



# Mark Scheme (Results)

## November 2025

Pearson Edexcel International GCSE in Further Pure  
Mathematics

4PM1/01

## Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications are awarded by Pearson, the UK's largest awarding body. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information visit our qualifications websites at [www.edexcel.com](http://www.edexcel.com) or [www.btec.co.uk](http://www.btec.co.uk). Alternatively, you can get in touch with us using the details on our contact us page at [www.edexcel.com/contactus](http://www.edexcel.com/contactus)

### Pearson: helping people progress, everywhere

Pearson aspires to be the world's leading learning company. Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at:

[www.pearson.com/uk](http://www.pearson.com/uk)

November 2025

Question Paper Log Number P78964A

Publication Code 4PM1\_01\_2511\_MS

All the material in this publication is copyright

© Pearson Education Ltd

## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

### Types of mark

- M marks: method marks
- A marks: accuracy marks
- B marks: unconditional accuracy marks (independent of M marks)

### Abbreviations

- cao – correct answer only
- ft – follow through
- isw – ignore subsequent working
- SC – special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- awrt – answer which rounds to
- eeoo – each error or omission

- cas – correct answer scores full marks (unless from obvious incorrect working)
- wr – working required

### No working

If no working is shown then correct answers normally score full marks

If no working is shown then incorrect (even though nearly correct) answers score no marks.

### With working

If there is a wrong answer indicated on the answer line always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.

If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.

If a candidate misreads a number from the question. e.g., uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review. If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used.

If there is no answer on the answer line then check the working for an obvious answer.

### Ignoring subsequent work

It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.

It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect e.g. algebra.

Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

### Parts of questions

Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

## General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

### Method mark for solving a 3 term quadratic equation

#### 1. Factorisation:

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c| \text{ leading to } x = \dots$$

$$(ax^2 + bx + c) = (mx + p)(nx + q) \text{ where } |pq| = |c| \text{ and } |mn| = |a| \text{ leading to } x = \dots$$

#### 2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for  $a$ ,  $b$  and  $c$ , leading to  $x = \dots$

#### 3. Completing the square:

$$x^2 + bx + c = 0: \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0 \quad \text{leading to } x = \dots$$

### Method marks for differentiation and integration

#### 1. Differentiation

Power of at least one term decreased by 1. ( $x^n \rightarrow x^{n-1}$ )

#### 2. Integration:

Power of at least one term increased by 1. ( $x^n \rightarrow x^{n+1}$ )

### Use of a formula

Generally, the method mark is gained by **either** quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

**or**, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

### Answers without working

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show....")

### **Exact answers**

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

### **Rounding answers (where accuracy is specified in the question)**

Penalise only once per question for failing to round as instructed – i.e. giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the rule may allow the mark to be awarded before the final answer is given.

| Question             | Scheme   | Marks                 |
|----------------------|--|-----------------------|
| 1                    | $\left(1 - \frac{x}{4}\right)^{-2} = 1 + (-2)\left(-\frac{x}{4}\right) + \frac{(-2)(-3)\left(-\frac{x}{4}\right)^2}{2!} + \frac{(-2)(-3)(-4)\left(-\frac{x}{4}\right)^3}{3!}$ $= 1 + \frac{x}{2} + \frac{3x^2}{16} + \frac{x^3}{16}$ | M1<br><br>A1A1<br>[3] |
| <b>Total 3 marks</b> |  |                       |

| Mark | Notes   |
|------|---|
| M1   | <p>For applying a correct binomial expansion in unsimplified form.<br/>Minimum required:</p> <ul style="list-style-type: none"> <li>• The expansion begins with 1</li> <li>• The second term is correct (ignore sign)</li> <li>• At least one coefficient of the powers of <math>\left(-\frac{x}{4}\right)</math> must be correct.</li> <li>• The denominators are correct. Accept 2 for 2!</li> </ul> <p>Do not allow missing brackets unless there is recovery later.<br/>Ignore any terms higher than <math>x^3</math></p> |
| A1   | <p>Follows M1 – this is a general point of marking. A mark can only follow an awarded M mark.<br/>Either term in <math>x^2</math> or <math>x^3</math> correct and simplified</p>  |
| A1   | All fully correct and simplified in the correct order.  |

| Question             | Scheme   | Marks       |
|----------------------|--|-------------|
| <b>2 (a)</b>         | $f(x) = 2x^2 - 10x + 7 = 2(x^2 - 5x) + 7$                          | M1          |
|                      | $= 2\left(x - \frac{5}{2}\right)^2 - \frac{11}{2}$                 | M1          |
|                      | $\Rightarrow a = 2 \quad b = \frac{-5}{2} \quad c = \frac{-11}{2}$ | A1<br>[3]   |
| <b>(b)</b>           | (i) $\frac{-11}{2}$  | B1ft        |
|                      | (ii) $\frac{5}{2}$   | B1ft<br>[2] |
| <b>Total 5 marks</b> |  |             |

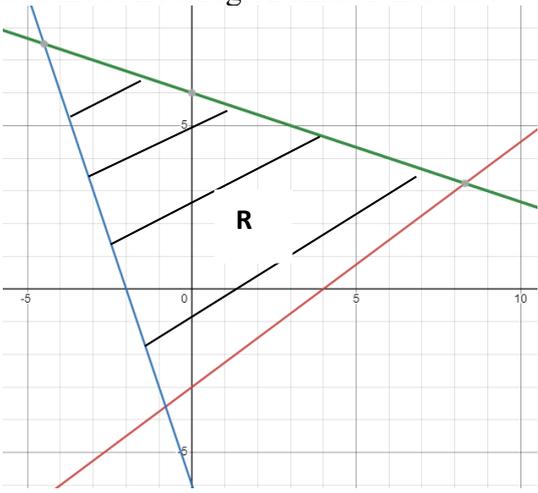
| Part    | Mark                              | Notes   |
|---------|-----------------------------------|---|
| 2 (a)   | M1                                | For factorising the given expression to achieve either:<br>$2(x^2 - 5x) + 7$ or $2\left(x^2 - 5x + \frac{7}{2}\right)$  |
|         | M1                                | For completing the square<br>[See General Guidance]   |
|         | A1                                | For fully correct values of $a$ , $b$ and $c$ . Accept embedded or written separately.  |
|         | <b>ALT – Equates coefficients</b> |   |
|         | M1                                | Multiplies out $a(x+b)^2 + c = ax^2 + 2abx + ab^2 + c$  |
|         | M1                                | Equates coefficients to obtain one of $a$ , $b$ or $c$<br>$a = 2 \Rightarrow$<br>$-10 = 2ab \Rightarrow b = \frac{-5}{2}$<br>$7 = ab^2 + c \Rightarrow c = \frac{-11}{2}$ |
|         | A1                                | For fully correct values of $a$ , $b$ and $c$ . Accept embedded or written separately.  |
| (b)(i)  | B1ft                              | For $-\frac{11}{2}$ [ft their value of $c$ ]  |
| (b)(ii) | B1ft                              | For $\frac{5}{2}$ [ft their value of $-b$ ]   |

| Question             | Scheme  | Marks                         |
|----------------------|---|-------------------------------|
| <b>3 (a)</b>         | $15^2 = 12^2 + x^2 - 2 \times 12 \times x \times \cos 60^\circ$<br>$\Rightarrow 225 = 144 + x^2 - 12x \Rightarrow x^2 - 12x - 81 = 0$ *   | M1<br><br>M1A1<br>cso<br>[3]  |
|                      | <b>Special Case in this part for using Sine Rule</b><br><b>The working must be CORRECT for the award of these marks.</b>  |                               |
|                      | $\frac{15}{\sin 60} = \frac{12}{\sin C} \Rightarrow C = 43.9^\circ$ AND $\frac{15}{\sin 60} = \frac{x}{\sin 76.1..}$<br>$\Rightarrow x = 16.8(16...), -4.8(16...)$ <b>Both roots required</b><br><br>$x^2 - 12x - 81 = 0 \Rightarrow x = 16.8..., 4.81...$ and concludes that $x$ must satisfy the given quadratic equation | B1<br><br>B1                  |
| <b>(b)</b>           | $x^2 - 12x - 81 = 0 \Rightarrow x = 6 + \sqrt{117} = [6 + 3\sqrt{13}]$<br><br>$A = \frac{1}{2} \times 12 \times (6 + 3\sqrt{13}) \times \sin 60^\circ$<br><br>$A = 3\sqrt{3} (6 + 3\sqrt{13}) = 9(2\sqrt{3} + \sqrt{39})$   | M1<br><br>M1<br><br>A1<br>[3] |
| <b>Total 6 marks</b> |   |                               |

| Part  | Mark  | Notes   |
|-------|---|---|
| 3 (a) | M1  | Applies a correct cosine rule to the given values   |
|       | M1  | Uses the exact value of $\cos 60$ and attempts to form a 3TQ  |
|       | A1<br>cso   | Obtains the given result with no errors seen.   |
|       | <b>SC Uses sine rule – the work must be correct for the award of these marks. The final A mark cannot be awarded using this method.</b> |   |
|       | M1  | For using sine rule twice to find <b>TWO</b> values for $x$ in either exact or inexact form.  |
|       | M1  | Solves the given equation finding <b>both</b> roots <b>AND</b> concludes that $x$ satisfies this equation.  |
| (b)   | M1  | <b>Work in part (b) must be exact only.</b><br>Solves the given 3TQ by any method. If the root(s) are incorrect only award this mark for full correct working seen. |
|       | M1  | Uses the correct formula for the area of a triangle using the correct values with their positive value for $x$ .  |
|       | A1  | For the correct exact area.   |

| Question             | Scheme   | Marks                     |
|----------------------|--|---------------------------|
| 4(a)                 | $\sum_{r=1}^n (4r + A) = \frac{n}{2}(2(4 + A) + (n-1)4) = \frac{n}{2}(4 + 2A + 4n)$  | M1                        |
|                      | Then $\frac{n}{2}(4 + 2A + 4n) = \frac{n}{2}(4n - 6)$<br>$4 + 2A = -6 \Rightarrow A = -5^*$  | M1<br>A1 cso<br>[3]       |
| (b) (i)              | $\sum_{21}^{60} (4r - 5) = \frac{60}{2}(4 \times 60 - 6) - \frac{20}{2}(4 \times 20 - 6) = [7020 - 740] = 6280$  | M1A1<br>[2]               |
|                      | <b>ALT 1</b><br>$U_{21} = 4 \times 21 - 5 = 79 \quad U_{60} = 4 \times 60 - 5 = 235$<br>$\sum_{21}^{60} (4r - 5) = \frac{40}{2}(79 + 235) = 6280$  | [M1A1]                    |
|                      | <b>ALT 2</b><br>$S_{40} = \frac{40}{2}(2(79) + (40-1) \times 4)$<br>$= 20 \times (158 + 156) = 6280$   | [M1A1]                    |
| (ii)                 | $\sum_{r=1}^n (4r - 5) < 3418 \Rightarrow \frac{n}{2}(4n - 6) < 3418 \Rightarrow 4n^2 - 6n - 6836 < 0$<br>$\Rightarrow 4n^2 - 6n - 6836 < 0 \Rightarrow n < 42.096\dots$<br>$\Rightarrow n = 42$ | M1<br><br>M1<br>A1<br>[3] |
| <b>Total 8 marks</b> |  |                           |

| Part    | Mark      | Notes  |
|---------|-----------|--|
| 4 (a)   | M1        | For using the correct summation formula for an arithmetic series with $d = 4$ and first term $4 + A$ |
|         | M1        | For equating values and solving the linear equation.   |
|         | A1<br>cso | For obtaining the given result with no errors.   |
| (b) (i) | M1        | For using any method shown with $n = 40$ to find the sum   |
|         | A1        | For the correct sum of 6280  |
| (ii)    | M1        | For forming an inequality with 3 terms.  |
|         | M1        | For solving their 3TQ  |
|         | A1        | For $n = 42$   |

| Question   | Scheme  | Marks                                    |              |            |             |           |  |          |       |   |
|--|---|--|--------------|------------|-------------|-----------|--|----------|-------|---|
| <p><b>5(a)</b></p> <p>(i)</p> <p>(ii)</p> <p>(iii)</p> | <p>For each line drawn correctly</p> <p><math>3x - 4y = 12</math> Intersections with axes at (0, -3) and (4, 0)</p> <p><math>y + 6 + 3x = 0</math> Intersections with axes at (0, -6) and (-2, 0)</p> <p><math>3y = 18 - x</math> Coordinates at (0, 6) (3, 5) (6, 4)</p> | <p>B1</p> <p>B1</p> <p>B1</p> <p>[3]</p> |              |            |             |           |  |          |       |   |
| <p><b>(b)</b></p>                                      | <p>For the correct region shaded in or out</p>    | <p>B1ft</p> <p>[1]</p>                   |              |            |             |           |  |          |       |   |
| <p><b>(c)</b></p>                                      | <p>For attempting to <b>read off</b> the correct coordinates of at least one intersection.</p> <table border="1" data-bbox="360 1249 1145 1305"> <tr> <td>Vertex</td> <td>(-0.8, -3.6)</td> <td>(8.3, 3.2)</td> <td>(-4.5, 7.5)</td> </tr> </table>                       | Vertex                                   | (-0.8, -3.6) | (8.3, 3.2) | (-4.5, 7.5) | <p>M1</p> |  |          |       |   |
| Vertex   | (-0.8, -3.6)  | (8.3, 3.2)                               | (-4.5, 7.5)  |            |             |           |  |          |       |   |
|  | <table border="1" data-bbox="360 1417 1145 1503"> <tr> <td><math>P = 3x - 2y</math></td> <td>4.8</td> <td>18.5</td> <td>-28.5</td> </tr> <tr> <td></td> <td></td> <td>Greatest</td> <td>Least</td> </tr> </table>   | $P = 3x - 2y$                            | 4.8          | 18.5       | -28.5       |           |  | Greatest | Least | <p>dM1</p> <p>A1caoA1cao</p> <p>[4]</p> |
| $P = 3x - 2y$  | 4.8   | 18.5                                     | -28.5        |            |             |           |  |          |       |   |
|  |   | Greatest                                 | Least        |            |             |           |  |          |       |   |
| <p><b>Total 8 marks</b></p>                            |   |  |              |            |             |           |  |          |       |   |

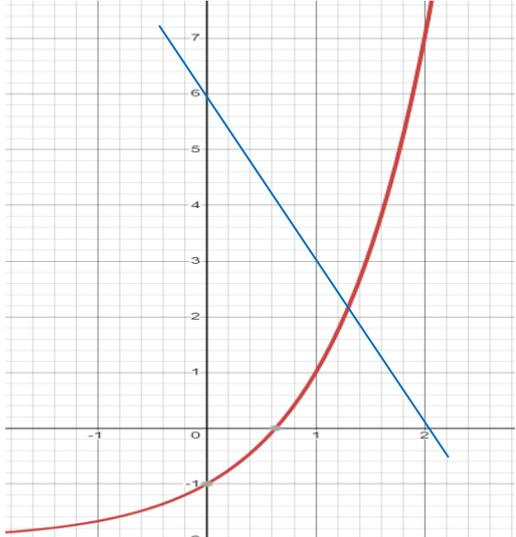
| Part           | Mark           | Notes  |                  |             |  |        |                |                |               |              |              |              |              |                |               |                |              |                  |
|----------------|----------------|--|------------------|-------------|--|--------|----------------|----------------|---------------|--------------|--------------|--------------|--------------|----------------|---------------|----------------|--------------|------------------|
| 5 (a)          | B3             | Award B1 for each line drawn correctly.  |                  |             |  |        |                |                |               |              |              |              |              |                |               |                |              |                  |
| (b)            | B1ft           | For the correct region shaded in or out.<br>You do not need to see $R$ marked. Follow through with candidate's lines, where a closed region has been created.  |                  |             |  |        |                |                |               |              |              |              |              |                |               |                |              |                  |
| (c)            | M1             | For at least one set of coordinates <b>read off</b> from their graph.<br>If fractional values such as $-4/5$ , $-18/5$ , $108/13$ etc award M0.  |                  |             |  |        |                |                |               |              |              |              |              |                |               |                |              |                  |
|                | dM1            | For finding at least one value for $P$ using their coordinates.  |                  |             |  |        |                |                |               |              |              |              |              |                |               |                |              |                  |
|                | A1cao          | For either $P_{\text{greatest}} = 18.5$ or $P_{\text{least}} = -28.5$  |                  |             |  |        |                |                |               |              |              |              |              |                |               |                |              |                  |
|                | A1cao          | For both $P_{\text{greatest}} = 18.5$ and $P_{\text{least}} = -28.5$<br>Mark should only be awarded for fully correct work with all three values correct.  |                  |             |  |        |                |                |               |              |              |              |              |                |               |                |              |                  |
|                |                | <p>Note on tolerances<br/>For each point on the graph, we accept candidate values within “half a square” of the correct value.<br/>Thus, for the first vertex, we accept <math>x</math> values in <math>(-0.9 - -0.7)</math> and corresponding <math>y</math> values within <math>(-3.7</math> to <math>-3.5)</math></p> <p>Co-ordinates:</p> <table border="1"> <thead> <tr> <th>Vertices</th> <th>Accept awfw</th> <th></th> <th>P awfw</th> </tr> </thead> <tbody> <tr> <td><math>(-0.8, -3.6)</math></td> <td><math>(-0.9, -0.7)</math></td> <td><math>(-3.7 -3.5)</math></td> <td><math>(4.3, 5.3)</math></td> </tr> <tr> <td><math>(8.3, 3.2)</math></td> <td><math>(8.2, 8.4)</math></td> <td><math>(3.1, 3.3)</math></td> <td><math>(18.0, 19.0)</math></td> </tr> <tr> <td><math>(-4.5, 7.5)</math></td> <td><math>(-4.6, -4.4)</math></td> <td><math>(7.4, 7.6)</math></td> <td><math>(-29.0, -28.0)</math></td> </tr> </tbody> </table> | Vertices         | Accept awfw |  | P awfw | $(-0.8, -3.6)$ | $(-0.9, -0.7)$ | $(-3.7 -3.5)$ | $(4.3, 5.3)$ | $(8.3, 3.2)$ | $(8.2, 8.4)$ | $(3.1, 3.3)$ | $(18.0, 19.0)$ | $(-4.5, 7.5)$ | $(-4.6, -4.4)$ | $(7.4, 7.6)$ | $(-29.0, -28.0)$ |
| Vertices       | Accept awfw    |  | P awfw           |             |  |        |                |                |               |              |              |              |              |                |               |                |              |                  |
| $(-0.8, -3.6)$ | $(-0.9, -0.7)$ | $(-3.7 -3.5)$  | $(4.3, 5.3)$     |             |  |        |                |                |               |              |              |              |              |                |               |                |              |                  |
| $(8.3, 3.2)$   | $(8.2, 8.4)$   | $(3.1, 3.3)$   | $(18.0, 19.0)$   |             |  |        |                |                |               |              |              |              |              |                |               |                |              |                  |
| $(-4.5, 7.5)$  | $(-4.6, -4.4)$ | $(7.4, 7.6)$   | $(-29.0, -28.0)$ |             |  |        |                |                |               |              |              |              |              |                |               |                |              |                  |

| Question             | Scheme   | Marks  |
|----------------------|--|--|
| 6                    | $\frac{dy}{dx} = 4e^{4x} \cos 3x - 3e^{4x} \sin 3x$ $\left[ \frac{dy}{dx} = 4y - 3e^{4x} \sin 3x \right]$ $\frac{d^2y}{dx^2} = 4 \frac{dy}{dx} - 3[4e^{4x} \sin 3x + 3e^{4x} \cos 3x]$ $\left\{ 12e^{4x} \sin 3x = 16y - 4 \frac{dy}{dx} \right\}$ $\frac{d^2y}{dx^2} = 4 \frac{dy}{dx} - 16y + 4 \frac{dy}{dx} - 9y \Rightarrow \frac{d^2y}{dx^2} + 25y = 8 \frac{dy}{dx}$ $A = 25 \quad B = 8$   | <p>M1A1A1</p> <p>M1A1</p> <p>M1</p> <p>M1A1cso<br/>[8]</p>   |
| ALT                  | $\frac{dy}{dx} = 4e^{4x} \cos 3x - 3e^{4x} \sin 3x$ $\frac{d^2y}{dx^2} = 16e^{4x} \cos 3x - 12e^{4x} \sin 3x - 12e^{4x} \sin 3x - 9e^{4x} \cos 3x$ $= 7e^{4x} \cos 3x - 24e^{4x} \sin 3x$ <p>Substituting into given equation:</p> $\frac{d^2y}{dx^2} + Ay = B \frac{dy}{dx} \Rightarrow$ $7e^{4x} \cos 3x - 24e^{4x} \sin 3x + Ae^{4x} \cos 3x = B(4e^{4x} \cos 3x - 3e^{4x} \sin 3x)$ <p>Equating coefficients</p> $7 + A = 4B \quad -24 = -3B \Rightarrow B = 8$ $7 + A = 4 \times 8 = 32$ $\Rightarrow A = 25$ | <p>M1A1A1</p> <p>M1A1</p> <p>M1</p> <p>M1<br/>A1<br/>[8]</p> |
| <b>Total 8 marks</b> |  |  |

| Mark       | Notes  |
|------------|--|
| 6          | For an attempt at product rule: <ul style="list-style-type: none"> <li>The formula used must be correct</li> <li><math>e^{4x} \rightarrow ke^{4x} \quad k \neq 0,1</math></li> </ul>   |
| M1         | <ul style="list-style-type: none"> <li><math>\cos 3x \rightarrow -l \sin 3x \quad l \neq 0,1</math></li> <li>Minimally acceptable attempt is:<br/><math>ke^{4x} \cos 3x - le^{4x} \sin x</math></li> </ul>   |
| A1         | At least one term must be correct  |
| A1         | Fully correct.   |
| A1         | Ignore poor notation for $\frac{dy}{dx}$   |
| M1         | For an attempt at the second derivative.   |
| A1         | For the correct second derivative - simplified or unsimplified.  |
| M1         | For converting $me^{4x} \sin 3x$ into $y$ 's and $\frac{dy}{dx}$ 's<br>i.e., $-me^{4x} \sin 3x = ny - p \frac{dy}{dx}$   |
| M1         | For changing the expression in terms of $y$ , $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ only   |
| A1also     | For the correct expression exactly as given in the question.   |
| <b>ALT</b> |  |
| M1         | For an attempt at product rule: <ul style="list-style-type: none"> <li>The formula used must be correct</li> <li><math>e^{4x} \rightarrow ke^{4x} \quad k \neq 0,1</math></li> <li><math>\cos 3x \rightarrow -l \sin 3x \quad l \neq 0,1</math></li> <li>Minimally acceptable attempt is:<br/><math>ke^{4x} \cos 3x - le^{4x} \sin x</math></li> </ul> |
| A1         | At least one term must be correct  |
| A1         | Fully correct.   |
| A1         | Ignore poor notation for $\frac{dy}{dx}$   |
| M1         | For attempting the second derivative. Ft their $\frac{dy}{dx}$ and look for exactly 4 terms, with an attempt to differentiate as for the first M1  |
| A1         | Fully correct second derivative – simplified or unsimplified. If it correct unsimplified, then isw and award this mark.  |
| M1         | For setting up the equation with $A$ and $B$ as shown in the main scheme.  |
| M1         | Equating <b>their</b> coefficients to produce two equations in $A$ and $B$   |
| A1         | For the correct values of $A$ and $B$ or embedded in the equation $\frac{d^2y}{dx^2} + 25y = 8 \frac{dy}{dx}$  |

| Question              | Scheme  | Marks  |
|-----------------------|---|--|
| 7(a)                  | $(\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) = \alpha^3 - \alpha^2\beta + \alpha\beta^2 + \beta\alpha^2 - \alpha\beta^2 + \beta^3 = \alpha^3 + \beta^3$  | B1<br>[1]  |
| (b)                   | $\alpha + \beta = -4 \quad \alpha\beta = \frac{-k}{2}$ $\left[ \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta \right]$ $\alpha^3 + \beta^3 = (\alpha + \beta)\left((\alpha + \beta)^2 - 2\alpha\beta - \alpha\beta\right) = (\alpha + \beta)\left((\alpha + \beta)^2 - 3\alpha\beta\right)$ $-94 = (-4)\left((-4)^2 - 3 \times \frac{-k}{2}\right) \Rightarrow \frac{47}{2} = 16 + \frac{3k}{2} \Rightarrow k = 5 \quad *$ <p><b>ALT</b></p> $\alpha + \beta = -4 \quad \alpha\beta = -\frac{k}{2}$ $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ $-94 = (-4)^3 - 3 \times -\frac{k}{2} \times -4 \Rightarrow -30 = -6k$ $k = 5 \quad *$ | B1<br><br>M1<br>M1A1<br>cso<br>[4]<br><br>[B1<br>M1<br><br>M1<br>A1] |
| (c) (i)               | $(\alpha - \beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2 = (\alpha + \beta)^2 - 4\alpha\beta$ $\alpha - \beta = \sqrt{(-4)^2 - 4 \times -\frac{5}{2}} = \sqrt{26} \quad *$   | M1<br><br>M1A1<br>cso<br>[3]   |
| (ii)                  | $\alpha^3 - \beta^3 = (\alpha - \beta)^3 + 3\alpha\beta(\alpha - \beta)$ $= ('\sqrt{26}')^3 + 3\left(\frac{-5}{2}\right)('\sqrt{26}')$ $= \frac{37\sqrt{26}}{2}$  | M1<br><br><br>A1<br>[2]  |
| <b>Total 10 marks</b> |   |  |

| Part    | Mark       | Notes  |
|---------|------------|--|
| 7 (a)   | B1         | A full expansion with no errors.   |
| (b)     | B1         | For the correct values of the product and sum  |
|         | M1         | For the correct algebra $\alpha^3 + \beta^3$ in a form that values for $\alpha + \beta$ and $\alpha\beta$ [in terms of $a$ ] can be substituted.   |
|         | M1         | For substituting their values of $\alpha + \beta$ and $\alpha\beta$ into their expression for $\alpha^3 + \beta^3$ and solving a <b>linear</b> equation to find the value of $k$ .   |
|         | A1<br>cso  | For the correct value of $k = 5$   |
|         | <b>ALT</b> |  |
|         | B1         | For the correct values of the product and sum  |
|         | M1         | For the correct algebra $\alpha^3 + \beta^3$ in a form that values for $\alpha + \beta$ and $\alpha\beta$ [in terms of $a$ ] can be substituted using $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ |
|         | M1         | For substituting their values of $\alpha + \beta$ and $\alpha\beta$ into their expression for $\alpha^3 + \beta^3$ and solving a <b>linear</b> equation to find the value of $k$ .   |
|         | A1<br>cso  | For the correct value of $k = 5$   |
| (c) (i) | M1         | For the correct algebra for $(\alpha - \beta)^2$   |
|         | M1         | For substituting the correct values for the sum and product and take the positive square root.   |
|         | A1<br>cso  | For the correct value with no errors seen.   |
| (ii)    | M1         | For the correct algebra to find $\alpha^3 - \beta^3$ in a form such that the sum and product can be substituted in directly.   |
|         | A1         | For the correct value of $\alpha^3 - \beta^3$  |

| Question              | Scheme  |      |    |      |      |      |   |     |   | Marks   |    |      |   |      |     |      |   |     |   |     |      |      |    |      |      |     |   |     |   |           |
|-----------------------|---|------|----|------|------|------|---|-----|---|---|----|------|---|------|-----|------|---|-----|---|-----|------|------|----|------|------|-----|---|-----|---|-----------|
| 8(a)                  | <table border="1"> <tr> <td><math>x</math></td> <td>-1</td> <td>-0.5</td> <td>0</td> <td>0.25</td> <td>0.5</td> <td>0.75</td> <td>1</td> <td>1.5</td> <td>2</td> </tr> <tr> <td><math>y</math></td> <td>-1.7</td> <td>-1.4</td> <td>-1</td> <td>-0.7</td> <td>-0.3</td> <td>0.3</td> <td>1</td> <td>3.2</td> <td>7</td> </tr> </table>  |      |    |      |      |      |   |     |   | $x$   | -1 | -0.5 | 0 | 0.25 | 0.5 | 0.75 | 1 | 1.5 | 2 | $y$ | -1.7 | -1.4 | -1 | -0.7 | -0.3 | 0.3 | 1 | 3.2 | 7 | B2<br>[2] |
| $x$                   | -1  | -0.5 | 0  | 0.25 | 0.5  | 0.75 | 1 | 1.5 | 2 |   |    |      |   |      |     |      |   |     |   |     |      |      |    |      |      |     |   |     |   |           |
| $y$                   | -1.7  | -1.4 | -1 | -0.7 | -0.3 | 0.3  | 1 | 3.2 | 7 |   |    |      |   |      |     |      |   |     |   |     |      |      |    |      |      |     |   |     |   |           |
| (b)                   |   |      |    |      |      |      |   |     |   | B2<br>[2]   |    |      |   |      |     |      |   |     |   |     |      |      |    |      |      |     |   |     |   |           |
| (c)                   | $\log_3 4.5$<br>$y = 3^x - 2 \Rightarrow 3^x = y + 2 \Rightarrow x = \log_3 (y + 2)$<br>$\log_3 4.5 = \log_3 (2.5 + 2) \Rightarrow y = 2.5$<br>$\Rightarrow x = 1.4$  |      |    |      |      |      |   |     |   | M1<br>M1<br>A1<br>[3]                                     |    |      |   |      |     |      |   |     |   |     |      |      |    |      |      |     |   |     |   |           |
| (d)                   | $3^x - 2 = Ax + B \Rightarrow 3^x = Ax + (B + 2) \Rightarrow x = \log_3 (Ax + (B + 2))$<br>$\Rightarrow \log_3 (Ax + (B + 2)) - x = 0$<br>$\log_3 (8 - 3x) - x = 0$<br>$\Rightarrow A = -3 \quad 8 = B + 2 \Rightarrow B = 6$<br>$\Rightarrow y = 6 - 3x$<br><br><b>ALT</b><br>$\log_3 (8 - 3x) - x = 0 \Rightarrow \log_3 (8 - 3x) = x$<br>Taking exponents<br>$8 - 3x = 3^x \Rightarrow 8 - 3x - 2 = 3^x - 2 = y$<br>$6 - 3x = y$<br><br>Draws the line $y = 6 - 3x$<br>$x = 1.3$ |      |    |      |      |      |   |     |   | M1<br>M1<br>A1<br>[M1]<br>[M1]<br>[A1]<br>M1<br>A1<br>[5] |    |      |   |      |     |      |   |     |   |     |      |      |    |      |      |     |   |     |   |           |
| <b>Total 12 marks</b> |   |      |    |      |      |      |   |     |   |   |    |      |   |      |     |      |   |     |   |     |      |      |    |      |      |     |   |     |   |           |

| Part  | Mark      | Notes  |
|-------|-----------|--|
| 8 (a) | B2        | B2 for all correct values<br>B1 for a minimum of two values  |
| (b)   | B1ft      | All points plotted correctly within half of one square.  |
|       | B1ft      | Their points joined by a smooth curve.   |
| (c)   | M1        | <b>In this part, there must be evidence of algebraic work, together with use of the graph.</b><br>For using log laws to rearrange the given expression to find $x = \log_3(y+2)$ |
|       | M1        | For deducing that the required value of $y$ is 2.5 and using their graph to find the value of $x$ at that point.   |
|       | A1        | For $x = 1.4$  |
| (d)   | M1        | For setting $3^x - 2 = Ax + B$ and obtaining $x = \log_3(Ax + (B+2))$  |
|       | M1        | For equating coefficients to deduce that the equation of the line is $y = 6 - 3x$  |
|       | A1        | For the correct equation of the line.<br>Minimally acceptable equation is $y = k - 3x$ where $k$ is a positive value   |
|       | ALT<br>M1 | For rearranging the given equation   |
|       | ALT<br>M1 | For taking exponents and rearranging.  |
|       | ALT<br>A1 | For the correct equation of the line.  |
|       | M1        | For drawing their line on the graph.<br>The correct line will cross the axes at (0, 6) and (2, 0)  |
|       | A1        | For a value of 1.3 <b>only</b><br>The calculator value is 1.291  |

| Question | Scheme   | Marks   |
|----------|--|---|
| 9(a)     | $V = 0.2t$ $r = h \tan 30 = \frac{h}{\sqrt{3}}$ $V = \frac{1}{3} \pi \left( \frac{h}{\sqrt{3}} \right)^2 h \quad \left( = \frac{\pi}{9} h^3 \right)$ $0.2t = \frac{\pi}{9} h^3 \Rightarrow h = \sqrt[3]{\frac{9t}{5\pi}} \quad *$  | B1<br>B1<br>B1<br>M1A1<br>cso<br>[5]  |
| (b)      | $\frac{dV}{dt} = 0.2$ $A = \pi r^2 = \frac{\pi h^2}{3}$ $\frac{dA}{dh} = \frac{2}{3} \pi h, \quad \frac{dV}{dh} = \frac{\pi h^2}{3}$ <p>when <math>t = 6</math></p> $h = \sqrt[3]{\frac{9 \times 6}{5\pi}} \quad (= 1.50923\dots)$ $\frac{dA}{dt} = \frac{dV}{dt} \times \frac{1}{\frac{dV}{dh}} \times \frac{dA}{dh} = \left[ \frac{1}{5} \times \frac{3}{\pi h^2} \times \frac{2\pi h}{3} \right]$ $\frac{dA}{dt} = \frac{2}{5h} = \frac{2}{5 \times 1.50923} \approx 0.265$ <p><b>ALT</b></p> $\frac{dV}{dt} = 0.2$ $A = \pi r^2 = \frac{\pi h^2}{3}$ $\frac{dA}{dh} = \frac{2}{3} \pi h$ $h^3 = \frac{9t}{5\pi} \Rightarrow t = \frac{5}{9} \pi h^3$ $\frac{dt}{dh} = \frac{5}{3} \pi h^2$ <p>When <math>t = 6</math></p> $h = \sqrt[3]{\frac{9 \times 6}{5\pi}} \quad (= 1.50923\dots).$ $\frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dt}$ | B1<br>B1<br>M1, M1<br>B1<br>M1<br>M1A1<br>[8]<br>[B1]<br>[B1]<br>[M1]<br>[M1]<br>[B1] |

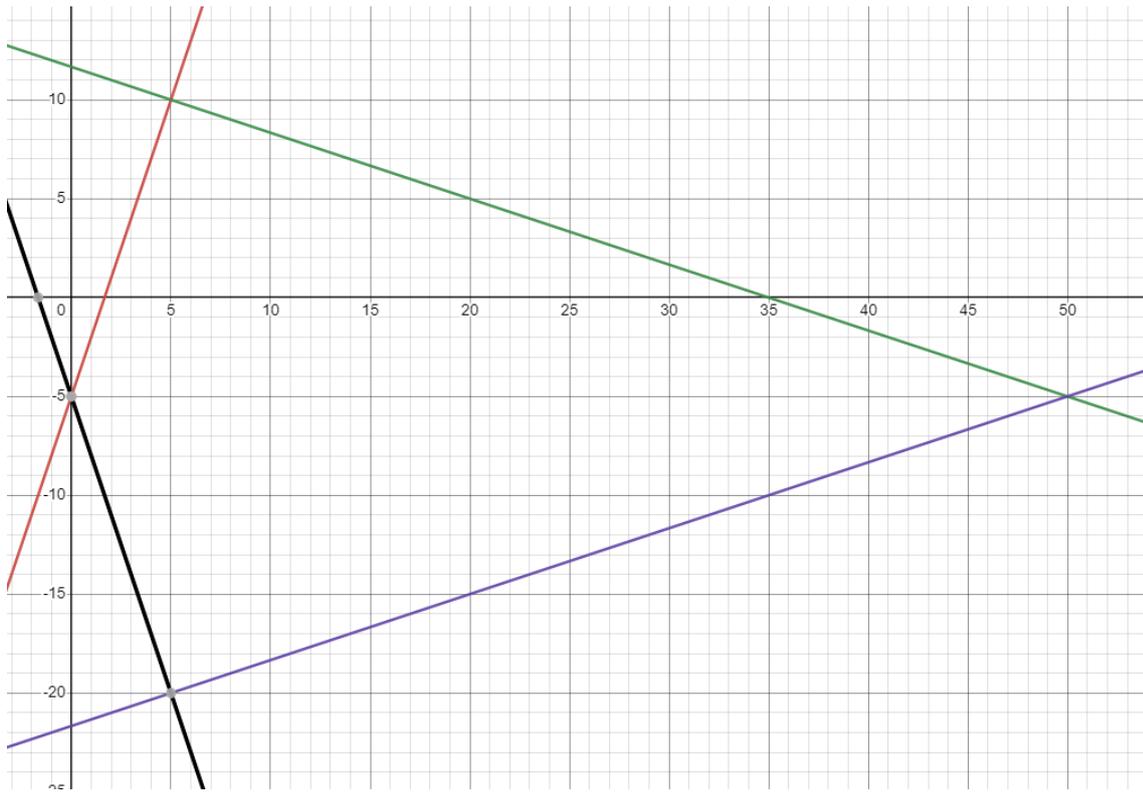
|                       |  |                           |
|-----------------------|--|---------------------------|
|                       | $= \frac{2}{3} \pi h \times \frac{1}{\frac{5}{2} \pi h^3} = \frac{2}{5h}$ $\frac{dA}{dt} = \frac{2}{5 \times 1.50923} \approx 0.265$ | [M1]<br><br>[M1A1]<br>[8] |
| <b>Total 13 marks</b> |  |                           |

| Part       | Mark                                | Notes   |
|------------|-------------------------------------|---|
| 9 (a)      | B1                                  | For stating $V = 0.2t$  |
|            | B1                                  | For finding $h$ in terms of $r$   |
|            | B1                                  | For finding the volume of the cone in terms of $h$ only                           |
|            | M1                                  | Equates both expressions for $V$  |
|            | A1<br>cso                           | For obtaining the required expression   |
| (b)        | B1                                  | For stating $\frac{dV}{dt} = 0.2$ anywhere in their response, including part (a). |
|            | B1                                  | For finding the area of the surface of liquid in terms of $h$ only.               |
|            | M1                                  | For the correct $\frac{dA}{dh}$   |
|            | M1                                  | For the correct $\frac{dV}{dh}$   |
|            | B1                                  | For using the correct expression for $h$ at $t = 6$                               |
|            | M1                                  | For a correct expression of chain rule in terms of $V, A, h$ and $t$ only         |
|            | M1                                  | For substituting all the values   |
|            | A1                                  | For the correct value of awrt 0.265   |
| ALT<br>(b) |                                     |   |
|            | B1                                  | For stating $\frac{dV}{dt} = 0.2$   |
|            | B1                                  | For finding the area of the surface of liquid in terms of $h$ only.               |
|            | M1                                  | For the correct $\frac{dA}{dh}$   |
|            | M1                                  | For rearranging the expression in (a) and finding $\frac{dt}{dh}$                 |
|            | B1                                  | For using the correct expression for $h$ at $t = 6$                               |
|            | M1                                  | For a correct expression of chain rule in terms of $V, A, h$ and $t$ only         |
|            | M1                                  | For substituting all the values   |
| A1         | For the correct value of awrt 0.265 |   |



|                       |   |  |
|-----------------------|---|--|
|                       | $\text{Area} = \frac{50 \times (10 - (-20))}{2} = 750$ <p><b>ALT 1</b><br/>Correctly uses determinant method with their co-ordinates</p> $\text{Area} = \frac{1}{2} \begin{vmatrix} 0 & 5 & 50 & 5 & 0 \\ -5 & 10 & -5 & -20 & -5 \end{vmatrix} = 750$ <p><b>ALT 2</b><br/>Correctly uses formulae for two triangles ABD and ACD using their lengths</p> $\text{Area} = 2 \times \left( \frac{1}{2} \times 15 \times 50 \right) = 750$ <p>Or</p> $\text{Area} = \left( \frac{1}{2} \times 30 \times 5 \right) + \left( \frac{1}{2} \times 45 \times 30 \right) = 750$ | dM1A1<br>[3]<br><br>[M1<br>dM1A1]<br><br>[M1<br><br>dM1A1] |
| <b>Total 17 marks</b> |   |  |

| Part    | Mark  | Notes  |
|---------|-------|--|
| 10 (a)  | M1    | Substitutes the given coordinates in the equation  |
|         | A1cso | For $a = 5$  |
| (b)     | B1    | For the correct coordinates.   |
| (c) (i) | B1    | For finding the gradient of both lines   |
|         | M1    | Multiplies the gradients together.   |
|         | A1    | For a correct proof with no errors seen.   |
| (ii)    | M1    | For any correct equation of the line using their $m_2$   |
|         | A1    | For a correct equation in the required form  |
| (d)     | M1    | For a correct equation involving the gradient in terms of $m$ and $n$ using their co-ordinates from (b). |
|         | M1    | For a correct expression for length of the line in terms of $n$ and $m$                                  |
|         | M1    | For eliminating $n$ from either equation and forming a 3TQ   |
|         | M1    | For solving their 3TQ  |
|         | A1    | For $m = 5$  |
|         | A1    | For both $m = 5$ and $n = -20$   |
| ALT (d) | M1    | For a correct equation involving the gradient in terms of $m$ and $n$ using their co-ordinates from (b). |
|         | M1    | For a correct expression for length of the line in terms of $n$ and $m$                                  |
|         | M1    | For eliminating $m$ from either equation and forming a 3TQ   |
|         | M1    | For solving their 3TQ  |
|         | A1    | For $n = -20$  |
|         | A1    | For both $m = 5$ and $n = -20$   |
| (e)     | M1    | For selecting a correct method for finding the area of a quadrilateral.                                  |
|         | dM1   | For the correct calculations using their correct method and values only                                  |
|         | A1    | For the correct area of 750  |



| Question              | Scheme  | Marks                                       |
|-----------------------|---|---|
| <b>11(a)</b>          | $\frac{1 - \cos 2A}{1 + \cos 2A} = \frac{1 - (\cos^2 A - \sin^2 A)}{1 + (\cos^2 A - \sin^2 A)}$   | M1  |
|                       | $= \frac{1 - (1 - \sin^2 A - \sin^2 A)}{1 + (\cos^2 A - (1 - \cos^2 A))}$   | M1  |
|                       | $= \frac{2 \sin^2 A}{2 \cos^2 A} = \tan^2 A *$  | A1cso<br>[3]                                |
| <b>(b)</b>            | $\frac{3 - 3 \cos 4x}{1 + \cos 4x} + \frac{5 \sin 2x}{\cos 2x} = 2 \Rightarrow 3 \tan^2 2x + 5 \tan 2x - 2 = 0$ $\Rightarrow (3 \tan 2x - 1)(\tan 2x + 2) = 0 \Rightarrow \tan 2x = \frac{1}{3}, -2$ $\Rightarrow 2x = -161.565, 18.43, -63.43, 116.56$ $\Rightarrow x = -80.8, -31.7, 9.2, 58.3$ Accept awrt 1dp | M1, M1<br><br>M1A1<br><br>M1<br>A1A1<br>[7] |
| <b>Total 10 marks</b> |   |   |

| Part   | Mark  | Notes   |
|--------|-------|---|
| 11 (a) | M1    | Applies the formula for $\cos 2A$ as $(\cos^2 A - \sin^2 A)$ at least once  |
|        | M1    | Applies the Pythagorean formula at least once.  |
|        | A1cso | Applies the $\frac{\sin}{\cos} = \tan$ identity with no errors seen.  |
| (b)    | M1    | Uses the result from (a) to obtain $3 \tan^2 2x$  |
|        | M1    | Uses the $\frac{\sin}{\cos} = \tan$ to obtain $5 \tan 2x$ <b>AND</b> forms a 3TQ in terms of $\tan 2x$<br>{the terms can be in any order} |
|        | M1    | Uses any method to solve their 3TQ to obtain a value for $\tan 2x$  |
|        | A1    | Obtains either correct value for $\tan 2x$  |
|        | M1    | Obtains at least one correct value for $2x$ from their $\tan 2x$  |
|        | A1    | At least two correct values   |
|        | A1    | All correct values with no extra values within range.<br>Ignore extra values outside of the range.  |