Mark Scheme (Results)

Summer 2025

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1) Paper 02

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks – can only be awarded when relevant M marks have been gained
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - cso – correct solution only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**

  If no working is shown, then correct answers may score full marks
  If no working is shown, then incorrect (even though nearly correct) answers score no marks.

- **With working**

  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.

  If a candidate misreads a number from the question: e.g. uses 252 instead of 255; follow through their working and deduct 2A marks from any gained provided the work has not been simplified. (Do not deduct any M marks gained.)

  If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used

  Examiners should send any instance of a suspected misread to review (but see above for simple misreads).

- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: e.g. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect e.g. algebra.

- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking
(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:
   $\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right)$ where $|pq|=|c|$ leading to $x=\ldots$
   $\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=\ldots$

2. Formula:
   Attempt to use the **correct** formula (shown explicitly or implied by working) with values for a, b and c, leading to $x=\ldots$

3. Completing the square:
   $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=\ldots$

Method marks for differentiation and integration:

1. Differentiation
   Power of at least one term decreased by 1. $(x^n\rightarrow x^{n-1})$
2. Integration:
   Power of at least one term increased by 1. $(x^n\rightarrow x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either**
quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values
or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks". General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this e.g. in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - i.e. giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question number	Scheme	Marks
1	$\sqrt{3}\tan A = 2\sin A \Rightarrow \dfrac{\sqrt{3}\sin A}{\cos A} = 2\sin A$	M1
	$\Rightarrow \sqrt{3}\sin A - 2\sin A\cos A = 0 \Rightarrow \sin A\left(\sqrt{3} - 2\cos A\right) = 0$	M1
	$\Rightarrow \sin A = 0 \quad \text{or} \quad \cos A = \dfrac{\sqrt{3}}{2}$	M1
	$\Rightarrow A = \sin^{-1} 0 = 0^{\circ}, 180^{\circ}, 360^{\circ} \quad , \quad \cos A = \dfrac{\sqrt{3}}{2} \Rightarrow A = 30^{\circ}, 330^{\circ}$	A1A1 [5]
	Total 5 marks	

Mark	Additional Guidance
M1	Uses identity $\tan = \dfrac{\sin}{\cos}$
M1	Rearranges to obtain $\sqrt{3}\sin A - 2\sin A\cos A = 0$ or it's equivalent **AND** factorises to obtain $\sin A\left(\sqrt{3} - 2\cos A\right) = 0$ This step can be implied by further correct work. If they cancel through by $\sin A$ this is M0
M1	Finds any one value for either sin *A* or cos *A*
A1	For at least any two correct angles obtained from either cos *A* or sin *A*
A1	For all five correct angles Ignore any values outside of the range. Withold this mark for any extra angles within range.

Question number	Scheme	Marks
2(a)	$x < \frac{1}{2}$ o.e	B1 (1)
(b)	$10x^2 + 7x - 12 (< 0)$ $(5x-4)(2x+3)(< 0)$	M1
	$x = \frac{4}{5}$ $x = -\frac{3}{2}$	
	$-\frac{3}{2} < x < \frac{4}{5}$ o.e	M1A1 (3)
(c)	$-\frac{3}{2} < x < \frac{1}{2}$	B1FT (1)
	Total 5 marks	

Part	Mark	Additional Guidance
(a)	B1	$x < \frac{1}{2}$ o.e
(b)	M1	Rearranges to a 3TQ and attempts to solve. See general guidance for what constitutes an attempt to solve. If they use a calculator to solve the 3TQ, the 3TQ must be correct and the values of *x* also correct. They must obtain **two** values of *x* for this mark
	M1	For forming an **inside** region with their critical values.
	A1	For the correct inequality. May be seen described in set notation.
(c)	B1FT	For $-\frac{3}{2} < x < \frac{1}{2}$ FT their (a) and (b) provided they overlap and it is an inside region. **NB** The final answer given must be consistent with their parts (a) and (b)

Question	Scheme	Marks

number		
3	$\frac{a+b\sqrt{5}}{6-2\sqrt{5}}\times\frac{6+2\sqrt{5}}{6+2\sqrt{5}}=\frac{6a+10b+2a\sqrt{5}+6b\sqrt{5}}{36-20}=\left[\frac{(6a+10b)+\sqrt{5}(2a+6b)}{16}\right]$	M1
	$\frac{(6a+10b)+\sqrt{5}(2a+6b)}{16}=\left[\frac{(3a+5b)+\sqrt{5}(a+3b)}{8}\right]=\frac{9+4\sqrt{5}}{C}$	
	$\Rightarrow 9=6a+10b$ OR $9=3a+5b$	
	$\Rightarrow 4=2a+6b$ $\quad 4=a+3b$	M1
	$\Rightarrow a=\frac{7}{8},\quad b=\frac{3}{8}$ OR $\Rightarrow a=\frac{7}{4},\quad b=\frac{3}{4}$	M1
	$\frac{\frac{7}{8}+\frac{3}{8}\sqrt{5}}{6-2\sqrt{5}}=\frac{9+4\sqrt{5}}{2\times 8}\Rightarrow\left[\frac{\frac{7}{4}+\frac{3}{4}\sqrt{5}}{6-2\sqrt{5}}=\frac{9+4\sqrt{5}}{2\times 4}\right]=\frac{7+3\sqrt{5}}{6-2\sqrt{5}}=\frac{9+4\sqrt{5}}{2}$	M1
	$\Rightarrow a=7, b=3$ and $c=2$	A1 [5]
	ALT	
	$c\left(a+b\sqrt{5}\right)=\left(9+4\sqrt{5}\right)\left(6-2\sqrt{5}\right)$	[M1
	$=54-18\sqrt{5}+24\sqrt{5}-40$	M1
	$c\left(a+b\sqrt{5}\right)=14+6\sqrt{5},=2\left(7+3\sqrt{5}\right)$	M1,M1
	$\Rightarrow c=2\quad a=7\quad b=3$	A1]
	Total 5 marks	

Mark	Additional Guidance

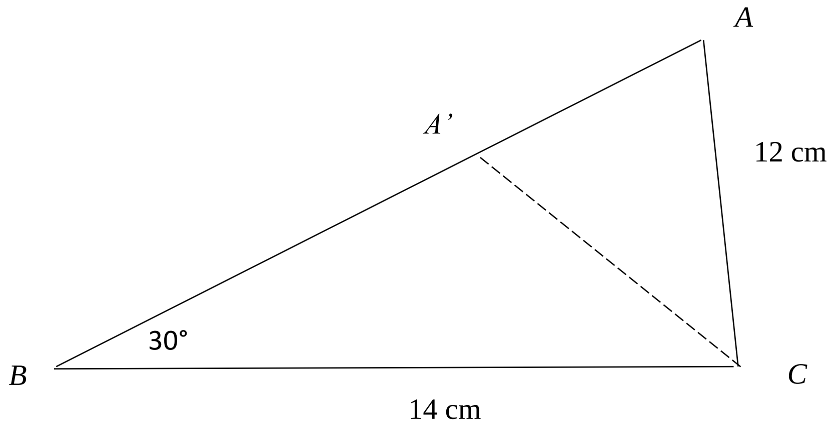
M1	Multiplies numerator and denominator by $6+2\sqrt{5}$ with no errors
M1	Equates numerator coefficients to find two simultaneous equations in *a* and *b*
M1	Solves their simultaneous equations by any method provided the equations of the form; $ma+nb=...$ and $pa+qb=...$ where $m \neq p$ and $n \neq q$ They must reach both a value for *a* and a value for *b* for this mark
M1	For multiplying through by 8 or 4 to give a prime number in the denominator of $\frac{9+4\sqrt{5}}{c}$
A1	For the correct values of *a*, *b* and *c* Allow embedded values.
ALT	
M1	'Cross multiplies' through by $\left(6-2\sqrt{5}\right)$ **AND** by *c*
M1	Multiplies out both sides with no errors
M1	Collects up like terms to obtain $c\left(a+b\sqrt{5}\right)=A+B\sqrt{5}$
M1	Factorises their expression to obtain $P\left(Q+R\sqrt{5}\right)$ where <u>*P*</u>, *Q* and *R* are integers or deduces by inspection that *a* = 7 and *c* = 2
A1	For the correct values – accept if embedded in the expression.

Question number	Scheme	Marks
4 (a) (i)	$\frac{ar^6}{ar^2}=\frac{40}{10}$	M1
	$r=\sqrt[4]{\frac{40}{10}}=(\pm\sqrt{2})$	M1
	$r=-\sqrt{2}$	A1
(ii)	$ar^2=10 \Rightarrow a=\frac{10}{'2'}$	M1
	$a=5$	A1 (5)
(b)	$\lvert r\rvert>1$ so the series is not convergent.	B1FT (1)
	Total 6 marks	

Part	Mark	Additional Guidance
(a)(i)	M1	For $\frac{ar^6}{ar^2}=\frac{40}{10}$ or $\frac{ar^2}{ar^6}=\frac{40}{10}$
	M1	For rearranging to find a value for r $r=\sqrt[4]{\frac{40}{10}}=(\pm\sqrt{2})$ or $\left(\pm\sqrt[4]{4}\right)$
	A1	For the correct value of $r=-\sqrt{2}$ Accept $-\sqrt[4]{4}$
(a)(ii)	M1	For attempting to find a value for a using their r $ar^2=10 \Rightarrow a=\frac{10}{'2'}$
	A1	For the correct value of $a=5$ Allow this mark from the use of $r=\sqrt{2}$
(b)	B1FT	$\lvert r\rvert>1$ so the series is not convergent. Follow through their value of r provided it is greater than 1.

Question number	Scheme	Marks
5(a)	$12^2 = 14^2 + x^2 - 2\times 14x\cos 30$	M1
	$x^2 - 14\sqrt{3}x + 52 = 0$	M1 A1
	$x = \dfrac{14\sqrt{3} \pm \sqrt{(14\sqrt{3})^2 - 4(1)(52)}}{2(1)}$	M1
	$x = 7\sqrt{3} \pm \sqrt{95}$	A1cso [5]
(b)	Area $= \dfrac{1}{2}\times 14\times\left(7\sqrt{3}\pm\sqrt{95}\right)\times\sin 30$ or uses Area $= \dfrac{1}{2}\times 12\times\left(7\sqrt{3}\pm\sqrt{95}\right)\times\dfrac{7}{12}$	M1
	Difference in areas $D = \dfrac{1}{2}\times 14\times\left(7\sqrt{3}+\sqrt{95}\right)\times\sin 30 - \dfrac{1}{2}\times 14\times\left(7\sqrt{3}-\sqrt{95}\right)\times\sin 30$	dM1
	$D = 7\sqrt{95}$	A1 [3]
ALT Part (a) – sine rule		
	$\dfrac{12}{\sin 30} = \dfrac{14}{\sin A} \Rightarrow \sin A = \dfrac{7}{12}$	{M1
	$\left(\dfrac{7}{12}\right)^2 + \cos^2 A \equiv 1 \Rightarrow \cos A = \sqrt{1-\left(\dfrac{7}{12}\right)^2}$	M1
	$\sin A = \dfrac{7}{12}, \cos A = \pm\dfrac{\sqrt{95}}{12}$	A1
	$x = 12\cos A + 14\cos 30 \Rightarrow x = 12\times\dfrac{\pm\sqrt{95}}{12} + 14\times\dfrac{\sqrt{3}}{2}$	M1
	$x = 7\sqrt{3} \pm \sqrt{95}$ *	A1 Cso [5]}
		Total 8 marks

USEFUL SKETCH



Part	Mark	Additional Guidance
(a)	M1	Use the cosine rule to form an equation. Rule to be correct but may be in any rearranged form.
	M1	Rearrange to obtain a 3TQ. Terms in any order. = 0 may be missing.
	A1	Correct 3TQ Allow poor notation – missing = 0
	M1	Attempt to solve their 3TQ by use of the formula or completing the square. See general guidance for what constitutes an attempt to solve. Use of calculators: If the 3TQ is correct, accept correct solutions without working.
	A1	Correct values for *x* $x = 7\sqrt{3} \pm \sqrt{95}$
(b)	M1	For use of area of triangle $= \frac{1}{2}ac\sin B$ with $a = 14,\ B = 30°$ and *their x* May use either value found for *x*. Or uses $= \frac{1}{2}bc\sin A$ with $c = 12$ and $\sin A = \frac{7}{12}$
	dM1	For finding the difference between their two areas provided both areas are positive This M mark is dependent on the previous M mark
	A1	Correct difference in areas (must be exact value)

	ALT Part (a) – Uses sine rule	
	M1	Use of the sine rule to find sin*A*
	M1	Use of $\sin^2 A + \cos^2 A \equiv 1$ to find an exact value for cos *A*
	A1	Correct exact values for sin*A* and cos *A*
	M1	Method to find *x* $x = 12\cos A + 14\cos 30$ Allow *their* cos *A*
	A1	Correct values for *x* $x = 7\sqrt{3} \pm \sqrt{95}$

Question number	Scheme	Marks
6 (a)	$700\pi = 2\pi r^2 + 2\pi rh$	B1
	$h = \dfrac{350 - r^2}{r}$ or $rh = 350 - r^2$	M1
	$V = \pi r^2 h = \pi r^2\left(\dfrac{350 - r^2}{r}\right) = \left\{\pi r\left(350 - r^2\right)\right\} = \pi r\left(350 - r^2\right)$ *	M1A1* cso [4]
(b)	$V = 350\pi r - \pi r^3$	
	$\dfrac{dV}{dr} = 350\pi - 3\pi r^2 = 0$	M1
	$\Rightarrow 350\pi = 3\pi r^2 \Rightarrow r = \sqrt{\dfrac{350}{3}} = 10.8$ (3sf)	M1A1
	$\dfrac{d^2V}{dr^2} = -6\pi r$	M1
	$\dfrac{d^2V}{dr^2} < 0 \therefore$ max	A1 [5]
(c)	$h = \dfrac{350 - r^2}{r} = \dfrac{350 - \frac{350}{3}}{\sqrt{\frac{350}{3}}} = 21.6$ (3 s.f.)	B1 [1]
		Total 10 marks

Part	Mark	Additional Guidance
(a)	B1	For a **correct** equation for the surface area in terms of the radius and height
	M1	For rearranging to find h in terms of r which must be as a minimum $h=\frac{k-lr^2}{mr}$ where k, l and m are integers OR $rh=k-lr^2$ where k and l are integers
	M1	For a correct expression for volume using *their* h in terms of r
	A1* cso	For rearranging and obtaining the given result with no errors or omissions and must include $V = \ldots$ [Allow Volume = …. or Vol = …..]
(b)	M1	For an attempt to differentiate the **given** expression for V which must be of the form $\frac{\mathrm{d}V}{\mathrm{d}r}=350\pi-K\pi r^2$ where K is a constant Using Product Rule: $\frac{\mathrm{d}V}{\mathrm{d}r}=\pi r\left(-2r\right)+\pi\left(350-r^2\right)\Rightarrow\frac{\mathrm{d}V}{\mathrm{d}r}=\left[350\pi-K\pi r^2\right]$
	M1	For setting their $\frac{\mathrm{d}V}{\mathrm{d}r}=0$ **and** attempting to find a value for r.
	A1	$r=\sqrt{\frac{350}{3}}$ $[=10.801...]$ accept awrt 10.8
	M1	For finding $\frac{\mathrm{d}^2V}{\mathrm{d}r^2}$ Accept only $\frac{\mathrm{d}^2V}{\mathrm{d}r^2}=\pm k\pi r$ where k is a constant See general guidance for what constitutes an attempt to differentiate.
	A1	For the correct conclusion derived from the correct working throughout.
(c)	B1	For obtaining the value of h correct to 3 s.f. Accept awrt 21.6

<table>
<tr><th>Question number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>7 (a)</td><td>

x	0	0.5	1	1.5	2	2.5	3	3.5
y	4.26	**4.14**	4	**3.83**	3.63	**3.37**	3	2.37

</td><td>B2
(2)</td></tr>
<tr><td>(b)</td><td>Points plotted within half a square.
Points joined with a smooth curve.</td><td>B1ft
B1ft
(2)</td></tr>
<tr><td>(c)</td><td>$3^{2x-5}=(4-x)^3 \Rightarrow 2x-5=3\log_3(4-x)$
$\frac{2x-5}{3}+3=\log_3(4-x)+3 \Rightarrow \frac{2x+4}{3}=\log_3(4-x)+3$
ALT
$\log_3(4-x)+3=mx+c$
$\Rightarrow \log_3(4-x)=mx+(c-3)$
$\Rightarrow (4-x)=3^{mx+(c-3)}$
$\Rightarrow (4-x)^3=3^{(mx+(c-3))3}=3^{3mx+3c-9}$
$\Rightarrow 2x-5=3mx+3c-9 \Rightarrow 3m=2$ and $-5=3c-9$
$\Rightarrow m=\frac{2}{3} \quad c=\frac{4}{3}$

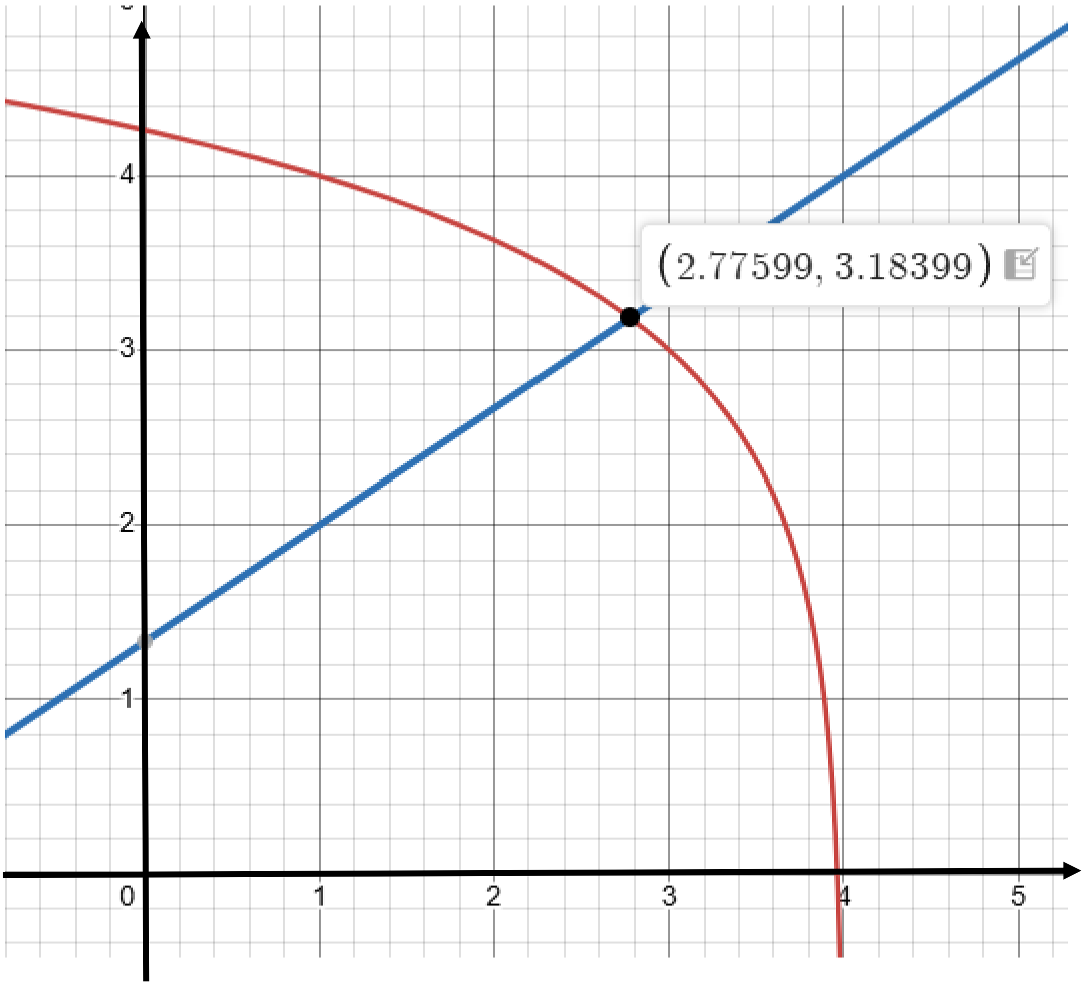
Graph of $y=\frac{2x+4}{3}$ drawn. Intersection point is $x=2.8$</td><td>M1M1
M1A1

[M1
M1
M1
A1]

M1A1
(6)
[10]</td></tr>
</table>

Part	Mark	Additional Guidance
(a)	B2	All three points correct (B1 for 2 points correct)
(b)	B1ft	Points plotted ft their table. Allow half a square tolerance.
	B1ft	Points joined together with a smooth curve. Allow half a square tolerance on joining up their points. FT their table
(c)	M1	For taking logs base 3 of both sides $2x-5=\log_3(4-x)^3$
	M1	For applying the power law $2x-5=3\log_3(4-x)$
	ddM1	For attempting to rearrange to the required form An attempt is defined as dividing through by 3 and adding 3 to both sides.
	A1	For the correct line, in any form $\frac{2x-5}{3}+3=\log_3(4-x)+3 \Rightarrow \left[(y=)\frac{2}{3}x+\frac{4}{3}\right]$
	ALT for first 4 marks	
	M1	Sets up the equation to obtain $\log_3\left(4-x\right)+3=mx+c$ And removes the log to obtain $\left(4-x\right)=3^{mx+(c-3)}$
	M1	Raises **both** sides to a power of 3. Allow one error in this on the LHS
	M1	Compares coefficients to obtain an equation in *m* and an equation in *c*
	A1	Obtains the values $m=\frac{2}{3}$ $c=\frac{4}{3}$
	Draws the line	
	M1	For their line drawn provided it is of the form $y=kx\pm\frac{4}{3}$ for $k\neq 0,\ 1$
	A1	$x=2.8$ or 2.7 Calculator value is 2.776

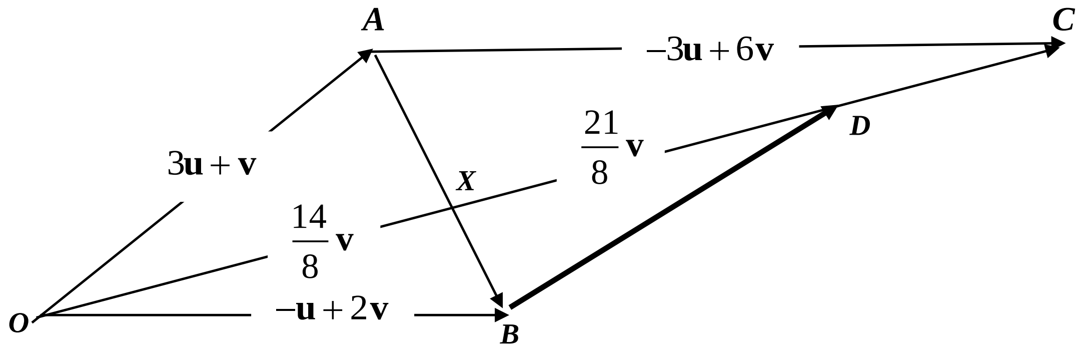
USEFUL GRAPH



Question number	Scheme	Marks
8 (a)	$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}, = -(3\mathbf{u}+\mathbf{v}) + (2\mathbf{v}-\mathbf{u}) = -4\mathbf{u}+\mathbf{v}$	M1,A1
		(2)
(b)	$\overrightarrow{AC} = \lambda(2\mathbf{v}-\mathbf{u})$	M1
	$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = (3\mathbf{u}+\mathbf{v}) + \lambda(2\mathbf{v}-\mathbf{u}) = \mu\mathbf{v}$	M1
	$\mathbf{u}: 0 = 3-\lambda \qquad \mathbf{v}: \mu = 1+2\lambda$	M1
	$\mu = 7 \qquad [\lambda = 3]$	A1
		(4)
(c)	$\overrightarrow{OX} = \overrightarrow{OA} + \alpha\overrightarrow{AB} = 3\mathbf{u}+\mathbf{v}+\alpha(-4\mathbf{u}+\mathbf{v})$ or $\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AC} + \beta\overrightarrow{CO} = 3\mathbf{u}+\mathbf{v}+3(2\mathbf{v}-\mathbf{u})+\beta(-7\mathbf{v})$	M1
	$\overrightarrow{OX} = \frac{7}{4}\mathbf{v}$	A1
	$\overrightarrow{OD} = \frac{5}{2}\overrightarrow{OX}$ or $\overrightarrow{XD} = \frac{3}{2}\overrightarrow{OX}$	M1
	$\overrightarrow{OD} = \frac{35}{8}\mathbf{v}$ or $\overrightarrow{XD} = \frac{21}{8}\mathbf{v}$	M1
	$\overrightarrow{BD} = \mathbf{u}-2\mathbf{v}+\frac{35}{8}\mathbf{v}$ or $\overrightarrow{BD} = \mathbf{u}-2\mathbf{v}+\frac{7}{4}\mathbf{v}+\frac{21}{8}\mathbf{v}$	A1
	$\overrightarrow{BD} = \mathbf{u}+\frac{19}{8}\mathbf{v}$	(5)
	Total 11 marks	

USEFUL SKETCH

$$\overrightarrow{OD} = \frac{35}{8}\mathbf{v} \quad \text{and} \quad \overrightarrow{AX} = -3\mathbf{u}+\frac{3}{4}\mathbf{v}$$



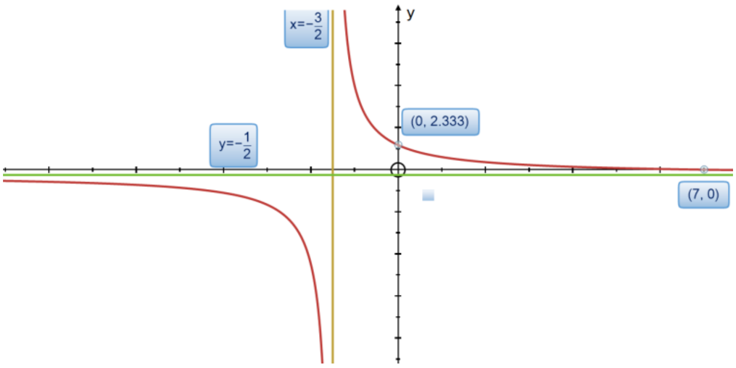
Part	Mark	Additional Guidance
(a)	M1	Use of $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$
	A1	$-4\mathbf{u} + \mathbf{v}$
(b)	M1	For use of $\overrightarrow{AC}$ is parallel to $\overrightarrow{OB}$ to form an expression for $\overrightarrow{AC}$
	M1	For use of their expression for $\overrightarrow{AC}$ to form an expression for $\overrightarrow{OC}$
	M1	For equating for **u** and for **v**
	A1	For $\mu = 7$
(c)		**General principles for marking part (c)** For example: You may see the vector path for $\overrightarrow{BD}$ first - score the 4th M mark • Candidates will need to find vector $\overrightarrow{OX}$ [M1A1] • Candidates will need to use the ratio of bases to find $\overrightarrow{OD} = \frac{5}{2}\overrightarrow{OX}$ or $\overrightarrow{XD} = \frac{3}{2}\overrightarrow{OX}$ **leading** to establish $\overrightarrow{OD} = \frac{35}{8}\mathbf{v}$ or $\overrightarrow{XD} = \frac{21}{8}\mathbf{v}$ [M1] • Check any path they choose on the diagram below. For example, $\overrightarrow{BD} = \overrightarrow{BO} + \overrightarrow{OX} + \overrightarrow{XD}$ or [M1] $\overrightarrow{BD} = \overrightarrow{BO} + \overrightarrow{OA} + \overrightarrow{AX} + \overrightarrow{XD}$ If a candidate chooses to take a longer path and need to find for example, $\overrightarrow{AX}$ they will need to find a vector. There is no separate mark for this. • For the correct vector
	M1	For an expression for $\overrightarrow{OX}$ including an unknown constant
	A1	For $\overrightarrow{OX} = \frac{7}{4}\mathbf{v}$
	M1	For recognising that triangle *OBX* and triangle *OBD* share a base *OB* and using this to obtain a ratio for $\overrightarrow{OX}$ to $\overrightarrow{OD}$ together with an attempt to find $\overrightarrow{OD}$ OR For recognising that triangle *OBX* and triangle *BXD* share a base *BX* and using this to obtain a ratio for $\overrightarrow{OX}$ to $\overrightarrow{XD}$ together with an attempt to find $\overrightarrow{XD}$
	M1	For method to find $\overrightarrow{BD}$ Award this for any valid path given
	A1	$\overrightarrow{BD} = \mathbf{u} + \frac{19}{8}\mathbf{v}$

Question number	Scheme	Marks
9 (a)	$\frac{dy}{dt} = -4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t \Rightarrow 2e^{-4t}\sin 2t = -\frac{dy}{dt} - 4y$ *	M1A1A1 Cso [3]
(b)	**Method A**	
	$\frac{d^2y}{dt^2} = -4(-4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t) + 8e^{-4t}\sin 2t - 4e^{-4t}\cos 2t$	M1
	$\frac{d^2y}{dt^2} = -4\frac{dy}{dt} + 8e^{-4t}\sin 2t - 4y$	M1
	$2e^{-4t}\sin 2t = -\frac{dy}{dt} - 4e^{-4t}\cos 2t \Rightarrow 8e^{-4t}\sin 2t = -4\frac{dy}{dt} - 16y$	M1
	$\frac{d^2y}{dt^2} = -4\frac{dy}{dt} + 8e^{-4t}\sin 2t - 4y \Rightarrow \frac{d^2y}{dt^2} = -4\frac{dy}{dt} + \left\{-4\frac{dy}{dt} - 16y\right\} - 4y$ $\Rightarrow \frac{d^2y}{dt^2} = -8\frac{dy}{dt} - 20y \Rightarrow \frac{d^2y}{dt^2} + 8\frac{dy}{dt} + 20y = 0$	M1
	$\frac{d^2y}{dt^2} + M\frac{dy}{dt} + Ny = 0 \Rightarrow M = 8, N = 20$	A1 [5]
	Method B	
	$\frac{d^2y}{dt^2} = -4(-4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t) + 8e^{-4t}\sin 2t - 4e^{-4t}\cos 2t$ $\left(= 16e^{-4t}\sin 2t + 12e^{-4t}\cos 2t\right)$	[M1
	$\frac{d^2y}{dt^2} + M\frac{dy}{dt} + Ny$ $= 16e^{-4t}\sin 2t + 12e^{-4t}\cos 2t + M(-4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t) + Ne^{-4t}\cos 2t$ $= (16 - 2M)e^{-4t}\sin 2t + (12 - 4M + N)e^{-4t}\cos 2t$	M1 M1
	$16 - 2M = 0$ and $12 - 4M + N = 0$	M1
	$M = 8,\ N = 20$	A1]
	Method C By inspection – please see additional guidance.	
		Total 8 marks

Part	Mark	Additional Guidance

(a)	M1	For an attempt to differentiate the given expression. • There needs to be an acceptable attempt to differentiate both terms. $e^{-4t}\cos 2t \to$ $ke^{-4t}\sin 2t + le^{-4t}\cos 2t$ with $k, l \neq 0$ • There need to be two terms added. $\frac{dy}{dt} = -4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t$
	A1	One term completely correct. $\frac{dy}{dt} = -4e^{-4t}\cos 2t + '-2e^{-4t}\sin 2t$ OR $\frac{dy}{dt} = '-4e^{-4t}\cos 2t' - 2e^{-4t}\sin 2t$
	A1	Fully correct differentiated expression and rearranged into the required form $\frac{dy}{dt} = -4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t \Rightarrow 2e^{-4t}\sin 2t = -\frac{dy}{dt} - 4y$ Accept also, For example: $-\left(-4e^{-4t}\cos 2t' - 2e^{-4t}\sin 2t\right) - 4\left(e^{-4t}\sin 2t\right) = 2e^{-4t}\sin 2t$ With any unambiguous indication that a conclusion has been reached, e.g., QED, hash sign, double underline, RHS = LHS etc.
		Method A
(b)	M1	For an attempt to find $\frac{d^2y}{dt^2}$ Minimally acceptable attempt is $ke^{-4x}\sin 2t + le^{-4t}\cos 2t \to k(me^{-4t}\sin 2t + ne^{-4t}\cos 2t) + l(pe^{-4t}\cos 2t + qe^{-4t}\sin 2t)$ k, l as in their first derivative, $m, n, p, q \neq 0$ $\frac{d^2y}{dt^2} = -4(-4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t) + 8e^{-4t}\sin 2t - 4e^{-4t}\cos 2t$
	M1	For substituting y and $\frac{dy}{dt}$ into their $\frac{d^2y}{dt^2}$ $\frac{d^2y}{dt^2} = -4\frac{dy}{dt} + 8e^{-4t}\sin 2t - 4y$
	M1	For preparing to eliminate cos2t by rearranging their $\frac{dy}{dt}$ $2e^{-4t}\sin 2t = -\frac{dy}{dt} - 4e^{-4t}\cos 2t \Rightarrow 8e^{-4t}\sin 2t = -4\frac{dy}{dt} - 16y$ Allow errors in arithmetic but not mathematically incorrect process.
	M1	For an unsimplified expression only in terms of $y, \frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$ $\frac{d^2y}{dt^2} = -4\frac{dy}{dt} + \left\{-4\frac{dy}{dt} - 16y\right\} - 4y$
	A1 [8]	For finding the value of M and the value of N. $M = 8,\ N = 20$
		Method B

	M1	For an attempt to find $\dfrac{d^2y}{dt^2}$ Minimally acceptable attempt is $ke^{-4t}\sin 2t + le^{-4t}\cos 2t \to k(me^{-4t}\sin 2t + ne^{-4t}\cos 2t) + l(pe^{-4t}\cos 2t + qe^{-4t}\sin 2t)$ k, l as in their first derivative, $m, n, p, q \neq 0$ $\dfrac{d^2y}{dt^2} = -4(-4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t) + 8e^{-4t}\sin 2t - 4e^{-4t}\cos 2t$
	M1	For substituting y, $\dfrac{dy}{dt}$ and $\dfrac{d^2y}{dt^2}$ into the equation. $\dfrac{d^2y}{dt^2} + M\dfrac{dy}{dt} + Ny$ $= 16e^{-4t}\sin 2t + 12e^{-4t}\cos 2t + M(-4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t) + Ne^{-4t}\cos 2t$
	M1	For collecting terms $\dfrac{d^2y}{dt^2} + M\dfrac{dy}{dt} + Ny = ('16' - 2M)e^{-4t}\sin 2t + ('12' - 4M + N)e^{-4t}\cos 2t$
	M1	For identifying two equations from their working and a method to solve them simultaneously For example, $'16' - 2M = 0$ and $'12' - 4M + N = 0$
	A1	For finding the value of M and the value of N. $M = 8, N = 20$
		Method C (By inspection)
	M1	For an attempt to find $\dfrac{d^2y}{dt^2}$ Minimally acceptable attempt is $ke^{-4t}\sin 2t + le^{-4t}\cos 2t \to k(me^{-4t}\sin 2t + ne^{-4t}\cos 2t) + l(pe^{-4t}\cos 2t + qe^{-4t}\sin 2t)$ k, l as in their first derivative, $m, n, p, q \neq 0$ $\dfrac{d^2y}{dt^2} = -4(-4e^{-4t}\cos 2t - 2e^{-4t}\sin 2t) + 8e^{-4t}\sin 2t - 4e^{-4t}\cos 2t$
	M1	For simplifying $\dfrac{d^2y}{dt^2} = e^{-4t}16\sin 2t + e^{-4t}12\cos 2t = \left[e^{-4t}\left(16\sin 2t + 12\cos 2t\right)\right]$
	M1	For comparing $\dfrac{d^2y}{dt^2}$ with $\dfrac{dy}{dt}$ $[\sin 2t]$ and deducing that $'16' + \left(\underline{8} \times -2\right) = 0 \Rightarrow M = 8$
	M1	For comparing $\dfrac{d^2y}{dt^2}$ with $\dfrac{dy}{dt}$ and y and deducing that $'12' - \left('8' \times 4\right) + \underline{20} = 0 \Rightarrow N = 20$
	A1	For both M = 8 and N = 20
		Total 8 marks

Question number	Scheme	Marks
10 (a)	(i) $x = -\frac{3}{2}$ (ii) $y = -\frac{1}{2}$	B1 B1 [2]
(b)	$(7, 0)$ $\left(0, \frac{7}{3}\right)$	B1 B1 [2]
(c)		B1ft curve B1ft asymptotes B1ft (intersections with *x*- and *y*-axes) [3]
(d)	$\frac{-1(2x+3)-2(7-x)}{(2x+3)^2} = \left[\frac{-17}{(2x+3)^2}\right]$ $\frac{-1\left(2\times\frac{1}{2}+3\right)-2\left(7-\times\frac{1}{2}\right)}{\left(2\times\frac{1}{2}+3\right)^2} = -\frac{17}{16}$ ALT – product rule $-1(2x+3)^{-1}+(7-x)(-1)(2)(2x+3)^{-2} = \left[\frac{-17}{(2x+3)^2}\right]$ $-1\left(2\times\frac{1}{2}+3\right)^{-1}+\left(7-\frac{1}{2}\right)(-1)(2)\left(2\times\frac{1}{2}+3\right)^{-2} = -\frac{17}{16}$	M1 M1A1* cso [3] M1 M1A1* cso

(e)	$-\frac{17}{(2x+3)^2}=-\frac{17}{16}$	M1
	$(2x+3)^2=16\Rightarrow 2x+3=\pm\sqrt{16}\Rightarrow x=0.5$ or -3.5	dM1
	$y=\frac{7--3.5}{2\times-3.5+3}\left(=-\frac{21}{8}\right)$	ddM1
	$\left(-\frac{7}{2},-\frac{21}{8}\right)$	A1
	$y--\frac{21}{8}=-\frac{17}{16}\left(x--\frac{7}{2}\right)$	M1A1
	$34x+32y+203=0$	A1 [7]
	Total 17 marks	

Part	Mark	Additional Guidance
For parts (a) and (b) look for their responses in the body of the question. In part (a), if the responses are not labelled (i) and (ii) accept them in order.		
(a)	(i) B1	For $x=-\frac{3}{2}$ oe
	(ii) B1	For $y=-\frac{1}{2}$ oe
(b)	B1 B1	First B1 for either correct, second B1 for both correct Condone if not given as coordinates e.g. $x=7$ and/or $y=\frac{7}{3}$ given
(c)	B1ft	Two branches drawn in the correct two "quadrants" created by the two asymptotes. Mark intention, allow poor curves, but do not allow the curve to bend back on itself or touch any asymptotes.
	B1ft	Two clearly marked asymptotes, ft their (a), labelled as described, there must be one section of the curve present, tending towards these asymptotes.
	B1ft	Two clearly labelled intersections with the axes, ft their (b), at least one section of their curve must pass through one of these intersections. Intersections must be labelled correct way around. If additional intersections seen then B0
(d)	M1	Attempt the quotient rule. Numerator must be of the form $A.(2x+3)-B.(7-x)$, $B>1$. Denominator must be of the form $(2x+3)^2$
	M1	Substitutes $x=\frac{1}{2}$ into their derivative **NB: This is an A mark in Epen**
	A1* cso	Obtains the required value of gradient $\left(-\frac{17}{16}\right)$ with no errors **NB: This is a B mark in Epen**
	ALT – product rule	
	M1	For an attempt at Product Rule. Must be a sum of two products. Must have the form $c(2x+3)^{-1}+d(7-x)(2x+3)^{-2}$ for constants c, d.
	M1	Substitutes $x=\frac{1}{2}$ into their derivative **NB: This is an A mark in Epen**
	A1* cso	Obtains the required value of gradient $\left(-\frac{17}{16}\right)$ with no errors **NB: This is a B mark in Epen**
(e)	M1	For equating the gradient of the tangent to their $\frac{dy}{dx}$ from (d)
	dM1	For rearranging to obtain $(2x+3)^2=16$ and attempt to solve or for rearrangement to a 3TQ and method to solve the 3TQ. See general guidance for acceptable methods to solve.

	ddM1	For substitution of their x into the original equation to obtain y. This must **not** be $x = \frac{1}{2}$ This M mark is dependent on the **two** previous M marks
	A1	For correct coordinate of P $\left(-\frac{7}{2}, -\frac{21}{8}\right)$
	M1	For substitution of their coordinates of P and the gradient $-\frac{17}{16}$ to obtain an equation of a straight line using any valid method. If $y = mx + c$ is used a value of c must be found **and** an equation needs to be formed. For example; $'-\frac{21}{8}' = -\frac{17}{16} \times '-\frac{7}{2}' + c \Rightarrow c = -\frac{203}{32} \Rightarrow y = -\frac{17}{16}x - \frac{203}{32}$ [
	A1	For any correct equation of the tangent with the correct values of x and y in any form.
	A1	For the correct equation of tangent given in the required form. $34x + 32y + 203 = 0$

Question number	Scheme	Marks
11 (a)	$\cos^4\theta-\sin^4\theta=(\cos^2\theta-\sin^2\theta)(\cos^2\theta+\sin^2\theta)$	M1
	$=(\cos^2\theta-\sin^2\theta)\times 1=\cos 2\theta$ *	M1M1A1 cso [4
	ALT example	
	$\cos^4\theta-\sin^4\theta=\cos^4\theta-\sin^2\theta\sin^2\theta$	
	$=\cos^4\theta-\left(1-\cos^2\theta\right)\left(1-\cos^2\theta\right)$	2nd M1
	$=\cos^4\theta-\left(1-2\cos^2\theta+\cos^4\theta\right)$	1st M1
	$=\left(1-2\cos^2\theta\right)$	3rd M1
	$=\cos 2\theta$	A1
(b)	$8\cos^2\left(2\theta+\frac{\pi}{4}\right)-3=2[\cos^4\left(\theta+\frac{\pi}{8}\right)-\sin^4\left(\theta+\frac{\pi}{8}\right)]$	M1
	$=2\cos\left(2\theta+\frac{\pi}{4}\right)$	A1
	$8\cos^2\left(2\theta+\frac{\pi}{4}\right)-2\cos\left(2\theta+\frac{\pi}{4}\right)-3=0$	M1
	$\left(4\cos\left(2\theta+\frac{\pi}{4}\right)-3\right)\left(2\cos\left(2\theta+\frac{\pi}{4}\right)+1\right)[=0]$	
	$\cos\left(2\theta+\frac{\pi}{4}\right)=\frac{3}{4}\quad \cos\left(2\theta+\frac{\pi}{4}\right)=-\frac{1}{2}$	A1
	$2\theta+\frac{\pi}{4}=0.72273...\quad 2\theta+\frac{\pi}{4}=\frac{2\pi}{3}$	dM1
	$\theta=2.39, 3.11, 0.65, 1.70$ accept $\theta=\frac{5\pi}{24},\frac{13\pi}{24}$	A1A1 [7]
(c)	$\int_{\frac{\pi}{16}}^{\frac{\pi}{8}}\left(\cos^4 2x-\sin^4 2x-8\sin 4x\right)\mathrm{d}x=\int_{\frac{\pi}{16}}^{\frac{\pi}{8}}\left(\cos 4x-8\sin 4x\right)\mathrm{d}x$	M1
	$=\left[\frac{1}{4}\sin 4x+\frac{8}{4}\cos 4x\right]_{\frac{\pi}{16}}^{\frac{\pi}{8}}$	M1
	$=\frac{1}{4}\left[\sin\left(4\times\frac{\pi}{8}\right)-\sin\left(4\times\frac{\pi}{16}\right)\right]+2\left[\cos\left(4\times\frac{\pi}{8}\right)-\cos\left(4\times\frac{\pi}{16}\right)\right]$	M1
	$=\frac{1}{4}\left[1-\frac{\sqrt{2}}{2}\right]+2\left[0-\frac{\sqrt{2}}{2}\right]=\frac{1}{4}-\frac{9\sqrt{2}}{8}$	A1 [4]
		Total 15 marks

Part	Mark	Additional Guidance
General Principles for marking part (a) • First M mark is for either applying the difference of two squares or multiplying out their expression and collecting terms. • Second M mark is for applying $\cos^2\theta+\sin^2\theta=1$ in any form. This can be implied from correct work. However, do not award for just $\cos^4\theta-\sin^4\theta=\cos^2\theta-\sin^2\theta$ seen. • Third M mark is for applying any identity for $\cos 2\theta$ • A1 For obtaining the required result with no errors.		
(a)	M1	For factorising as the difference of two squares $(\cos^2\theta-\sin^2\theta)(\cos^2\theta+\sin^2\theta)$
	M1	For use of $\cos^2\theta+\sin^2\theta)=1$
	M1	For use of $\cos^2\theta-\sin^2\theta=\cos(2\theta)$
	A1 cso	Obtain the given result with no errors in the working.
(b)	M1	For using the given result to obtain a quadratic in $\left(2\theta+\frac{\pi}{4}\right)$ Note: another variable may be used in place of $\left(2\theta+\frac{\pi}{4}\right)$ for this mark
	A1	For rearranging to the correct quadratic
	M1	For a valid attempt to solve their **3TQ**. See General Guidance for what constitutes an attempt to factorise. If candidates use a calculator [so no working shown] then award this mark only if the 3TQ is correct with the correct solutions.
	A1	For obtaining correct values for $\cos\left(2\theta+\frac{\pi}{4}\right)$ Note: another variable may be used in place of $\left(2\theta+\frac{\pi}{4}\right)$ for this mark
	dM1	For achieving any valid value **in radians** for their $2\theta+\frac{\pi}{4}$ i.e. 0.7227…or $\frac{2}{3}\pi$ Accept values out of range for this mark. This is dependent on the previous M mark.
	A1	For two correct solutions in range
	A1	For all four correct solutions in range and no others. Extra solutions within range scores A0 Ignore solutions outside of range. Exact solutions when given must be given as multiples of π.
(c)	M1	For use of the given result to obtain $\int_{\frac{\pi}{16}}^{\frac{\pi}{8}}\cos 4x\ \mathrm{d}x$
	M1	For an attempt to integrate $\cos kx\to l\sin kx$ where $k,l\neq 1$
	M1	For substituting limits into a changed expression
	A1	For the correct answer in the required form.

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