scheme	Marks
9 (i)	$\left(3(\log_a 9 + \log_a 27) = 1\right)$ $3(2\log_a 3 + 3\log_a 3) = 1$ oe eg $2\log_a 3 + 3\log_a 3 = \frac{1}{3}$	M1
	$\log_a 3 = \frac{1}{15}$	
	$a^{\frac{1}{15}} = 3$ or $\log_3 a = 15$	M1
	$(a =)3^{15}$	A1 [3]
	ALT	
	$3(\log_a (9\times 27) = 1$ oe eg $3(\log_a (3^5)) = 1$ $\log_a (9\times 27) = \frac{1}{3}$ oe eg $\log_a (3^5) = \frac{1}{3}$	M1
	$a^{\frac{1}{3}} = 9\times 27$	M1
	$(a =)3^{15}$	A1 [3]
(ii)	$\left(\log_4 p + \log_p 256 = -4\right)$	
	$\log_4 p + \frac{\log_4 256}{\log_4 p} = -4$ or $\frac{\log_p p}{\log_p 4} + \log_p 256 = -4$	M1
	$\left(\log_4 p\right)^2 + \log_4 256 = -4\log_4 p$ or $\left(\log_p p\right) + \left(\log_p 256\right)\left(\log_p 4\right) = -4\log_p 4$	dM1
	$\left(\log_4 p\right)^2 + 4\log_4 p + 4(= 0\ \ )$ or $4\left(\log_p 4\right)^2 + 4\log_p 4 + 1(= 0)$	A1
	$\left(\left(\log_4 p\right) + 2\right)^2 (= 0\ )$ or $\left(\left(2\log_p 4\right) + 1\right)^2 (= 0)$	M1
	$\log_4 p = -2 \rightarrow p =$ or $\log_p 4 = -\frac{1}{2} \rightarrow p =$	M1
	$(p =)\frac{1}{16}$ oe	A1 [6]
		Total 9 marks

Part	Mark	Additional Guidance
When marking (i) stick to 1 ALTERNATIVE only – eg it's not possible to gain M2 for an application of a power rule and an addition rule with logs. **Look out for equivalent working, particularly the use of** $\log_{a^b} c = \frac{1}{b}\log_a c$ **Send any work that you may think uses a different method worth partial marks to Review**		
(i)	M1	For correct use of the rule $\log_a A^b = b\log_a A$ This mark may be awarded anywhere the power rule is correctly used **once** in working, in either direction, even if there's other contradictory work.
	M1	For a rearrangement of $3(b\log_a 3 + c\log_a 3) = 1$ to $a^{\frac{1}{3R}} = 3$ or $\log_3 a = 3R \quad R > 1$ seen anywhere in the working, even if there's other contradictory work. Allow errors in rearrangement. May be implied by any correct value for *a* not in the required form eg $14348907, \left(3^5\right)^3, \left(3^3\right)^5, 9^{7.5}$
	A1	$(a =) 3^{15}$
ALT	M1	For correct use the rule $\log_a A + \log_a B = \log_a AB$ This mark may be awarded anywhere the addition rule is correctly used **once** in working, in either direction, even if there's other contradictory work.
	M1	For a rearrangement of $3(\log_a (C \times D)) = 1$ *to* $a^{\frac{1}{3}} = CD$ seen anywhere in the working, even if there's other contradictory work. Allow errors in rearrangement. May be implied by any correct value for *x* not in the required form eg $14348907, \left(3^5\right)^3, \left(3^3\right)^5, 9^{7.5}$
	A1	$(a =) 3^{15}$
(ii)	M1	Correct use of change of base of logs. This mark may be awarded anywhere a change of base of logs is used correctly **once** in working, even if there's other contradictory work.
	dM1	Rearranges their equation to a 3 term quadratic (= 0). Allow errors in rearrangement so long as reach an expression of the form $p\left(\log_4 p\right)^2 + q\log_4 p + r = 0$ or $s\left(\log_p 4\right)^2 + t\log_p 4 + u = 0 \quad p, q, r, s, t, u \neq 0$
	A1	Correct 3 term quadratic(= 0) as shown.
	M1	A minimally acceptable attempt to solve their 3 term quadratic, see general guidance. Students may have used an alternative variable here for $\log_4 p$ or $\log_p 4$ The mark is for a minimally acceptable attempt to solve a 3TQ – see general guidance. The correct value from the correct quadratic implies this mark, otherwise method must be shown.
	M1	Correctly coverts their log equation into a value for *p*. ie $\log_4 p = F \rightarrow p =$ or $\log_p 4 = G \rightarrow p =$
	A1	For $(p =)\frac{1}{16}$ oe

Question number	Scheme	Marks
10(a) Mark (i) and (ii) together	$-\frac{1}{4}=-\frac{2}{b}\to b=8$ * $-\frac{4}{3}a+8=0$ oe or $-\frac{b}{a}=-\frac{4}{3}$ $a=6$	B1*cso M1 A1 [3]
(b)	$y=\frac{5}{"6"}$	B1ft [1]
(c)	$\left(\frac{2}{5},0\right)$	B1 [1]
(d)	$\frac{5(6x+8)-6(5x-2)}{(6x+8)^2}$ $\frac{52}{(6x+8)^2}$ oe eg $\frac{52}{36x^2+96x+64}$ $\frac{52}{(6x+8)^2}$ and correct conclusion ALT using product rule $(5x-2)(6x+8)^{-1}$ $5(6x+8)^{-1}+(-1)(6)(5x-2)(6x+8)^{-2}$ $\frac{52}{(6x+8)^2}$ oe eg $\frac{52}{36x^2+96x+64}$ $\frac{52}{(6x+8)^2}$ and correct conclusion	M1 dM1 A1 A1 [4]
(e)		B1 (curve) B1ft (asymptotes) B1ft (intersections with axes) [3]
(f)	$\frac{13}{100}=\frac{52}{(6x+8)^2}$ oe $(6x+8)^2=400$ $3x^2+8x-28=0$ oe $x=2, y=\frac{2}{5}$ $x=-\frac{14}{3}, y=\frac{19}{15}$ oe $\sqrt{\left(("2")-\left("-\frac{14}{3}"\right)\right)^2+\left(\left("\frac{2}{5}"\right)-\left("\frac{19}{15}"\right)\right)^2}$ 6.72	M1 dM1 A1 A1 dM1 A1 [6]
		Total 18 marks

Part	Mark	Additional Guidance
(a) (i) and (ii)	B1*cso	For a minimum of the steps shown in the MS to find b, no errors.
	M1	For correct substitution of $x=-\frac{4}{3}$ and b = 8 into $ax + b = 0$
	A1	Correct a.
		Students can complete this in a different order $-\frac{4}{3}a+8=0 \Rightarrow a=6$ $-\frac{b}{a}=-\frac{4}{3}\left(\text{or } -\frac{b}{6}=-\frac{4}{3}\right)\Rightarrow b=8$ B1 M1 A1
(b)	B1ft	Follow through their value of a. $y=\frac{5}{\text{"their 6"}}$
(c)	B1	$\left(\frac{2}{5},0\right)$ allow $x=\frac{2}{5}$ $y=0$ $x=\frac{2}{5}$ alone is insufficient
(d)	M1	Attempt the quotient rule. Numerator must be the difference of two terms of the form $A\times(ax+8)-B\times(5x-2)$ or $B\times(5x-2)-A\times(ax+8)$, A and B >1. Denominator must be of the form $(ax+8)^2$ $a\neq 0$ Allow follow through of their value of a (general principle of method mark).
	dM1	Either term on the numerator fully correct (either way round), dependent on previous method mark, doesn't need to be simplified. Denominator must be of the form $(ax+8)^2$ Allow follow through of their value of a (general principle of method mark).
	A1	$\frac{52}{(6x+8)^2}$ oe eg $\frac{52}{36x^2+96x+64}$ eg $\frac{13}{(3x+4)^2}$ A simplified expression (denominator expanded acceptable) Dependent on both previous method marks – question states must use calculus, so some method must be shown.
	A1	$\frac{52}{(6x+8)^2}$ and correct conclusion, for example, (the numerator is a positive number) eg the denominator is always positive eg the $(ax+8)^2>0$ and therefore (the fraction) is always positive. No incorrect work for this mark eg don't accept incorrectly simplified gradient.
ALT	M1	Attempt the product rule. $C\times(ax+8)^{-1}+(5x-2)(-(1))(D)\ (ax+8)^{-2}$, C and D >1. Allow follow through of their value of a (general principle of method mark).
	M1	Either term correct (either way round), dependent on previous method mark, doesn't need to be simplified
	A1 A1	As main scheme

(e)	B1	For a negative reciprocal curve drawn anywhere in the grid – there **must** be two branches present, they **must not** cross any asymptotes drawn or implied or cross each other and **must not <u>obviously</u>** 'bend back' on themselves. Mark intention.
	B1ft	Two clearly marked asymptotes $x=-\frac{8}{\text{their }a}, y=\frac{5}{\text{their }a}$, ft their value for a, labelled as equations or clearly passing through the correctly labelled numbers on the axis. There must be one section of a negative reciprocal curve present, tending towards these asymptotes. This branch must not obviously cross or bend back from any asymptotes drawn or implied Ignore any other branch/curve/line drawn if there is one branch fulfilling these conditions.
	B1ft	Single curve (or even line) passing through $\left(0,-\frac{1}{4}\right)$ and $\left(\frac{2}{5},0\right)$ marked clearly on the graph as coordinates or crossing points on the axis. Ignore any other branch/curve/line present. Allow follow through only of their $(c,0)$ $c\neq 0$ from part (c)
(f)	M1	Sets their differentiated function from part (d) = $\frac{13}{100}$ Allow any function = $\frac{13}{100}$ coming from working in part (d) or re-started work to find the derivative. The function may not be the original function.
	dM1	For correctly rearranging an equation of the form $\frac{13}{100}=\frac{Q}{(cx+d)^2}$ to $(cx+d)^2=R$ using their a, or to the form $px^2+qx+r=0$ $p,q,r\neq 0$ with 2 out of 3 of p, q, r correct. leading to x =
	A1	For $x=2, y=\frac{2}{5}$ or $x=-\frac{14}{3}, y=\frac{19}{15}$ The pairing must be clear (may be implied in further working)
	A1	For $x=2, y=\frac{2}{5}$ and $x=-\frac{14}{3}, y=\frac{19}{15}$ The pairing must be clear (may be implied in further working)
	dM1	For correct use of the formula to find the length of a line, using their values for x and y. Dependent on **first** method mark only.
	A1	For awrt 6.72

Question number	Scheme	Marks
11 (a)	$(p=)3\mathrm{e}^{\frac{2}{3}}$ oe	B1 [1]
(b)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\mathrm{e}^{\frac{x}{3}} = \mathrm{e}^{\frac{2}{3}}$ oe	M1 A1
	$(m_{norm}=)-\frac{1}{"\mathrm{e}^{\frac{2}{3}}"}=\left(-\mathrm{e}^{-\frac{2}{3}}\right)$ oe	M1
	$y-"3\mathrm{e}^{\frac{2}{3}}"="-\frac{1}{"\mathrm{e}^{\frac{2}{3}}"}"(x-2)$ oe $\left(y=-\mathrm{e}^{-\frac{2}{3}}x+2\mathrm{e}^{-\frac{2}{3}}+3\mathrm{e}^{\frac{2}{3}}\right)$	M1
	$0-3\mathrm{e}^{\frac{2}{3}}=-\mathrm{e}^{-\frac{2}{3}}(x-2)$ oe	dM1
	$x=2+3\mathrm{e}^{\frac{4}{3}}$	A1
	(Area of triangle =) $\frac{1}{2}\left("3\mathrm{e}^{\frac{2}{3}}"\right)\left("2+3\mathrm{e}^{\frac{4}{3}}"-2\right)\left(=\frac{9}{2}\mathrm{e}^{2}\right)$	ddM1
	$\int_0^2 3\mathrm{e}^{\frac{x}{3}}\mathrm{d}x=\left[\frac{3\mathrm{e}^{\frac{x}{3}}}{\frac{1}{3}}\right]_0^2$	B1 M1
	$\left(\frac{3\mathrm{e}^{\frac{2}{3}}}{\frac{1}{3}}\right)-\frac{3\mathrm{e}^{0}}{\frac{1}{3}}$ $\left(=9\mathrm{e}^{\frac{2}{3}}-9\right)$	dM1
	$=\frac{9}{2}\mathrm{e}^{2}+9\mathrm{e}^{\frac{2}{3}}-9$	A1cso
ALT for ddM1	$\int_2^{"2+3\mathrm{e}^{\frac{4}{3}}"}\left(-\mathrm{e}^{-\frac{2}{3}}x+2\mathrm{e}^{-\frac{2}{3}}+3\mathrm{e}^{\frac{2}{3}}\right)\mathrm{d}x=-\mathrm{e}^{-\frac{2}{3}}\frac{x^2}{2}+2\mathrm{e}^{-\frac{2}{3}}x+3\mathrm{e}^{\frac{2}{3}}x$ $=\left[-\mathrm{e}^{-\frac{2}{3}}\left(\frac{"2+3\mathrm{e}^{\frac{4}{3}}"^2}{2}\right)+2\mathrm{e}^{-\frac{2}{3}}\left("2+3\mathrm{e}^{\frac{4}{3}}"\right)+3\mathrm{e}^{\frac{2}{3}}\left("2+3\mathrm{e}^{\frac{4}{3}}"\right)\right]$ $-\left[-\mathrm{e}^{-\frac{2}{3}}\left(\frac{"2^2"}{2}\right)+2\mathrm{e}^{-\frac{2}{3}}(2)+3\mathrm{e}^{\frac{2}{3}}(2)\right]$	ddM1 [11]
		Total 12 marks

Part	Mark	Additional Guidance
(a)	B1	For the correct value of p.
(b)	M1	Differentiates to get an expression of the form $a\mathrm{e}^{\frac{x}{3}}$ $a \neq 3$
	A1	For $\mathrm{e}^{\frac{2}{3}}$ This must be an exact value, but isw any following decimals.
	M1	Finds the negative reciprocal of their gradient. This is not a completely dependent mark but must come from any attempt at differentiation.
	M1	For a complete unsimplified attempt (if using $y - y_1 = m(x - x_1)$ to find the equation of l, using their value of p and their gradient, which can be any changed gradient after finding an expression and value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ If $y = mx + c$ is used, a complete attempt must involve correctly rearranging (using their values for p and their gradient) to find c. **Different to question 2, if the candidate is not working with correct values and the rearrangement to find c with exact values is not shown, this mark cannot be awarded**
	dM1	Sets their equation = 0 and attempts to find a value for x. Allow errors in rearrangement. Dependent on previous method mark. This must be an exact value, but isw any following decimals.
	A1	Finds the correct value of x
	ddM1	For correct substitution into the formula for the area of a triangle, using (their x-intercept – 2) and their answer for p Does not need to be simplified. Dependent on previous 2 method marks. Or for stating the integral, with their p and their x intercept as limits and correct integration of at least 2 terms. **We don't need to see substitution of limits for this mark.** Their integral must be of the minimum form $\int_2^{"f+g\mathrm{e}^{\frac{h}{3}}"}\left(a\mathrm{e}^{-\frac{c}{3}}x + b\mathrm{e}^{-\frac{d}{3}}\right)\mathrm{d}x \quad a,b,c,d,f,g,h \neq 0$ See notes for next B mark, can be awarded if limits are correct, within a combined integral.
	B1	For stating $\int_0^2 3\mathrm{e}^{\frac{x}{3}}$ Note: some candidates are listing this integral by adding or subtracting it from the equation of the line. ie $\int_a^{"b"}\left(-\mathrm{e}^{-\frac{2}{3}}x + 2\mathrm{e}^{-\frac{2}{3}} + 3\mathrm{e}^{\frac{2}{3}} \pm 3e^{\frac{x}{3}}\right)\mathrm{d}x$ - mark B1 if limits are correct and B0 if not and potentially next M1 dM1 for anything that is obviously their line $\pm 3e^{\frac{x}{3}}$
	M1	Integrates to get an expression of the form $a\mathrm{e}^{\frac{x}{3}}$ $a \neq 3$
	dM1	Substitution of correct limits into any changed expression, must see substitution of x = 2 and 0 (unless the 0 limit gives an evaluation of 0) Allow a correct final answer or a correct evaluation of this integral $9\mathrm{e}^{\frac{2}{3}} - 9$ to imply this mark, if no errors in working. Examiners are not expected to evaluate to check incorrect expressions integrated. Dependent on previous method mark.
	A1cso	For a full and correct worked solution with no errors.

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