Mark Scheme (Results)

November 2025

Pearson Edexcel International GCSE in Further Pure Mathematics

4PM1/02

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Types of mark

- M marks: method marks
- A marks: accuracy marks
- B marks: unconditional accuracy marks (independent of M marks)

Abbreviations

- cao – correct answer only
- ft – follow through
- isw – ignore subsequent working
- SC - special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- awrt – answer which rounds to
- eeoo – each error or omission

- cas – correct answer scores full marks (unless from obvious incorrect working)
- wr – working required

No working

If no working is shown then correct answers normally score full marks
If no working is shown then incorrect (even though nearly correct) answers score no marks.

With working

If there is a wrong answer indicated on the answer line always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
If a candidate misreads a number from the question. e.g., uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used.
If there is no answer on the answer line then check the working for an obvious answer.

Ignoring subsequent work

It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect e.g. algebra.
Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

Parts of questions

Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation

1. Factorisation:

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to $x=....$

$(ax^2+bx+c)=(mx+p)(nx+q)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=....$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and c, leading to $x=....$

3. Completing the square:

$x^2+bx+c=0$: $(x\pm\frac{b}{2})^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=....$

Method marks for differentiation and integration

1. Differentiation

Power of at least one term decreased by 1. $(x^n\rightarrow x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n\rightarrow x^{n+1})$

Use of a formula

Generally, the method mark is gained by **either**
quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working

The rubric states "Without sufficient working, correct answers may be awarded no marks".
General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this e.g. in a case of "prove or show...."

Exact answers

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

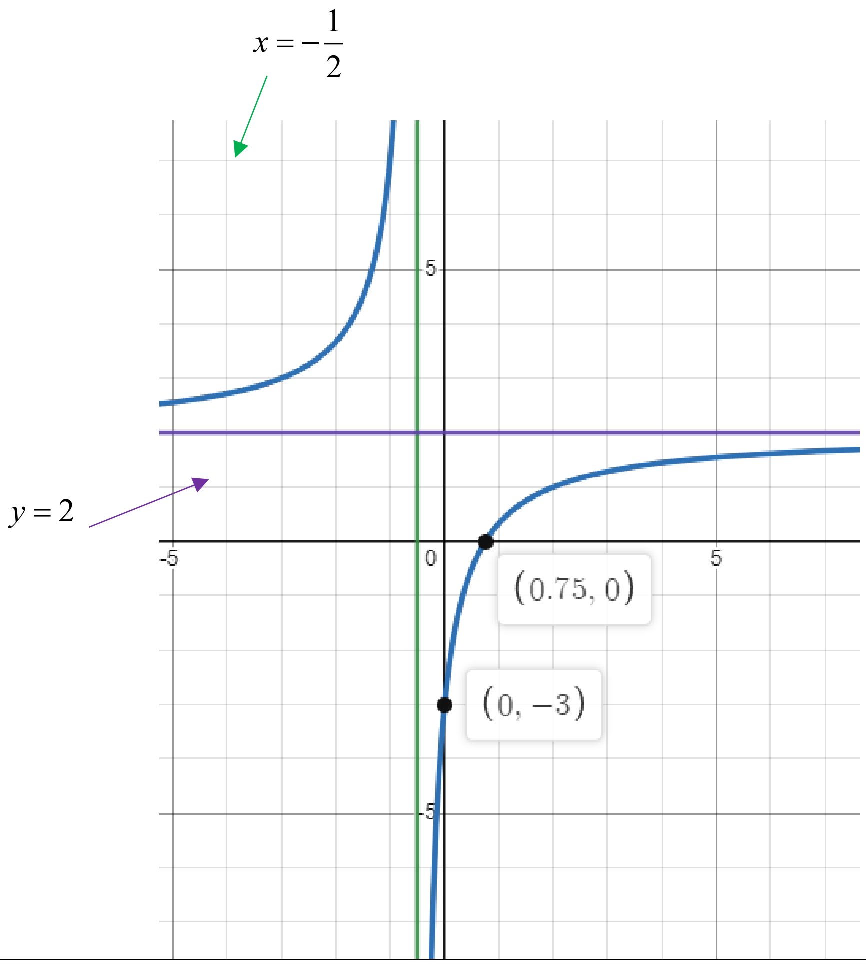
Penalise only once per question for failing to round as instructed – i.e. giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the is rule may allow the mark to be awarded before the final answer is given.

Question	Scheme	Marks
1	Let $\angle AOB = \theta$ $[49 = 14 + 14 + 14\theta \Rightarrow]$ $[\theta =]1.5$ oe $[A =]\frac{1}{2} \times 14^2 \times "1.5"$ oe $= 147\left[\text{cm}^2\right]$ In degrees: $\left[2\pi \times 14 \times \frac{\theta}{360} = (49 - 28)\right]$ $[\theta =]\frac{270}{\pi}$ $(85.9436...^{\circ})$ oe $[A =]\pi \times 14^2 \times \frac{"\frac{270}{\pi}"}{360}$ oe $= 147\left[\text{cm}^2\right]$	B1 M1A1 [3] B1 M1A1 [3]
	$[l = r\theta = 49 - 14 - 14 =]21$ $\left[A = \frac{1}{2}r(r\theta) =\right]\frac{1}{2} \times 14 \times 21$ oe $= 147\left[\text{cm}^2\right]$	B1 M1A1 [3]
		Total 3 marks

Mark	Notes
B1	For correct angle *AOB* in any form. If working in degrees, allow awrt 86
M1	Uses the correct formula/method to find the area of a sector, in radians or degrees. Allow the use of any angle, so long as it is clear they are using their angle. A final correct answer seen of 147 **(not any number seen rounded to 147)** will usually imply this mark (as can often be the case). Allow π to be 3.14 (or better) or $\frac{22}{7}$ for this mark.
A1	For the correct area only of 147. Allow working in degrees for B1 and M1. If working in degrees: • Any final answer **seen** rounded to 147 is A0 • If examiners see the angle in degrees written as 85…. or 86 and the final answer as 147, with no approximation from another number, we will assume the calculator has been used throughout and A1 may be awarded.
ALT B1	For the correct length of the arc Note, for this mark to be awarded, we need to see further work, indicating they are using this method. ie evidence of using the formula as $\frac{1}{2}r\left(r\theta\right)$ It may not be awarded for just 21.
M1	For correctly using the length of the arc in the correct formula for area of a sector.
A1	For the correct area.

Question	Scheme	Marks
2(a)	$3-2x=3x^2+9x-17 \Rightarrow 3x^2+11x-20[=0]$ oe eg $x^2+\frac{11}{3}x-\frac{20}{3}[=0]$ eg $(3x-4)(x+5)[=0] \Rightarrow x=\frac{4}{3}$ oe, -5	M1 M1A1 [3]
(b)	$\left[3-2x \leqslant 3x^2+9x-17 \Rightarrow 3x^2+11x-20 \leqslant 0\right]$ $x \leqslant "-5", \quad x \geqslant "\frac{4}{3}"$ $x \leqslant -5,$ $x \geqslant \frac{4}{3}$ oe	M1 A1 [2]
Total 5 marks		

Part	Mark	Notes
(a)	M1	Sets the line = curve and forms the correct 3TQ The correct 3TQ alone would imply this mark. We will condone a missing = 0
	M1	Solves their 3TQ by any minimally acceptable method (see general guidance) Correct values alone for x will **not** imply this and the previous method mark in this question, as the demand of the question, tells the candidates to 'use algebra' We will condone a missing = 0 Not a fully dependent method mark, but is dependent on correctly setting line = curve.
	A1	For both $x=\frac{4}{3}$ **and** -5 Accept for $\frac{4}{3}$, $1.\dot{3}$ or $1.33....$ or 1.3^r or 1.33 (or better) or any equivalent fraction. We need to see as a minimum, **both** the correct 3TQ written down **and** a correct method to solve this equation. As a minimum: Factorising - we must see $(3x-4)(x+5)$ or $(3x-4)$ **and** $(x+5)$ separately or **any** correct factorisation. We will condone, for example, $3x^2+11x-20$ followed by $\left(x-\frac{4}{3}\right)(x+5)$ Using the formula, we need a correct substitution into the formula. This doesn't need to be simplified, but allow one such as $\frac{-11\pm\sqrt{121+240}}{6}$ or $\frac{-11\pm\sqrt{361}}{6}$ as minimum. Completing the square, need to see $3\left(x+\frac{11}{6}\right)^2-\frac{361}{12}$ or $3\left(x+\frac{11}{6}\right)^2=\frac{361}{12}$ or $\left(x+\frac{11}{6}\right)^2=\frac{361}{36}$
(b)	M1	Chooses the outside region, for their values of x which if incorrect, must come clearly from part (a). It is possible to attain M1 A1 if the (ful)l working isn't shown in (a). For a re-start, possibly using a calculator, finding the correct values and choosing the region correctly. possible to be awarded M1 A1 for this approach also. For this mark allow $<$ for $\leqslant$ and $>$ for $\geqslant$ Allow $x\leqslant -5,\quad x\geqslant 1.3$ or $x<-5,\quad x>1.3$ Allow eg $\frac{4}{3}\leqslant x\leqslant -5$
	A1	For the correct region specified correctly. We do not need to see the word 'and'. Accept for $\frac{4}{3}$, $1.\dot{3}$ or $1.33....$ or 1.3^r or 1.33 (or better) Accept equivalences eg $(-\infty,-5]$ or $[-\infty,-5]$ and $\left[\frac{4}{3},\infty\right]$ or $\left[\frac{4}{3},\infty\right)$ For this mark do not allow $<$ for $\leqslant$ and $>$ for $\geqslant$ **Do not allow** $x\leqslant -5,\quad x\geqslant 1.3$ or $\frac{4}{3}\leqslant x\leqslant -5$

Question	Scheme	Marks
3(a)	$\left[y=2-\frac{5}{2x+1}=\right]\frac{4x-3}{2x+1}$ (i) $x=-\frac{1}{2}$ (ii) $y=2$	B1 B1 B1 [3]
(b)	(i) $(0,-3)$ (ii) $\left(\frac{3}{4},0\right)$ oe	B1ft B1ft [2]
(c)		B1 B1ft B1ft [3]
		Total 8 marks

Part	Mark	Notes
(a)	B1	For $\frac{4x-3}{2x+1}$ or $\frac{2(2x+1)}{2x+1}-\frac{5}{2x+1}$ or $\frac{4x+2-5}{2x+1}$ **Both** correct asymptote equations seen will imply this mark.
(i)	B1	For $x=-\frac{1}{2}$ Independent of method accuracy mark, **so ignore any incorrect attempt to form a single fraction**
(ii)	B1	For $y=2$ Independent of method accuracy mark, **so ignore any incorrect attempt to form a single fraction**
(b)	B1ft	For the coordinates of the y-intercept, follow through their a or b and c from any expression of the form $\frac{4x+a}{2x+1}$ $a \neq 0$ or $\frac{4x+b+c}{2x+1}$ $b, c \neq 0$ Condone eg just $y = -3$ or $y = a$ without $x = 0$ shown.
	B1ft	For the coordinates of the x-intercept, follow through their a as above. ie ft $(x=)-\frac{a}{4}$ Condone $x=\frac{3}{4}$ or $x=-\frac{a}{4}$ without $y = 0$ shown.
Note on labelling: Answers presented in the correct order without labelling – we would award all marks. If labelling not present, we can't assume the candidate's thinking.		
(c)	B1	For a negative reciprocal curve drawn anywhere in the grid – there **must** be two branches present, they **must not** cross any asymptotes drawn or implied or cross each other and **must not obviously** 'bend back' on themselves. Mark intention.
	B1ft	Two clearly marked asymptotes, ft their asymptotes listed from part (a). Labelled as equations or clearly passing through correctly labelled numbers on the axes. Must be one section of negative reciprocal curve, tending towards these asymptotes. This branch must not obviously cross or bend back from any asymptotes drawn or implied. Mark intention. Ignore any other branch/curve/line drawn if there is one branch fulfilling these conditions.
	B1ft	Single curve (or even line) passing through the correct intercepts. Ft their coordinates from part (b) marked clearly on the graph as coordinates or crossing points on the axes. Ignore any other branch/curve/line present.
	The answers to (a) and (b) cannot be retrieved from (c)	

Question	Scheme	Marks
4(a)	$\left(\frac{\mathrm{d}v}{\mathrm{d}t}=\right)8t-6=0\Rightarrow t=\left(\frac{6}{8}\right)$ oe $v=4\times\left("\frac{6}{8}"\right)^2-6\times\left("\frac{6}{8}"\right)+5=\frac{11}{4}\left[\mathrm{m/s}\right]$ oe	M1 dM1A1 [3]
ALT	$4\left(t-\frac{3}{4}\right)^2+\frac{11}{4}$ oe $\frac{11}{4}\left[\mathrm{m/s}\right]$ oe	M1 M1 A1 [3]
(b)	$18=8T-6\Rightarrow T=3$	M1A1 [2]
(c)	$\left[XY=\int_0^3\left(4t^2-6t+5\right)\mathrm{d}t=\right]$ $\left\|\left[\frac{4t^3}{3}-\frac{6t^2}{2}+5t\right]_0^3\right\|$ $XY=\left(\frac{4\times3^3}{3}-\frac{6\times3^2}{2}+5\times3\right)-\left(0\right)=24\text{ (m)}$	M1A1 M1A1 [4]
		Total 9 marks

Part	Mark	Notes
(a)	M1	For the **correct** derivative, setting = 0 and rearranging to find a value for t (allow an error in rearrangement)
	dM1	Substituting their value of t into the given expression for v. Dep on 1st M mark.
	A1	For the correct value of v Correct value of v will imply method marks.
ALT	M1	For completing the square with 2 of "4", " $-\frac{3}{4}$ " and " $\frac{11}{4}$ " correct.
	dM1	For correctly selecting their " $\frac{11}{4}$ "
	A1	For the correct value of v Correct value of v will imply method marks.
(b)	M1	For setting their expression for acceleration = 18 and **correctly** solving to find T
	A1	For the correct value of T Correct value of T will imply method mark.
(c)	M1	For a minimally acceptable attempt (see general guidance, but no power of t to decrease) to integrate the given expression. Limits do not need to be present. Condone + C if present.
	A1	For the correct integrated expression. Condone + C if present. Limits don't need to be present,
	M1	For substituting in the correct limits at least once into their integrated expression, to find a value for XY. Condone + C if present in the integration, but not as a (part of) a final value. This is not a dependent method mark. Substitution can be into any changed expression other than $8t - 6$ with a minimum of 2 terms. Substitution of 0 isn't necessary if the result of substituting is 0.
	A1	For the correct value of XY

In general, we will usually allow method marks to be implied by correct answers.
However, in this part of the question, we must see as a minimum

- a correct integration **or**
- the correct limits substituted into a correct integration (ie minimum working is as for 3rd method mark)

In either case, with this minimum shown and a correct answer, examiners may award M1 A1 M1 A1

Instructions to candidates state

- without sufficient working, correct answers **<u>may</u>** be awarded no marks.

Candidates stating simply $\int_0^3 \left(4t^2 - 6t + 5\right)\left[\mathrm{d}t\right] = 24$ with no work and a correct answer, may be awarded M1 A0 M0 A0 only. Allow dt to be missing.
Candidates who write down only 24 with no other work, will be awarded M0 A0 M0 A0

Question	Scheme	Marks
5(a)	$\left[S = 2\pi rh + 2\pi r^2 \Rightarrow 322\pi = 2\pi rh + 2\pi r^2\right]$ $\Rightarrow (h =)\dfrac{161 - r^2}{r}$ oe eg $\dfrac{161}{r} - r$ eg $\dfrac{322\pi - 2\pi r^2}{2\pi r}$ $(V = \pi r^2 h \Rightarrow)(V =)\pi r^2\left(\text{"}\dfrac{161 - r^2}{r}\text{"}\right) \Rightarrow$ $V = \pi r\left(161 - r^2\right)$ *	B1 M1 A1cso [3]
ALT	$\left(S = 2\pi rh + 2\pi r^2 \Rightarrow 322\pi = 2\pi rh + 2\pi r^2 \Rightarrow 161 = rh + r^2\right)$ $\Rightarrow (rh =)161 - r^2$ $(V = \pi r^2 h \Rightarrow)(V =)\pi r\left(161 - r^2\right)$ *	B1 M1 A1
(b)	$\left[\dfrac{\mathrm{d}V}{\mathrm{d}r} =\right]161\pi - 3\pi r^2$ oe $161\pi - 3\pi r^2 = 0 \Rightarrow r^2 = \dfrac{161}{3} \Rightarrow r = 7.3257... \Rightarrow r \approx 7.33$ $\left[\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2} =\right] - 6\pi r = \left[\left[-138.17, -138.09\right]\right]$ or -138 or -140 or awrt -44π < 0 hence maximum	M1 M1A1 M1A1 [5]
(c)	$V = \pi \times 7.3257\left(161 - 7.3257^2\right) = 2470$	M1A1 [2]
		Total 10 marks

Part	Mark	Notes
(a)	B1	For finding a correct expression for *h* in terms of *r* This expression doesn't need to be simplified. Independent of method mark, so mark the expression for *h* in terms of *r* for this mark. For the final A mark, must be no errors.
	M1	For substituting their expression for *h* in terms of *r* into a correct expression for *V* *V* = doesn't need to be present for this mark.
	A1* cso	For the correct formula exactly as written. *V* = must be shown at least once, throughout working. Minimum steps as shown in the mark scheme, no errors or omissions. Steps shown in square brackets are not required, but are provided as if present, must be correct and must be checked by Examiners. All additional steps must also be checked.
ALT	B1	For finding a correct expression for *rh* Independent of method mark, so mark the expression for r*h* in terms of *r* for this mark. For the final A mark, must be no errors.
	M1	For substituting their expression for *rh* in terms of *r* into correct expression for *V* *V* = doesn't need to be present for this mark.
	A1* cso	For the correct formula exactly as written. *V* = must be shown at least once, throughout working. Minimum steps as shown in the mark scheme, no errors or omissions. Steps shown in square brackets are not required, but are provided as if present, must be correct and must be checked by Examiners. All additional steps must also be checked.
(b)	M1	For a minimally acceptable attempt (see general guidance, but also no power or *r* to increase) to differentiate the given expression. Simplification is not necessary.
	M1	Sets their derivative = 0 and **correctly** rearranges to finds a value for *r* or r^2 Allow 7.3 without a full rearrangement to imply this mark. This isn't a completely dependent mark, but must follow from anything considered to be an attempt to differentiate and then rearrange an equation with a 2 term expression in *r* Ignore any value of negative *r*
	A1	For the correct value of *r*. Accept awrt 7.33 Ignore any value of negative *r* Because candidates are told to use calculus, a correct answer will not imply method marks.
	M1	For a minimally acceptable attempt (see general guidance, but also no power or *r* to increase) to differentiate their expression for the first derivative which must have 2 terms, with a least one term in *r*
	A1	For a correct conclusion. The value of the second derivative doesn't have to be calculated, but if it is, the numbers in the interval, or 138 or 140 must be seen. Must state < 0 and a brief conclusion which could be as short as shown or # Don't award if there is work to consider a negative value of *r*, unless discounted or abandoned. **All work must be correct throughout (b) for this mark.** Must be a clear conclusion referring to only the positive value of *r* for this mark.
(c)	M1	For substituting their value of *r* into the given expression for *V* May be implied by a correct value.
	A1	awrt 2470 but not from 7.3 or incorrect *r*

Question	Scheme	Marks
6(i)(a)	$\left[\frac{ar}{a}=r=\right]\frac{\sqrt{3}}{3+\sqrt{3}}\times\frac{3-\sqrt{3}}{3-\sqrt{3}}$ oe or $\frac{\sqrt{3}\left(3-\sqrt{3}\right)}{3^2-\left(\sqrt{3}\right)^2}$	M1
	$=\frac{3\sqrt{3}-3}{6}$ oe $\Rightarrow\frac{\sqrt{3}-1}{2}$ or $\frac{-1+\sqrt{3}}{2}$	A1 [2]
(b) (i)	$\frac{\sqrt{3}-1}{2}\left(=0.366\right)<1$ or $\lvert r\rvert<1$ so the series is convergent	B1ft [1]
(ii)	$S=\frac{8\left(1-0.6^n\right)}{1-0.6}$, $T=\frac{8}{1-0.6}=\left[20\right]$	M1, M1
	$\left["20"-"20"\left(1-0.6^n\right)<0.12\right]\Rightarrow 0.6^n<0.006$ oe eg $-0.6^n>-\frac{3}{500}$	M1
	$n\log_{10}0.6<\log_{10}0.006$ oe eg $n\log_{0.006}0.6<\log_{0.006}0.006$	
	$n>\frac{\log_{10}0.006}{\log_{10}0.6}$	dM1
	$\left(n>10.015\right)$ $n=11$	A1 [5]
		Total 8 marks

<table>
<tr><th>Part</th><th>Mark</th><th>Notes</th></tr>
<tr><td rowspan="2">(a)</td><td>M1</td><td>For dividing the second term by the first term and for a correct method stated to rationalise the denominator.
A correct answer may not imply this mark as candidates are told to show working.</td></tr>
<tr><td>A1</td><td>For $\frac{3\sqrt{3}-3}{6}$ oe and $\frac{\sqrt{3}-1}{2}$ or $\frac{-1+\sqrt{3}}{2}$
Accept $p = -1$ and $q = 2$
Minimum steps as shown in the mark scheme needed, with no errors or omissions.
Additional steps shown must be checked and correct.</td></tr>
<tr><td colspan="3">If students start completing 1st term divide 2nd
ie $\frac{3+\sqrt{3}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{3\sqrt{3}+3}{3}$
they must progress to
$\frac{3}{3\sqrt{3}+3}$ oe eg $\frac{1}{\sqrt{3}+1}$ before marks can be awarded.</td></tr>
<tr><td>(b) (i)</td><td>B1ft</td><td>For a correct explanation referring to r or the value being < 1 or $|r| < 1$
This may ft any value of r from their part (a) where $-1 < r < 1$
Accept "r is between 1 and – 1"</td></tr>
<tr><td rowspan="5">(ii)</td><td>M1</td><td>For the correct expression for S or T using the given values.
May be implied/embedded within the solution.</td></tr>
<tr><td>M1</td><td>For the correct expression for S and T using the given values.
May be implied/embedded within the solution.</td></tr>
<tr><td>M1</td><td>For an inequality of the form $0.6^n < A \quad A > 0$ or $-0.6^n > -B \quad B > 0$
$0.6^n < 0.006$ oe eg $-0.6^n > -\frac{3}{500}$ will definitely imply first 2 M marks if one or both are not explicitly stated.</td></tr>
<tr><td>dM1</td><td>For using logarithms correctly, including reversing the inequality to find a value for n
Dependent on previous method mark</td></tr>
<tr><td>A1</td><td>For the correct value of n following correct work.
Candidates must use logarithms and we must see all previous method marks.
If ($n = 11$) comes from incorrect logs work or incorrect reversal of this inequality, this will be A0.</td></tr>
</table>

Question	Scheme	Marks
7(a)	$\left[\mathrm{f}\left(-\frac{1}{2}\right)=\right]2\times\left(-\frac{1}{2}\right)^3+A\times\left(-\frac{1}{2}\right)^2+B\times\left(-\frac{1}{2}\right)-20=0\left[\Rightarrow A-2B=81\right]$ $\left[\mathrm{f}'(x)=\right]6x^2+2Ax+B$ $\left[\mathrm{f}'(1)=\right]6\times1^2+2A\times1+B=-27\left[\Rightarrow 2A+B=-33\right]$ $A-2B=81$ oe $2A+B=-33$ oe $\Rightarrow A=3^*,\quad B=-39$	M1 M1 M1 dddM1 A1 cso [5]
(b)	$\begin{array}{r} x^2+x-20 \\ 2x+1\overline{\smash{)}2x^3+3x^2-39x+20} \end{array}$ $x^2+x-20=(x-4)(x+5)$ $\Rightarrow[x=]-\frac{1}{2},4,-5$	M1 dM1 A1 [3]
(c)	$[A=]\int_{"-5"}^{"-\frac{1}{2}"}\left(2x^3+3x^2-39x-20\right)\mathrm{d}x+\left\|\int_{"-\frac{1}{2}"}^{"4"}\left(2x^3+3x^2-39x-20\right)\mathrm{d}x\right\|$ oe eg $\int_{"-5"}^{"-\frac{1}{2}"}\left(2x^3+3x^2-39x-20\right)\mathrm{d}x+\int_{"4"}^{"-\frac{1}{2}"}\left(2x^3+3x^2-39x-20\right)\mathrm{d}x$ $[A=]\left[\frac{2x^4}{4}+\frac{3x^3}{3}-\frac{39x^2}{2}-20x\right]_{[-5]}^{\left[-\frac{1}{2}\right]}+\left[\frac{2x^4}{4}+\frac{3x^3}{3}-\frac{39x^2}{2}-20x\right]_{\left[-\frac{1}{2}\right]}^{[4]}$ $[A=]\left\{\left[\frac{2\left(-\frac{1}{2}\right)^4}{4}+\frac{3\left(-\frac{1}{2}\right)^3}{3}-\frac{39\left(-\frac{1}{2}\right)^2}{2}-20\left(-\frac{1}{2}\right)\right]-\left[\frac{2(-5)^4}{4}+\frac{3(-5)^3}{3}-\frac{39(-5)^2}{2}-20(-5)\right]\right\}$ $[+]\left\|\left\{\left[\frac{2(4)^4}{4}+\frac{3(4)^3}{3}-\frac{39(4)^2}{2}-20(4)\right]-\left[\frac{2\left(-\frac{1}{2}\right)^4}{4}+\frac{3\left(-\frac{1}{2}\right)^3}{3}-\frac{39\left(-\frac{1}{2}\right)^2}{2}-20\left(-\frac{1}{2}\right)\right]\right\}\right\|$ oe $[A=]\frac{6561}{16}$ or 410.0625 oe	M1 M1 M1 A1 [4]
		Total 12 marks

Part	Mark	Notes
(a)	M1	Substitutes $-\frac{1}{2}$ into f(x) and sets = 0 Accept as a minimum $2\times\left(-\frac{1}{8}\right)+A\times\frac{1}{4}+B\times\left(-\frac{1}{2}\right)-20=0$ or $-\frac{1}{4}+A\times\frac{1}{4}+B\times\left(-\frac{1}{2}\right)-20=0$
	M1	For correctly differentiating f(x)
	M1	Substitutes 1 into their derivative, and sets = – 27 Accept $2A+B=-33$ with no errors. This isn't a dependent mark, but candidates must be substituting into a changed expression with at least 2 terms in x
	dddM1	Solves their linear simultaneous equations by any method, allow one error in solving. Minimal working here is fine, but there must be **some** working. Candidates who show no working to solve the simultaneous equations will not gain this mark (and therefore not the next mark) As this is a 'show' question, correct answers cannot imply this method mark. Dependent on previous 3 method marks.
	A1*cso	For the correct value of A [which is given in the question] and the correct value of B. Minimum steps as shown/stated, no errors or omissions.
(b)	M1	Award the mark if the candidate is seen to be attempting any method, such as algebraic division or equating coefficients (note this will often have minimal working) and attains: $x^2\pm Cx-20$ or $x^2+x\pm D \quad C,D\neq 0$ It is not sufficient to show simply a product of linear factors.
	dM1	For correctly factorising their quadratic factor or correctly using any method to solve their quadratic factor = 0 Dependent on previous method mark.
	A1	For all three roots. Candidates are told to use algebra, so correct answers alone cannot imply M marks for this question. This means correct answers alone will be 0 marks overall.

The majority of candidates are completing this question, using the factor of ($2x$ + 1) given.

Occasionally, we are seeing a re-start using a different factor – probably deduced from a calculator, but if sufficient algebra is shown, we will award the marks.

For (x + 5) need to see $2x^2-7x-4$ with $2x^2$ correct and either the – 7 or – 4 correct
For (x – 4) need to see $2x^2+11x+5$ with $2x^2$ correct and either the +11 or +5 correct

for the first M mark – the next 2 marks then follow.

(c)	M1	For a correct statement of the required area. Allow use of their limits from part (b), even if working wasn't shown. However, no limit may be 0 This mark may be implicit in further working and may even be awarded at the stage when a negative numerical area has been made positive.
	M1	For a minimally acceptable attempt at integration of an expression of the form $2x^3+3x^2+Px-20\ P\neq 0$. The modulus sign and the addition between the 2 calculations do not need to be seen for this mark. See general guidance, additionally, no power of x to decrease. Limits do no need to be present for this mark.
	M1	For a full and correct substitution of each of their limits (their roots from part (b), no limit 0) into any changed expression with a minimum of 2 terms. The modulus sign and the addition between the 2 calculations do not need to be seen for this mark. A correct final answer, if the integration is shown, may imply this mark.
	A1	For the correct exact area. The correct final answer may imply the first and third method marks, but may not imply the second method mark (ie must show integration). If no integration is shown, with a correct answer, the only possible marks are M1 M0 M1 A0 (if it's clear somewhere in working that the candidate has split the area). or M0 M0 M1 A0 (if the answer is correct with no reference to splitting the area).
If candidates work with $\int_{"-5"}^{"4"}\left(2x^3+3x^2-39x-20\right)\mathrm{d}x$ they **may** attain up to M0 M1 M1 A0		

Question	Scheme	Marks
8	$3\log_{27} x - \dfrac{\log_{27} y}{\log_{27} 3} = 2\left[\Rightarrow 3\log_{27} x - 3\log_{27} y = 2\right]$ or $3\dfrac{\log_3 x}{\log_3 27} - \log_3 y = 2\left[\Rightarrow \log_3 x - \log y = 2\right]$	M1M1
	$\log_{27}\left(\dfrac{x}{y}\right) = \dfrac{2}{3}$ or $\log_3\left(\dfrac{x}{y}\right) = 2$	M1
	$\Rightarrow \dfrac{x}{y} = 27^{\frac{2}{3}}$ or $\dfrac{x}{y} = 3^2\left[\Rightarrow \dfrac{x}{y} = 9\right]$ or $\left[\log_6(x+3y) = 3\right] \Rightarrow x + 3y = 6^3$	M1
	$\Rightarrow x = 9y$ oe	A1
	$\Rightarrow 9y + 3y = 216$ oe	M1
	$\Rightarrow y = 18$	A1
	$\Rightarrow x\left[= 9\times 18\right] = 162$	A1 [8]
ALT	We are seeing increased use of $\log_{a^b} c = \dfrac{1}{b}\log_a c$ During marking, this can be considered as 'a change of base of log' and in this question leads to the following solution. The notes can still be applied. If using this method, we must see a full change of base ie $\left[\log_{27} x = \log_{3^3} x =\right]\dfrac{1}{3}\log_3 x$ or $\left[3\log_{27} x = 3\log_{3^3} x =\right]\log_3 x$ $\left[\log_{27} x^3 = \log_{3^3} x^3 =\right]\dfrac{1}{3}\log_3 x^3$ or combining 1st 2 M marks with no errors $\log_{27} x^3 = \dfrac{3\log_3 x}{3}$ Leaving as $3\log_{3^3} x$ or $\log_{3^3} x^3$ is insufficient	
	$\left[3\log_{3^3} x - \log_3 y\right] = 2 \Rightarrow \dfrac{3\log_3 x}{3} - \log_3 y = 2\left[\Rightarrow \log_3 x - \log_3 y = 2\right]$	M1M1
	$\log_3\left(\dfrac{x}{y}\right) = 2$	M1
	$\Rightarrow \dfrac{x}{y} = 3^2$ or $\left[\log_6(x+3y) = 3\right] \Rightarrow x + 3y = 6^3$	A1
	Final M1 A1 A1 as main scheme.	
		Total 8 marks

Mark	Notes
M1	For any correct use of the power rule for logs – this mark may be awarded at any point in working for **any** correct change use of the power rule.
M1	For **any correct change of base of log** – this mark may be awarded at any point in working for **any** correct change of base of log.
M1	For applying the division law correctly – this mark may be awarded at any point in working for **any** correct application of the division law.
M1	For correctly converting a log equation into an exponential – – this mark may be awarded at any point in working for **any** correct conversion of a log equation into an exponential equation (particularly watch out for this also when the candidate is manipulating the second equation).
A1	For obtaining $x = 9y$ oe
M1	For solving a pair of **linear** simultaneous equations in x and y. Allow one error. There must be an attempt at a method. Candidates must have gained at least any 2 of the previous 4 method marks before they can be awarded this mark. Correct final answers would imply this method mark.
A1	For the correct value of x or y Candidates are told they must use an algebraic method, so we must see the work for all the previous marks for this to be awarded. Trial and error will score 0.
A1	For both correct values. Candidates are told they must use an algebraic method, so we must see the work for all the previous marks for this to be awarded. Trial and error will score 0.
If different approaches are used – apply the mark scheme and award marks accordingly.	

Question	Scheme	Marks
9(a)	$\left[\overrightarrow{BC}=\right]\overrightarrow{BA}+\overrightarrow{AC}$ or $-(2\mathbf{a}+4\mathbf{b})+(7\mathbf{a}+\mathbf{7b})$	B1 B1
	$=5\mathbf{a}+3\mathbf{b}$	
	$\overrightarrow{BC}\left[=\frac{1}{2}(10\mathbf{a}+6\mathbf{b})=\right]\frac{1}{2}\overrightarrow{AD}$ oe eg $\frac{1}{2}(10\mathbf{a}+6\mathbf{b})=5\mathbf{a}+3\mathbf{b}$ eg $10\mathbf{a}+6\mathbf{b}=2(5\mathbf{a}+3\mathbf{b})$	M1
	[therefore parallel] therefore trapezium or eg # or shown.	A1 [4]
(b)	$\left[\overrightarrow{BD}=-\overrightarrow{AB}+\overrightarrow{AD}=-(2\mathbf{a}+4\mathbf{b})+(10\mathbf{a}+6\mathbf{b})=\right]8\mathbf{a}+2\mathbf{b}$	B1
	$\left[\lvert BD\rvert=\right]\sqrt{8^2+2^2}\left[=\sqrt{68}=2\sqrt{17}\right]$	B1ft
	Unit vector is $\pm\frac{1}{\sqrt{68}}(8\mathbf{a}+2\mathbf{b})$ oe eg $\pm\frac{1}{\sqrt{17}}(\mathbf{4a}+\mathbf{b})$	M1A1 [4]

Part	Mark	Notes
(a)	B1	For the correct unsimplified vector in **a** and **b** for $\overrightarrow{BC}$ or for $\left[\overrightarrow{BC} =\right] \overrightarrow{BA} + \overrightarrow{AC}$
	B1	For the correct simplified vector in **a** and **b** for $\overrightarrow{BC}$
	M1	For $\overrightarrow{BC} = \frac{1}{2}\overrightarrow{AD}$ oe (note accept eg $2BC = AD$ ie without vector notation for this mark) Accept equivalences – see example MS.
	A1	For the correct conclusion, with all previous marks gained. Ignore work on other vectors. Do not accept eg $2BC = AD$ without vector notation for this mark Note B0 B1 M1 A0 is a potential score. $2BC = AD$ followed by eg $10\mathbf{a} + 6\mathbf{b} = 10\mathbf{a} + 6\mathbf{b}$ will get Aq
Watch for equivalences such as working out vector $\overrightarrow{CB}$ and stating $\overrightarrow{CB} = -\frac{1}{2}\overrightarrow{AD}$ Permit eg $\frac{\overrightarrow{AD}}{\overrightarrow{BC}} = 2$		
(b)	B1	For the correct simplified vector in **a** and **b** for $\overrightarrow{BD}$
	B1ft	For applying Pythagoras theorem fully and correctly to find the length BD Ft any vector believed to be their $\overrightarrow{BD}$. $8^2 + 2^2$ alone is insufficient.
	M1	For using their vector for $\overrightarrow{BD}$ with their length BD to attempt to form a unit vector
	A1	For the correct unit vector.

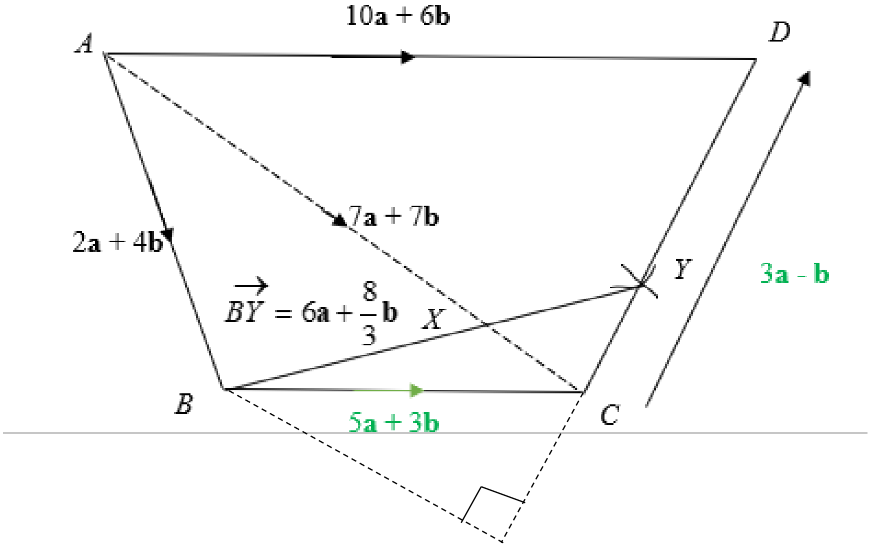
(c)(i)	$\Big[\left[\overrightarrow{CY}=\right]\frac{1}{3}(3\mathbf{a}-\mathbf{b})=\mathbf{a}-\frac{1}{3}\mathbf{b}$ oe eg $\overrightarrow{YC}$ or $\left[\overrightarrow{YD}=\right]\frac{2}{3}(3\mathbf{a}-\mathbf{b})=2\mathbf{a}-\frac{2}{3}\mathbf{b}$ oe eg $\overrightarrow{DY}$ Note: $\overrightarrow{BC}=5\mathbf{a}+4\mathbf{b}\Big]$	
	$\left[\overrightarrow{BY}=5\mathbf{a}+3\mathbf{b}+\mathbf{a}-\frac{\mathbf{b}}{3}=\right]6\mathbf{a}+\frac{8}{3}\mathbf{b}\Rightarrow$	M1
	$\left[\overrightarrow{BX}=\right]\lambda\left(6\mathbf{a}+\frac{8}{3}\mathbf{b}\right)$	dM1
	$\left[\overrightarrow{BX}=\right]-(2\mathbf{a}+4\mathbf{b})+\mu(7\mathbf{a}+7\mathbf{b})$ or $\left[\overrightarrow{BX}=\right](5\mathbf{a}+3\mathbf{b})+\alpha(-7\mathbf{a}-7\mathbf{b})$	
	$\left[\lambda\left(6\mathbf{a}+\frac{8}{3}\mathbf{b}\right)=-(2\mathbf{a}+4\mathbf{b})+\mu(7\mathbf{a}+7\mathbf{b})\text{ or }(5\mathbf{a}+3\mathbf{b})+\alpha(-7\mathbf{a}-7\mathbf{b})\right]$	
	$\Big[6\lambda=7\mu-2$ and $\frac{8}{3}\lambda=7\mu-4$ or $\frac{8}{3}\lambda=-7\alpha+3$ and $6\lambda=-7\alpha+5\Big]$	ddM1
	$\left[\lambda=\frac{3}{5};\mu=\frac{4}{5};\alpha=\frac{1}{5}\Rightarrow\overrightarrow{BX}=\right]\frac{18}{5}\mathbf{a}+\frac{8}{5}\mathbf{b}\Rightarrow[BX:XY=]3:2$	A1 [4]
ALT	First mark as main scheme.	M1
	$\left[\overrightarrow{XY}=\right]\beta\left(6\mathbf{a}+\frac{8}{3}\mathbf{b}\right)$	
	$\left[\overrightarrow{XY}=\right]\gamma(7\mathbf{a}+7\mathbf{b})+\mathbf{a}-\frac{1}{3}\mathbf{b}$ or	
	$\left[\overrightarrow{XY}=\right]\delta(-7\mathbf{a}-7\mathbf{b})+10\mathbf{a}+6\mathbf{b}-\left(2\mathbf{a}-\frac{2}{3}\mathbf{b}\right)$	dM1
	$\left[\beta\left(6\mathbf{a}+\frac{8}{3}\mathbf{b}\right)=\gamma(7\mathbf{a}+7\mathbf{b})+\mathbf{a}-\frac{1}{3}\mathbf{b}\text{ or }\delta(-7\mathbf{a}-7\mathbf{b})+10\mathbf{a}+6\mathbf{b}-\left(2\mathbf{a}-\frac{2}{3}\mathbf{b}\right)\right]$	
	$\Big[6\beta=7\gamma+1$ and $\frac{8}{3}\beta=7\gamma-\frac{1}{3}$ or $\frac{8}{3}\beta=-7\delta+6+\frac{2}{3}$ and $6\beta=-7\delta+10-2\Big]$	ddM1
	$\left[\beta=\frac{2}{5};\gamma=\frac{1}{5};\delta=\frac{4}{5}\Rightarrow\overrightarrow{XY}=\right]\frac{12}{5}\mathbf{a}+\frac{16}{15}\mathbf{b}[BX:XY=]3:2$	A1 [4]

Part	Mark	Notes
(c)(i) Main and ALT	M1	Correct simplified vector in **a** and **b** for ± $\overrightarrow{BY}$
	dM1	Correct vector in **a** and **b** with parameter using $\overrightarrow{CY}$ or $\overrightarrow{YC}$ for $\overrightarrow{BX}$ / $\overrightarrow{XB}$ or $\overrightarrow{XY}$ / $\overrightarrow{YX}$ **and** correct vector in **a** and **b** with a different parameter along a second distinct path using $\overrightarrow{AC}$ or $\overrightarrow{CA}$ for the same vector. Dependent on previous method mark.
	ddM1	For the correct simultaneous equations. Any one correct parameter or $\overrightarrow{BX} = \frac{18}{5}\mathbf{a} + \frac{8}{5}\mathbf{b}$ will imply this mark. Dependent on both previous method marks.
	A1	For the required ratio $BX : XY = 3:2$

Note, correct method can include the opposite signs in vectors multiplied by a parameter, as the parameter will just be the opposite sign when calculated and a fully correct solution is possible.

Eg $\left[\overrightarrow{BX} = \right](5\mathbf{a}+3\mathbf{b})+\alpha(7\mathbf{a}+7\mathbf{b})$ would just make $\alpha = -\frac{1}{5}$

Vectors questions notoriously have different methods.
Correct answer, not from obviously incorrect working, full marks may be awarded **if method is shown.**
(This is because the demand of this question says students must use a vector method).
Min method must involve finding two correct different distinct relevant paths.



(ii) **Eg 1**	$\left[\frac{CXY}{BXC}=\frac{2}{3}\ \text{and}\ \frac{BAC}{BXC}=\frac{5}{1}\Rightarrow\right]\ \frac{CXY}{BAC}=\frac{2}{15}$ oe eg $\frac{BAC}{CXY}=\frac{15}{2}$	M1
	$\frac{ABCD}{BAC}=\frac{3}{1}$ oe eg $\frac{BAC}{ABCD}=\frac{1}{3}$	M1
	$\frac{CXY}{ABCD}=\frac{\frac{2}{15}}{3}\left[=\frac{2}{45}\right]$ oe eg $\frac{ABCD}{CXY}=\frac{3}{\frac{2}{15}}\left[=\frac{45}{2}\right]$	M1
	$\left[\Rightarrow CXY:ABCD=\right]2:45$	A1 [4]
Eg 2	$\frac{CXY}{BCY}=\frac{2}{5}$ and $\left[\frac{BCY}{BCD}=\frac{1}{3}\right]\ \frac{BCY}{BCA}=\frac{1}{3}$ oe	M1
	$\frac{ABCD}{BAC}=\frac{3}{1}$ oe eg $\frac{BAC}{ABCD}=\frac{1}{3}$	M1
	$\frac{BCY}{ABCD}=\frac{\frac{1}{3}}{3}\left[=\frac{1}{9}\right]$ oe and $\frac{CXY}{ABCD}=\frac{\frac{2}{5}}{9}\left[=\frac{2}{45}\right]$ oe	M1
	$\left[\Rightarrow CXY:ABCD=\right]2:45$	A1 [4]
		Total 16 marks

Part	Mark	Notes
(ii)	M1	For any correct ratio, in any form, stated between a pair of triangles.
	M1	For any correct second ratio, in any form, stated between a different pair of triangles.
	M1	For any correct ratio, in any form between a triangle and the trapezium.
	A1	For the correct ratio only.
Any one of the ratios listed may be implied by a correct answer with no obviously incorrect working leading to this answer. Examples of other correct ratios: $\frac{BCD}{ABCD}=\frac{1}{3}$ $\frac{ABX}{CXY}=\frac{1}{6}$ **To be checked** Not carrying incorrect ratios through.		

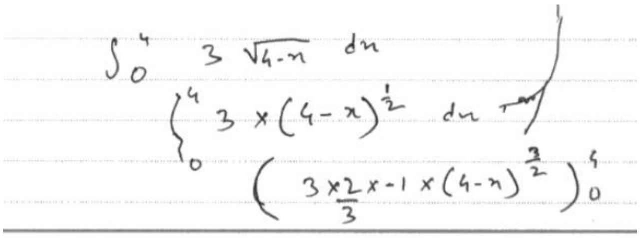
Question	Scheme	Marks
10(a)	At A:	
	$\left[y=3(4-0)^{\frac{1}{2}}=3\times 2=\right]6$	B1
	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]3\times\frac{1}{2}\times -1\times(4-x)^{-\frac{1}{2}}\left[=-\frac{3}{2}(4-x)^{-\frac{1}{2}}\right]$ oe	M1
	When $x=0$:	
	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{3}{2}(4-0)^{-\frac{1}{2}}=\right]-\frac{3}{4}$ oe	A1
	Equation of l:	
	eg $y-6=-\frac{3}{4}(x-0)$ or $y=-\frac{3}{4}x+6$ or $4y=-3x+24$ oe	M1A1 [5]
(b)	When $y=0$:	
	$\left[\text{Curve:}\quad 0=y=3(4-x)^{\frac{1}{2}}\Rightarrow x=\right]4$	B1ft
	$[\text{Line:}\quad 4\times 0+3x-24=0\Rightarrow x=]8$	
	$[V=]\pi\int_0^{"8"}\left("-\frac{3}{4}x+6"\right)^2\,\mathrm{d}x-\pi\int_0^{"4"}\left(3\sqrt{4-x}\right)^2\,\mathrm{d}x$ or	M1
	$[V=]\frac{1}{3}\times\pi\times 6^2\times"8"-\pi\int_0^{"4"}\left(3\sqrt{4-x}\right)^2\,\mathrm{d}x$	
	$\left[\quad=\pi\int_0^{"8"}\left(\frac{9}{16}x^2-9x+36\right)\mathrm{d}x-\pi\int_0^{"4"}(36-9x)\,\mathrm{d}x\right]$	
	$=[\pi]\left[\frac{3}{16}x^3-\frac{9}{2}x^2+36x\right]_{[0]}^{["8"]}-[\pi]\left[36x-\frac{9x^2}{2}\right]_{[0]}^{["4"]}$	M1M1 A1ft
	$\left[\quad=\pi\left[\frac{3}{16}\times"8"^3-\frac{9}{2}\times"8"^2+36\times"8"\right]-\pi\left[36\times"4"-\frac{9\times"4"^2}{2}\right]\right.$	
	$\left.\left[\quad=96\pi-72\pi=\right]24\pi\right]$	[5]
		Total 10 marks

Part	**Mark**	**Notes**
(a)	B1	For [$y =$] 6
	M1	For the correct derivative, doesn't need to be simplified.
	A1	For $-\frac{3}{4}$ oe If $-\frac{3}{4}$ oe from incorrect work – send to Review.
	M1	For correctly forming an equation of l using their values for y and $\frac{\mathrm{d}y}{\mathrm{d}x}$ Not a dependent mark, but there must be some attempt to have substituted $x = 0$ into a changed function to find a gradient. We will also accept if a candidate has done this and then found the negative reciprocal. If using $y = mx + c$, a correct rearrangement must be shown to find c and the equation then fully formed. If the rearrangement is not shown and the final equation is incorrect, c will need to be checked by examiners and must be correct for their values.
	A1	Award this mark for **any correct version** of the equation, simplified or not, rearranged or not, seen at any point. **It does not need to be given in the form stated by the question.**

In a complete break with convention, that must not be applied to any other question on this paper, examiners should:

- mark crossed out work, even if replaced
- mark multiple attempts (including crossed out work)
- award the highest mark from all attempts

For the usual conventions on marking crossed out work and multiple attempts, see General Guidance in the Mark Scheme.

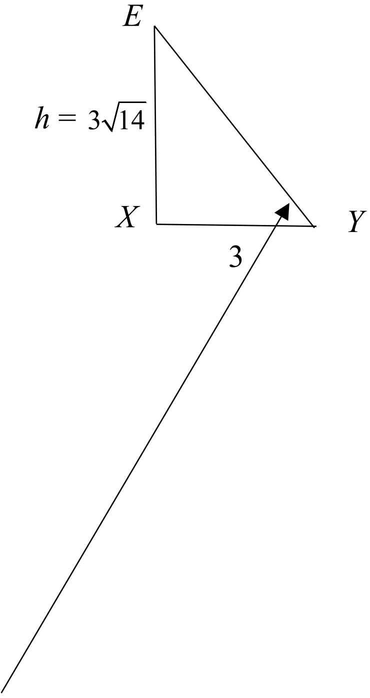
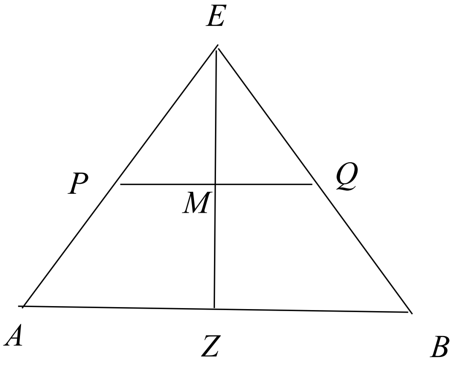
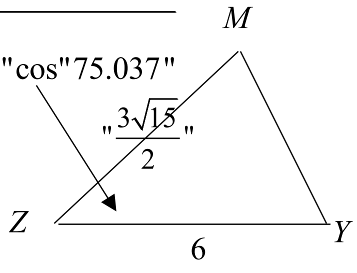
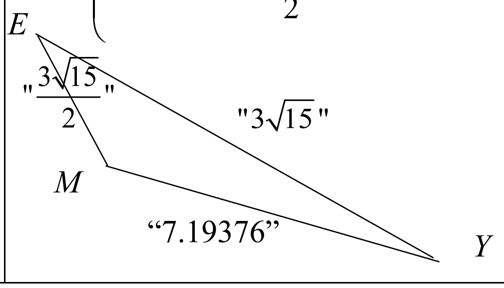
(b)	**For part (b) – allow candidates to follow through *y* = *mx* + *c* selected from (any multiple attempts) in part (a).**	
	B1ft	For [*x* =]4 **and** [*x* =]"8" ft the correct value of *x* for their $y = mx + c$ when $y = 0$ For example if $y=\frac{3}{4}x+6$ is used $x = -8$ must be listed.
	M1	For a correct statement for the volume of revolution. Condone $\left[V=\right]\pi\int_0^4\left(3\sqrt{4-x}\right)^2-\pi\int_0^{"8"}\left("-\frac{3}{4}x+6"\right)^2\,\mathrm{d}x$ or $\left[V=\right]\pi\int_0^4\left(3\sqrt{4-x}\right)^2-\frac{1}{3}\times\pi\times6^2\times"8"-\mathrm{d}x$ This mark will ft their $y = mx + c$
	M1	For a minimally acceptable integration of their expressions, no power of *x* to decrease. There must be a minimum of 2 terms to integrate. π and limits do not need to be present. Simplification of terms after integration not necessary. If students have chosen to write down and find the volume of a cone, this is for a minimally acceptable integration of their $\left(3\sqrt{4-x}\right)^2$ Sometimes students are forgetting to square expressions. If you see this correct integration, this is M1 here.  $\int_0^4 3\sqrt{4-x}\ \mathrm{d}x$ $\int_0^4 3\times(4-x)^{\frac{1}{2}}\ \mathrm{d}x$ $\left(3\times\frac{2}{3}x-1\times(4-x)^{\frac{3}{2}}\right)_0^4$
	M1	For a fully correct integration of $\left(3\sqrt{4-x}\right)^2$ π and limits do not need to be present. Simplification of terms not necessary.
	A1ft	For all of $\pi\int_0^{"8"}\left("\text{their } mx+c"\right)^2\,\mathrm{d}x-\pi\int_0^4\left(3\sqrt{4-x}\right)^2\,\mathrm{d}x$ or condone $\pi\int_0^4\left(3\sqrt{4-x}\right)^2-\pi\int_0^{"8"}\left(\text{their } mx+c\right)^2\,\mathrm{d}x$ seen **and** fully and correctly expanded/integrated – terms do not have to be simplified. If writing down and using the formula for the volume of a cone, for their ft value of *x* from their equation, this must be evaluated (simplified) correctly as $r\pi \quad r \neq 0$ - examiners to check and there must be a fully correct integration of $\pi\int_0^4\left(3\sqrt{4-x}\right)^2$. Examiners to check expansion/integration of $\left(\text{their } mx+c\right)^2$, π must be present at least once on each integration. We need to see the limits on all integration statements, ft their value of *x* for the intercept of the line but we do not need to see them substituted unless they are not present eg at the end of square brackets, as is convention. **We do not need to see and are not checking the final answer.**

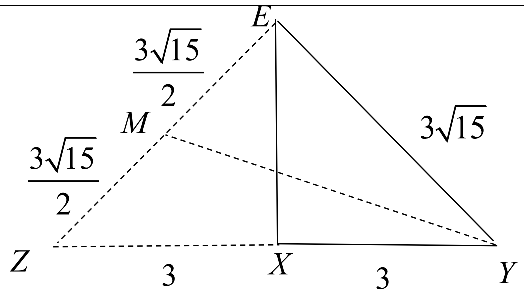
Note you may also see

$$\pi\int_0^4\left("-\frac{3}{4}x+6"\right)^2\,\mathrm{d}x-\pi\int_0^4\left(3\sqrt{4-x}\right)^2\,\mathrm{d}x\ +\pi\int_4^{"8"}\left("-\frac{3}{4}x+6"\right)^2$$

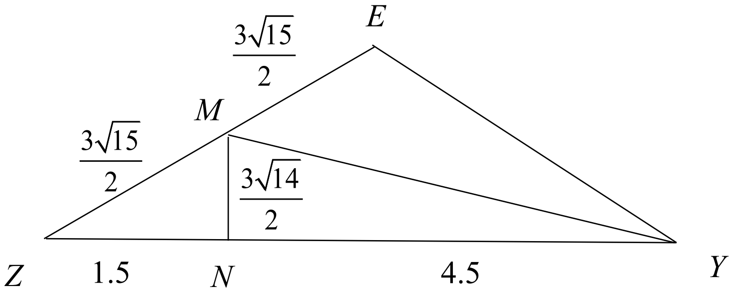
Question	Scheme	Marks
Look carefully at all diagrams, including the one provided. **Any contradictions with working, working takes precedence.**		
11(a)	Let the intersection of AC and DB be X $[AC=]\sqrt{6^2+6^2}\left[=\sqrt{72}\Rightarrow AX=3\sqrt{2}\right]$ oe or $[AX=]\sqrt{3^2+3^2}\left[=\sqrt{18}=3\sqrt{2}\right]$ $[h=]\sqrt{12^2-"\left(3\sqrt{2}\right)"^2}=3\sqrt{14}$ oe*	M1 M1A1 cso [3]
ALT1	Let Z be the midpoint of AB $[EZ=]\sqrt{12^2-3^2}\left[=\sqrt{135}=3\sqrt{15}\right]$ oe $[h=]\sqrt{\left("3\sqrt{15}"\right)^2-(3)^2}=3\sqrt{14}$ oe*	M1 M1A1 cso [3]
ALT2	$[h=]\sqrt{3^2+3^2+h^2}=12$ or $3^2+3^2+h^2=144$	M1M1 A1
(b)	Required angle $=[\angle EAX]\sin^{-1}\left(\frac{3\sqrt{14}}{12}\right)$ or $\tan^{-1}\left(\frac{3\sqrt{14}}{3\sqrt{2}}\right)$ or $\cos^{-1}\left(\frac{3\sqrt{2}}{12}\right)=69.295...^{\circ}$ or $\cos^{-1}\left(\frac{\left(6\sqrt{2}\right)^2+12^2-12^2}{2\times6\sqrt{2}\times12}\right)\approx 69.3^{\circ}$	M1A1 [2]

Part	Mark	Notes
(a)	M1	For correctly applying Pythagoras theorem to find *AC* or *AX* Full method, must include square rooting.
	M1	For correctly applying Pythagoras theorem to find h Allow follow through of their *AX/AC*. Full method, must include square rooting. Not a dependent method mark, but must see M1 or be correctly using $3\sqrt{2}$ M0 M1 A0 will occur. We will permit eg $3\sqrt{2}^2$ if the intention is clear
	A1*cso	For the correct length only. Minimum steps shown, no errors or omissions.
ALT1	M1	For correct applying Pythagoras theorem to find *EZ* Full method, must include square rooting.
	M1	For correctly applying Pythagoras theorem to find h Allow follow through of their *EZ*. Full method, must include square rooting. Not a dependent method mark, but must see M1 or be correctly using $3\sqrt{15}$ M0 M1 A0 will occur. We will permit eg $3\sqrt{15}^2$ if the intention is clear
	A1*cso	For the correct length only. Minimum steps shown, no errors or omissions.
ALT2	M1M1	For a fully correct statement using 3D Pythagoras, each side shown. M0 M0 for any part wrong
	A1*cso	For the correct length only. Minimum steps shown, no errors or omissions.
(b)	M1	For using the given value of h and applying correct trigonometry to find the required angle. Allow ft of $3\sqrt{2}$ if from correct method. If any other method used, the **method** must be complete and correct ie leaving as sin = is not complete. Note equivalent angles (because of symmetry) eg $\angle EBD$
	A1	For the correct angle awrt 69.3°

(c)	Let *M* be the midpoint of *PQ*, *Y* be the midpoint of *DC* and *Z* be the midpoint of *AB*	
	$\left[EY=EZ=\right]\sqrt{3^2+\left(3\sqrt{14}\right)^2}\left[=3\sqrt{15}\right]$	M1
	$\left[EM=\right]\dfrac{"3\sqrt{15}"}{2}$	M1
	  	
	Angle between planes *EAB* & *ABC* = $\left[\left[\angle EYX=\right]\tan^{-1}\left(\dfrac{3\sqrt{14}}{3}\right)\left[=75.0367^\circ\right] \text{ or } \cos^{-1}\left(\dfrac{"\left(3\sqrt{15}\right)"^2+6^2-"\left(3\sqrt{15}\right)"^2}{2\times"3\sqrt{15}"\times6}\right) \text{ or } \cos^{-1}\left(\dfrac{3}{"3\sqrt{15}"}\right) \text{ or } \sin^{-1}\left(\dfrac{3\sqrt{14}}{"3\sqrt{15}"}\right)\right]$	M1
	$\left[MY=\right]\sqrt{6^2+\left("\dfrac{3\sqrt{15}}{2}"\right)^2-2\times6\times"\dfrac{3\sqrt{15}}{2}"\,"\cos"75.037"}$ $=7.19376...$  	A1 (M1 on ePen)
	[Required angle = $\angle EMY$ =] $\cos^{-1}\left(\dfrac{\left("\dfrac{3\sqrt{15}}{2}"\right)^2+"7.19376"^2-\left("3\sqrt{15}"\right)^2}{2\times"\dfrac{3\sqrt{15}}{2}"\times"7.19376"}\right)=126.3133=126$	dddM1 A1
	 	[6]

3rd M mark and 1st A mark – ALTs to find *MY*		
	 $\left[\left[\angle ZEY = 2\times\angle XEY =\right]2\times\tan^{-1}\left(\frac{3}{3\sqrt{15}}\right)\left[=2\times 14.9632^\circ\right]\right]$ or $\left[\left[\angle ZEY =\right]\cos^{-1}\frac{"\left(3\sqrt{15}\right)"^2+"\left(3\sqrt{15}\right)"^2-"(6)"^2}{2\times"3\sqrt{15}"\times"\left(3\sqrt{15}\right)"}\left[=29.9264\right]\right]$ $MY=\sqrt{\left("3\sqrt{15}"\right)^2+\left("\frac{3\sqrt{15}}{2}"\right)^2-2\times"3\sqrt{15}"\times"\frac{3\sqrt{15}}{2}"\cos 29.9264}$ $=7.19376...$	M1 A1
		Total 11 marks

Part	Mark	Notes
.(c)	M1	For the correct and complete **method** to find *EY* or *EZ* If students use any other method than the simple Pythagoras shown, the **method** (not withstanding incorrect simplifications which can be carried forward) must be shown in full & be correct at all steps & must lead to *EY*, not eg $(EY)^2$ **If candidates have worked out *EY* or *EZ* in part (a), this must be written down or used in part (c) to gain this mark.**
	M1	For the correct and complete **method** to find *EM* If students use any other method other than dividing their *EY/EZ* by 2 (allow follow through of their *EY*), the **method** must be shown in full and be correct at all steps.
	M1	For the full and correct **method** to find *MY* Whichever method students use, it must be correct and complete. Allow follow through of their $3\sqrt{15}$ **if it has come from a correct method.**
	A1 (M1 on ePen)	For the length *MY* Accept awrt 7.2 Note "7.19376" = $\frac{3\sqrt{23}}{2}$
	dddM1	Uses a correct method to find the required angle. Dependent on previous 3 method marks.. If students reach cos ($\angle EMY$) = awrt – 0.6 or eg cos ($\angle EMY$) = $\frac{\left("\frac{3\sqrt{15}}{2}"\right)^2+"7.19376"^2-\left("3\sqrt{15}"\right)^2}{2\times"\frac{3\sqrt{15}}{2}"\times"7.19376"}$ (ie not quite a complete method), this mark can be awarded
	A1	For awrt 126°

Question	Scheme	Marks
11c ALT	Dropping a perpendicular from *M* to the base of the pyramid, which hits at point *N*	
		
	$\left(MN=\right)\frac{3\sqrt{14}}{2}$ $\left(\text{Note}: MZ=\sqrt{\left(\frac{3\sqrt{14}}{2}\right)+1.5^2}=\frac{3\sqrt{15}}{2}\right)$	M1
	$\left(\angle ZMN=\right)\tan^{-1}\left(\frac{1.5}{\frac{3\sqrt{14}}{2}}\right)$ or $\cos^{-1}\left(\frac{\frac{3\sqrt{14}}{2}}{\frac{3\sqrt{15}}{2}}\right)$ or $\sin^{-1}\left(\frac{1.5}{\frac{3\sqrt{15}}{2}}\right)\left(=14.96321743\right)$	M1
	$\left(\angle YMN=\right)\tan^{-1}\left(\frac{4.5}{\frac{3\sqrt{14}}{2}}\right)$ $\left(=38.7220711\right)$	M1
	$\left(\angle YMZ=14.96321743+38.7220711=\right)53.68528853$	A1
	$180-\text{"}53.68528853\text{"}=126.3$	dddM1A1 [6]

Part	Mark	Notes
ALT	M1	For correctly stating (explicitly or in working) the height of *MN*
	M1	For the full and correct **method** to find $\angle ZMN$ They may follow through *MZ* if found and from a correct method.
	M1	For the full and correct **method** to find $\angle YMN$ They may follow through *MZ* if found and from a correct method.
	A1	For the correct angle $\angle YMZ$ allow awrt 39
	dddM1	Dependent on all previous method marks, for subtracting their angle from 180.
	A1	For awrt 126°